

**Math 1901**  
**Freshman Seminar**  
**Mathematical Climate Models**

Fall 2024  
**1:00 - 2:15 Mondays and Wednesdays**  
**Vincent Hall 213**

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<https://www-users.cse.umn.edu/~mcgehee/Course/Math1901/>

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**Math 1901**

*Why do we think that the increasing atmospheric CO<sub>2</sub> is the cause of the increasing temperature?*

*Why do we think that human activity is causing the increasing atmospheric CO<sub>2</sub>?*

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**Math 1901**

*Why do we think that the increasing atmospheric CO<sub>2</sub> is the cause of the increasing temperature?*

**The Greenhouse Effect**

Greenhouse gases (CO<sub>2</sub>, H<sub>2</sub>O, CH<sub>4</sub>) are transparent to visible light, but opaque to infrared light. The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.

**Svante Arrhenius**, "On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground," *Philosophical Magazine and Journal of Science (Fifth Series)* 41, pp. 237-276, 1896.



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**Math 1901**

**Arguments from Climate Skeptics**

*Earth's climate has changed many times in the past.*  
*Why do we think humans are responsible now?*

*Humans haven't been around very long in geologic time.*  
*They couldn't have been the cause of climate changes in the distant past.*

*Humans have been burning fossil fuels for only about two hundreds years. But the ice ages occurred long before that, so humans couldn't have been the cause of those climate changes.*

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**Math 1901**

**Earth's climate has changed many times in the past.**

**Paleoclimatology**

**Paleoclimatology** is the scientific study of climates predating the invention of meteorological instruments, when no direct measurement data were available. As instrumental records only span a tiny part of Earth's history, the reconstruction of ancient climate is important to understand natural variation and the evolution of the current climate.

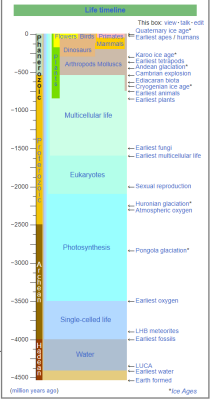
<https://en.wikipedia.org/wiki/Paleoclimatology>

