

Math 5421
An Introduction to
Mathematical Climate Models

Spring 2025
 1:25 – 3:20 Tuesdays and Thursdays
 Blegen Hall 155

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course website
<https://www-users.cse.umn.edu/~mcgehee/Course/Math5421/>

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2

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Global Mean Temperature

Is the climate changing?

Global Average Temperature 1850 - 2024

Land data prepared by Berkeley Earth and combined with ocean data adapted from the UK Hadley Centre
 Global temperature anomalies relative to 1850-1900 average
 Vertical lines indicate 95% confidence intervals

It seems to be getting hotter.

Why?

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3

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Greenhouse Effect

GREENHOUSE EFFECT
 A prerequisite for life on earth, the greenhouse effect occurs when infrared radiation [heat] is retained within the atmosphere.

Gary Stix, *Scientific American* September 2006, pp.46-49

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4

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Greenhouse Effect

The Greenhouse Effect

Greenhouse gases (CO_2 , H_2O , CH_4) are transparent to visible light, but opaque to infrared light. The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.

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5

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Who discovered the greenhouse effect?

1. A meteorologist
2. A physicist
3. A chemist
4. A mathematician
5. Al Gore

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6

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The Greenhouse Effect

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Who discovered the greenhouse effect?

A mathematician!

Joseph Fourier (1827), *Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires, Mémoires de l'Académie Royale des Sciences*, t. vii., p. 569.




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7


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Greenhouse Effect

The Greenhouse Effect!

Joseph Fourier, *Mémoires de l'Académie des Sciences de l'Institut de France*, t. vii. 1827.



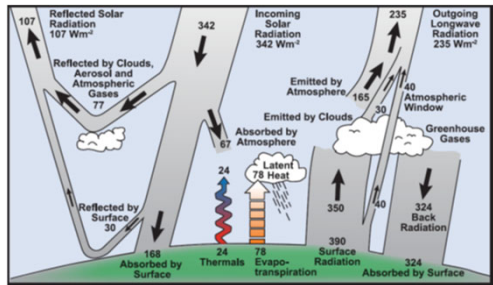
Svante Arrhenius, "On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground," *Philosophical Magazine and Journal of Science (Fifth Series)* 41, pp. 237-276, 1896.



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8

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Greenhouse Effect



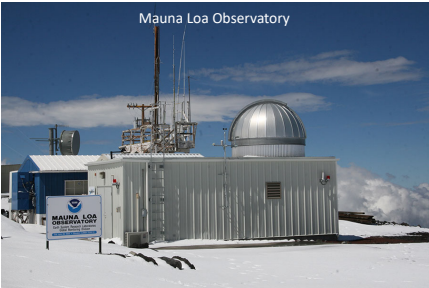
Historical Overview of Climate Change Science, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

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9

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Atmospheric CO2

Can we measure greenhouse gasses?



Mauna Loa Observatory

https://research.noaa.gov/Portals/0/EasyDNNews/1502/2000600p587EDNmain10061mlc_sigm_miller.jpg

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10

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Atmospheric CO2

Can we measure greenhouse gasses?

<https://keelingcurve.ucsd.edu/>

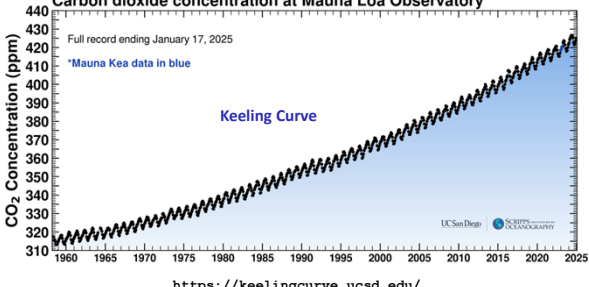
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11

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Atmospheric CO2

Can we measure greenhouse gasses?

Carbon dioxide concentration at Mauna Loa Observatory*



CO₂ Concentration (ppm)

Full record ending January 17, 2025

*Mauna Kea data in blue

Keeling Curve

UC San Diego | Scripps Institution of Oceanography

<https://keelingcurve.ucsd.edu/>

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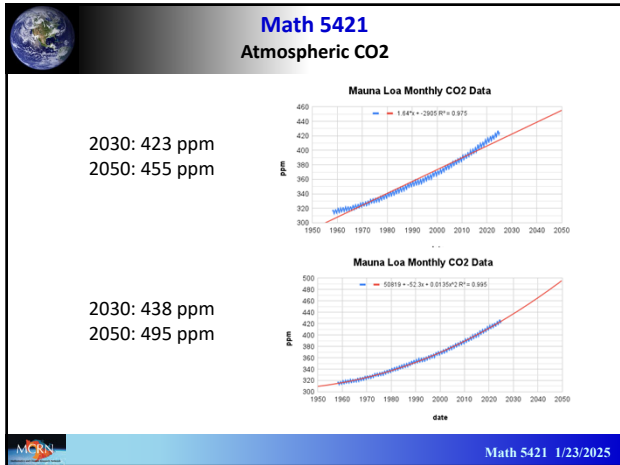
12

