Math 1901
Freshman Seminar
Mathematical Climate Models

Fall 2024
1:00 - 2:15 Mondays and Wednesdays
Vincent Hall 213

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course website
https://www-users.cse.umn.edu/~mcgehee/Course/Math1901/

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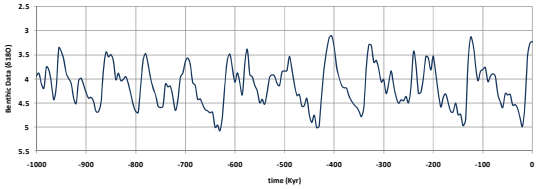
Glacial Cycles

Glacial Cycles

^{18}O in Foraminifera Fossils During the Past 1.0 Myr



Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

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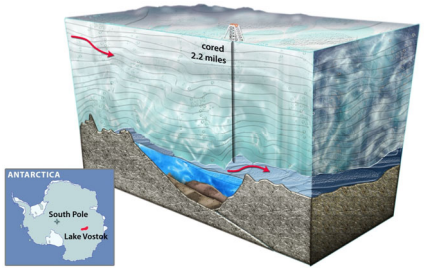
Lake Vostok



http://www.nsf.gov/news/mmg/media/images/lake_vostok_nsf_h.jpg

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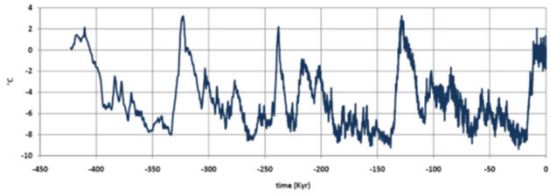
Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data



J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* 399, 429-436.

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What Causes Glacial Cycles?

Widely Accepted Hypothesis

The glacial cycles are driven by the variations in the Earth's orbit (***Milankovitch Cycles***), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms and/or triggering mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work are not fully understood.

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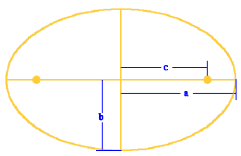
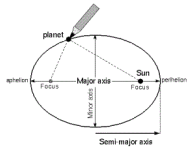
Earth's Orbit

Kepler's First Law: The orbit of every planet is an ellipse with the Sun at one of the two foci.



Eccentricity = c/a

Johannes Kepler (1571-1630)



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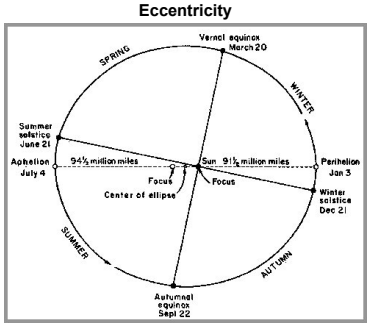
Eccentricity



John Imbrie & Katherine Palmer Imbrie, *Ice Ages: Solving the Mystery,* Harvard Univ. Press, 1979.

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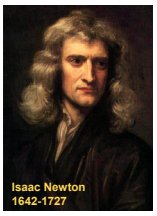
Planetary Motion

$$m_i \frac{d^2 x_i}{dt^2} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{G m_i m_j \left(x_j - x_i\right)}{\left|x_j - x_i\right|^3}$$



Jacques Laskar (1955-)

The orbits of all the planets can be computed (both forward and backward in time) for billions of years.

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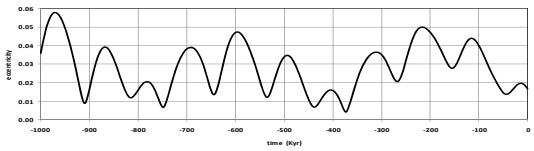
Earth's Eccentricity



Note periods of about 100 kyr and 400 kyr.

As the eccentricity varies between 0 and 0.06, the incoming solar radiation varies by about 0.2% or about **0.7 W/m²**.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

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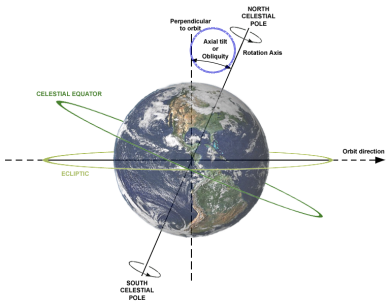
Earth's Obliquity



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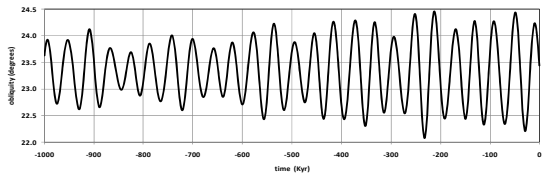
Earth's Obliquity



Note period of about 41 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

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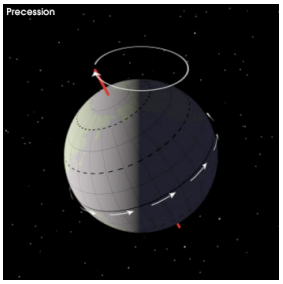
Precession



http://earthobservatory.nasa.gov/Library/Giants/Milankovitch/milankovitch_2.html

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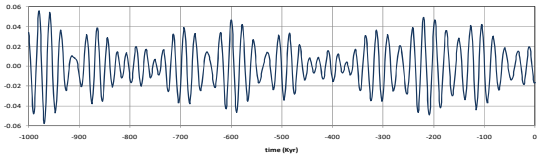
Earth's Precession Index



index $= e \sin\rho$, where e = eccentricity and ρ = precession angle (measured from spring equinox)

Note period of about 23 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

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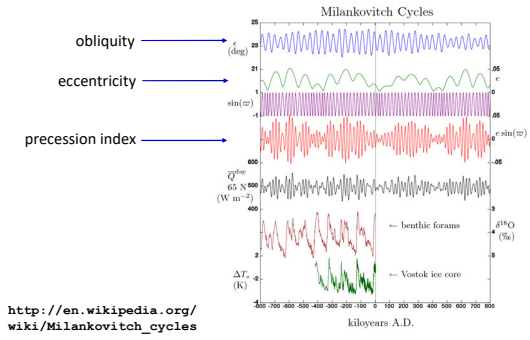

`http://en.wikipedia.org/wiki/Milankovitch_cycles`