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**Paleoclimate**

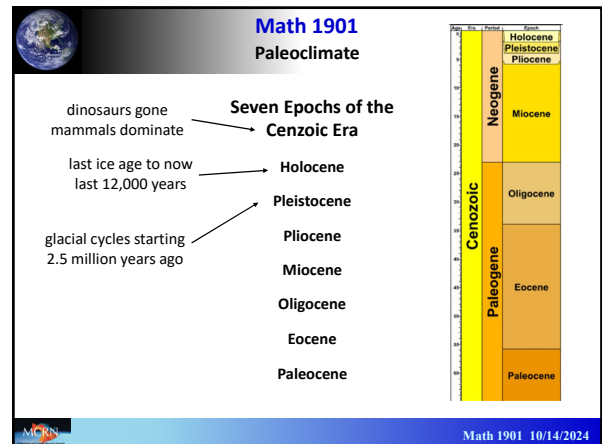
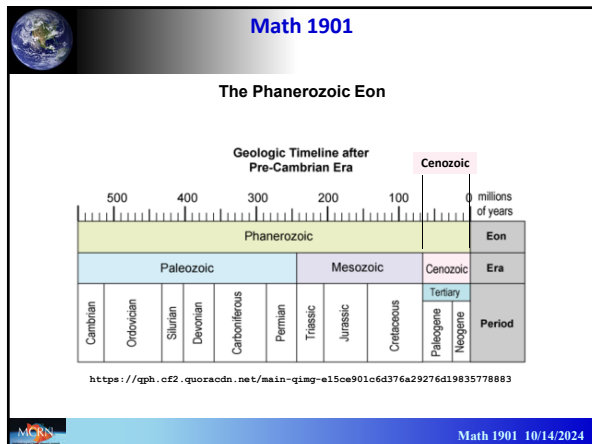
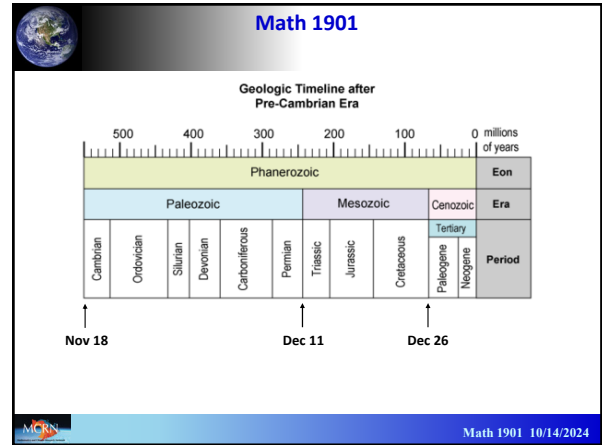
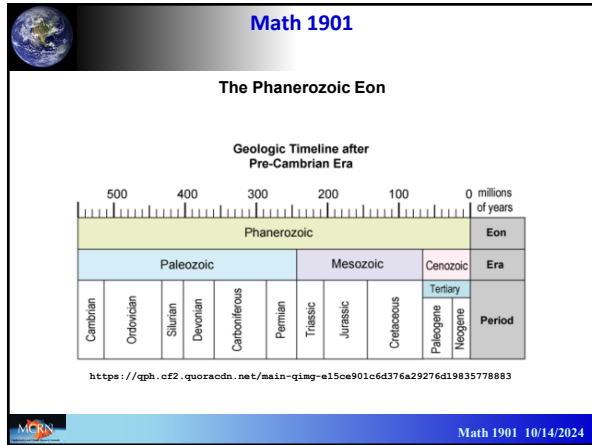
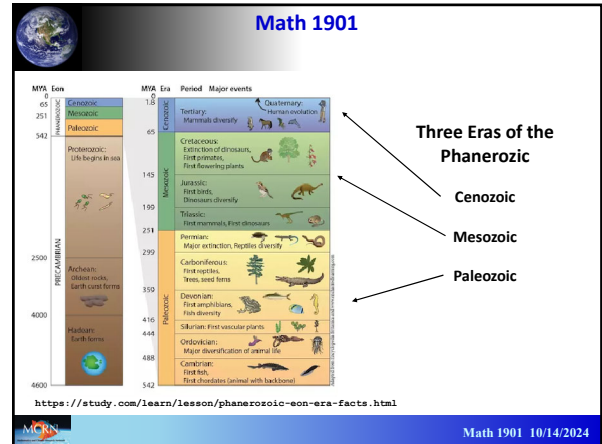
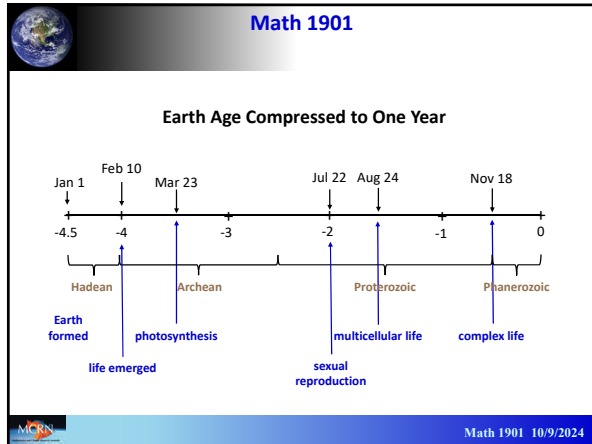
**Four Geologic Eons**