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Atmospheric CO2

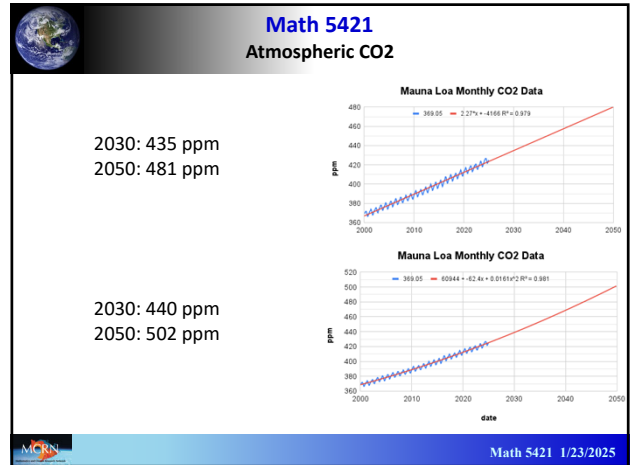
<https://www-users.cse.umn.edu/~mcgehee/Course/Math5421/assignments/A02.html>

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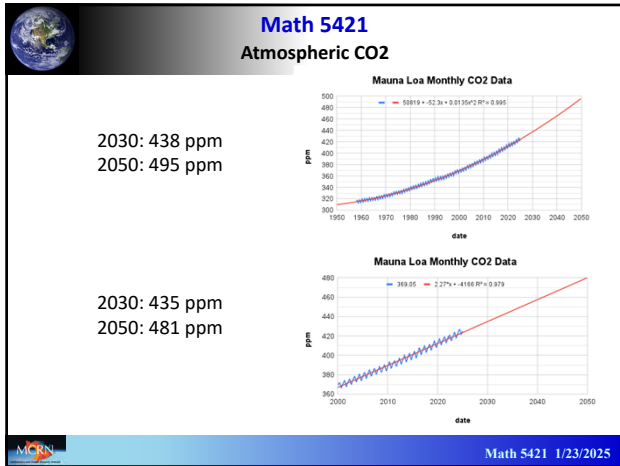
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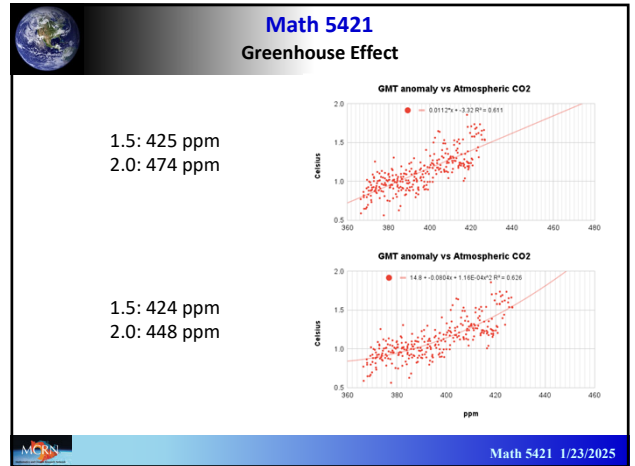
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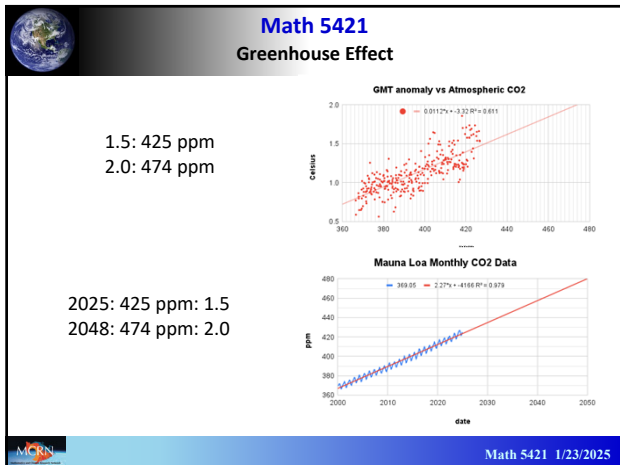
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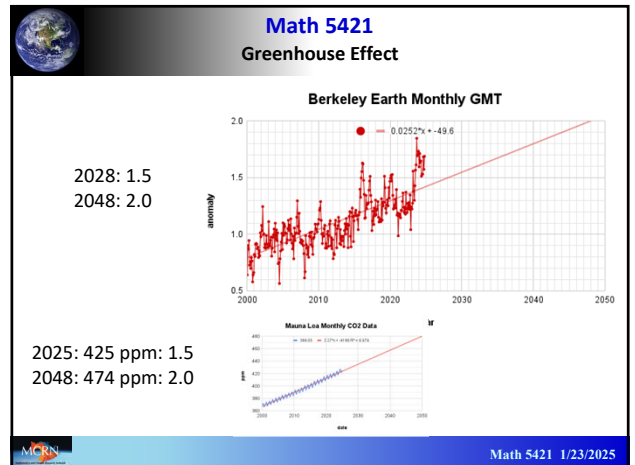
16




17



18



19



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Greenhouse Effect


We see that the atmospheric CO₂ and the global mean temperature are correlated.

Why do we think that the increasing atmospheric CO₂ is causing the temperature increase, and not the other way around?

The greenhouse effect,
a well-established theory originating about 200 years ago.

Also, we see an obvious source of the increasing CO₂ (us), but we see no reason for the temperature to be increasing.

Are we sure that we are the problem?
Maybe there is another explanation.



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20