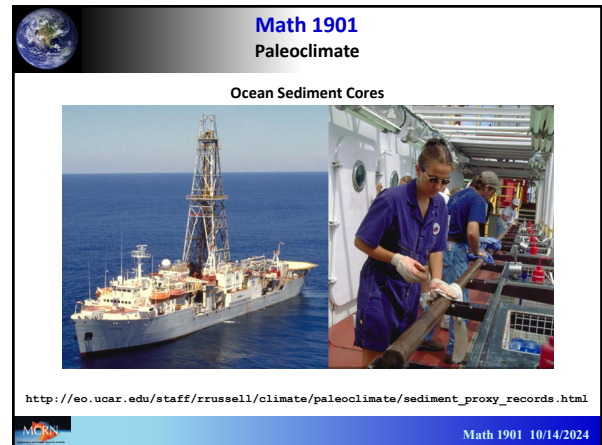
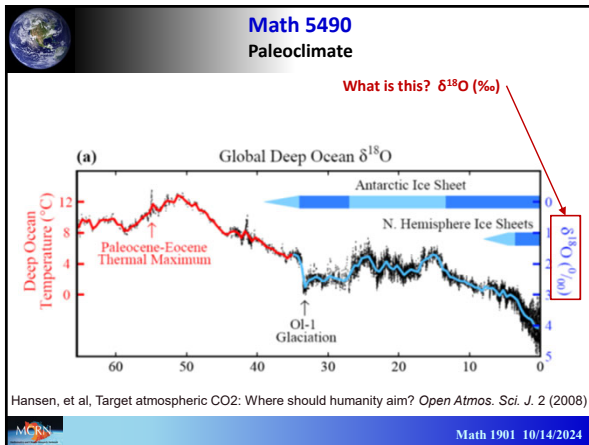
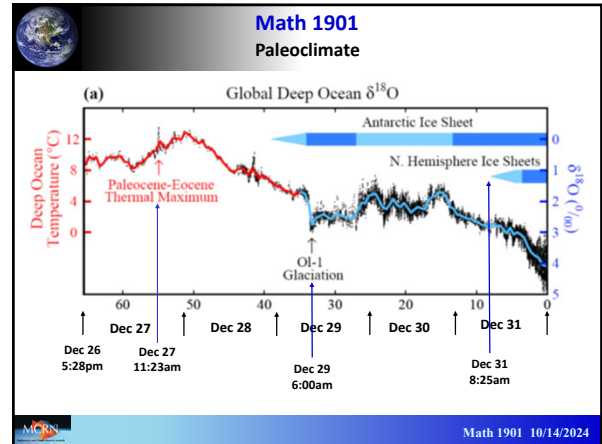
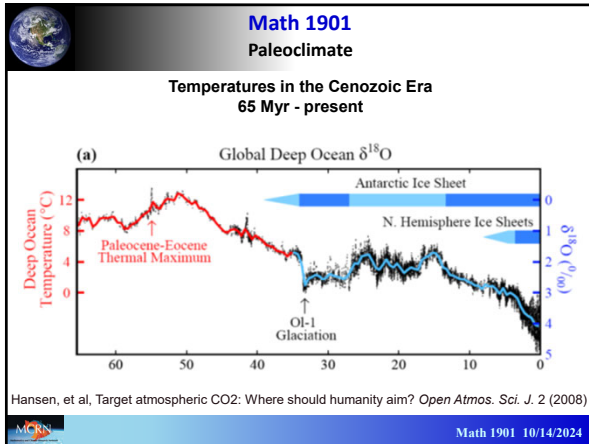
- Phanerozoic
- Proterozoic
- Archean
- Hadean



<https://en.wikipedia.org/wiki/Phanerozoic>

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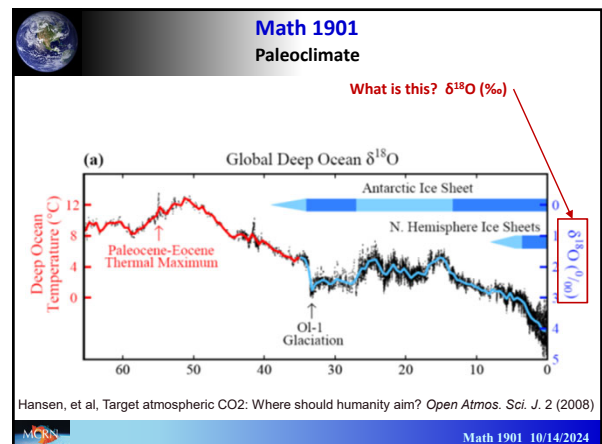
**$^{18}\text{O}$  as a Climate Proxy**

The isotope  $^{18}\text{O}$  preferentially evaporates from the ocean and is sequestered in glaciers, leaving the heavier isotope  $^{18}\text{O}$  more highly concentrated in the ocean. Thus oceanic concentration of the isotope  $^{18}\text{O}$  is higher during glacial periods.

Foraminifera absorb more  $^{18}\text{O}$  into their skeletons when the water temperature is lower and when more  $^{18}\text{O}$  is in the water.

Thus higher concentrations of  $^{18}\text{O}$  in foraminifera fossils indicate lower ocean temperatures and higher glacier volume.

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**Math 1901 Paleoclimate**

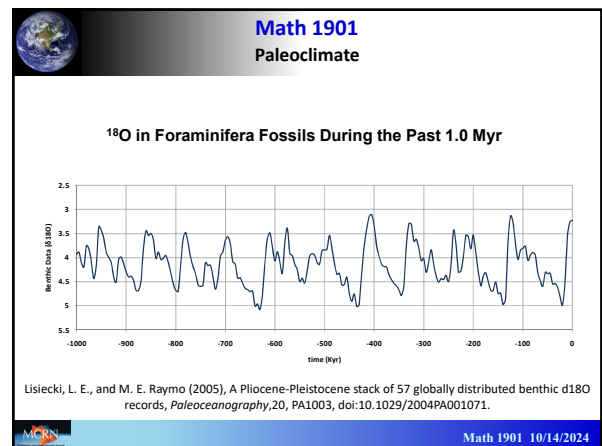
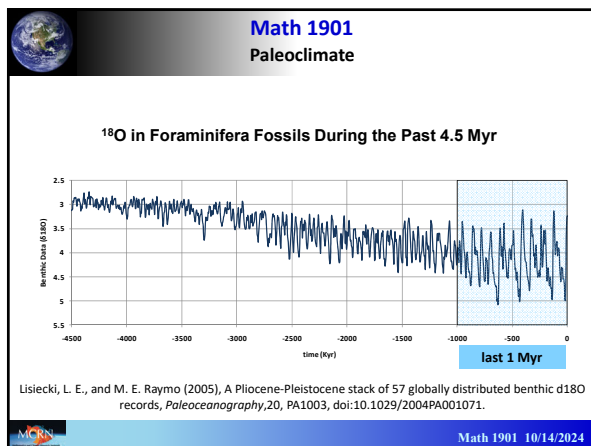
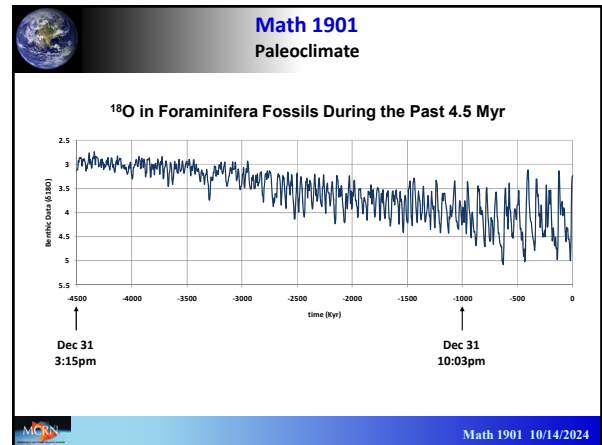
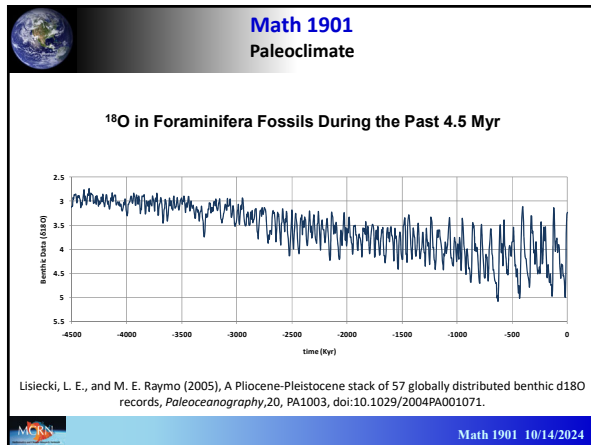
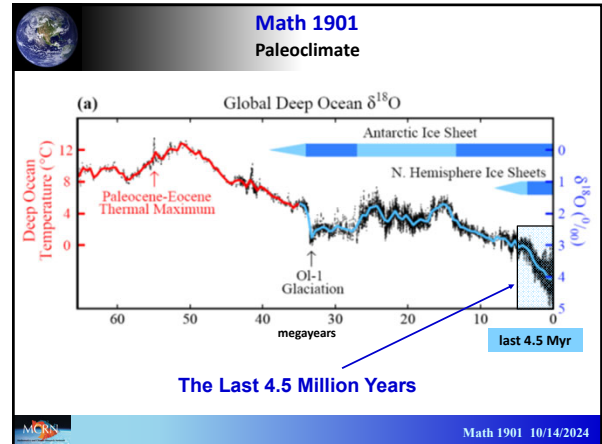
**What is this?  $\delta^{18}\text{O}$  (‰)**

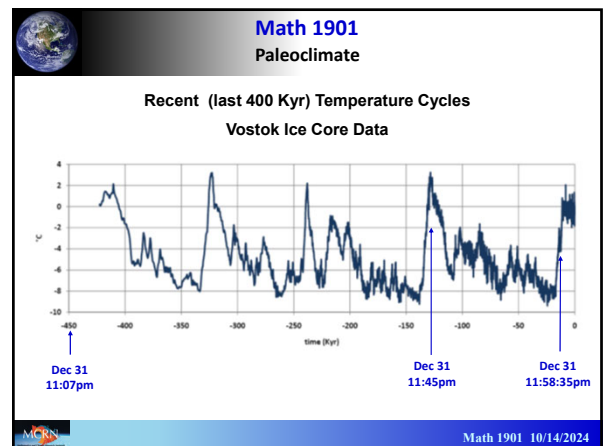
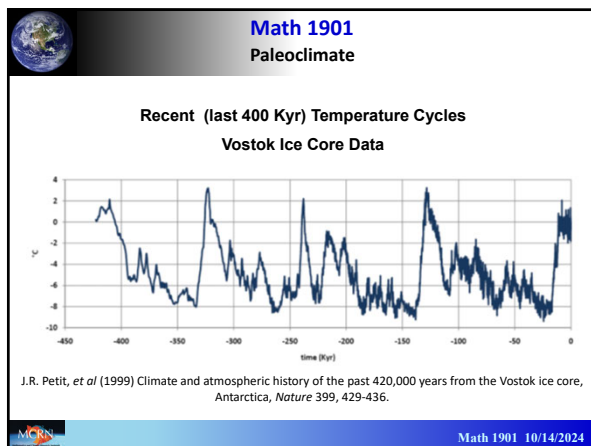
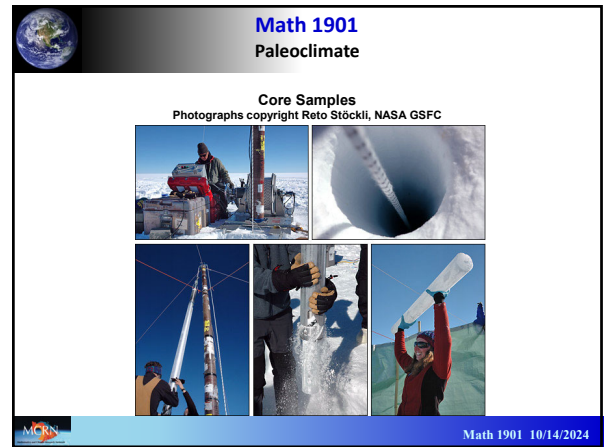
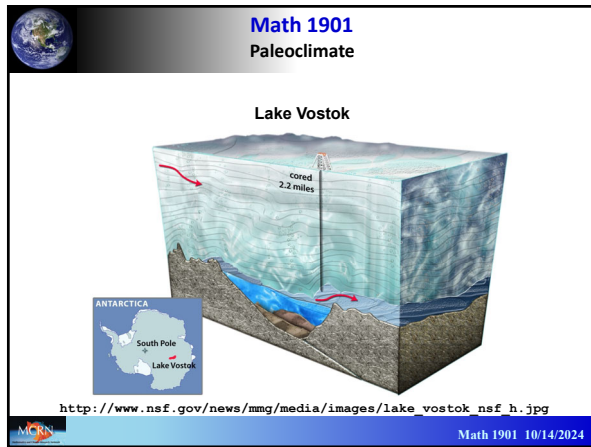
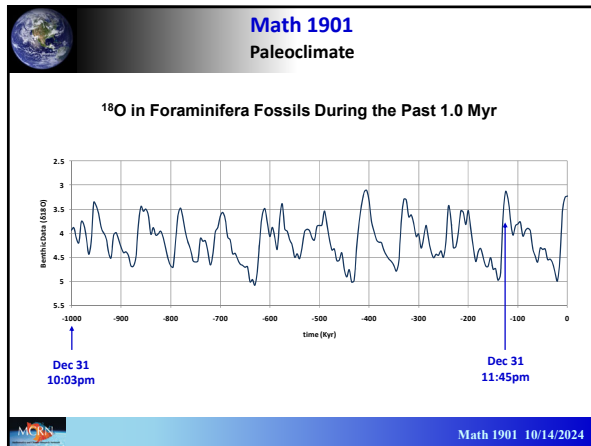
‰ : "per mil," "per thousand"

1000‰ = 100% = 1  
 10‰ = 1% = 0.01  
 1‰ = 0.1% = 0.001

$^{18}\text{O}$ : Oxygen 18: 8 protons 8 electrons 10 neutrons  
 $^{17}\text{O}$ : Oxygen 17: 8 protons 8 electrons 9 neutrons  
 $^{16}\text{O}$ : Oxygen 16: 8 protons 8 electrons 8 neutrons  
 Most of the oxygen atoms on Earth are  $^{16}\text{O}$ .  
 About 1 in 500 atoms is  $^{18}\text{O}$ . About 1 in 2500 is  $^{17}\text{O}$ .  
 There are other oxygen isotopes, but they are unstable.

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**Math 1901 Paleoclimate**

event	years before present	Earth age = 1 year
end of last ice age	12,000 ybp	- 84 seconds
First Egyptian pyramid	4,800 ybp	-34 seconds
First Chinese dynasty	4,000 ybp	-28 seconds
0 AD	2,024 ybp	-14 seconds
Norman Conquest	956 ybp	-7 seconds
Declaration of Independence	248 ybp	-1740 milliseconds
Industrialization (280ppm)	163 ybp	-1140 milliseconds
Atomic Age	79 ybp	-550 milliseconds
Mauna Loa (315ppm)	66 ybp	-460 milliseconds
350 ppm	37 ybp	-260 milliseconds
400 ppm	8 ybp	-56 milliseconds

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### What Causes Glacial Cycles?

**Widely Accepted Hypothesis**

The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

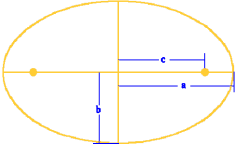
The additional hypothesis is that there are feedback mechanisms and/or triggering mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work are not fully understood.

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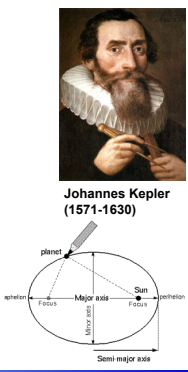
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### Earth's Orbit

**Kepler's First Law: The orbit of every planet is an ellipse with the Sun at one of the two foci.**



**Eccentricity =  $c/a$**



**Johannes Kepler (1571-1630)**

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