Math 5421

An Introduction to Mathematical Climate Models

Spring 2025
1:25 – 3:20 Tuesdays and Thursdays
Blegen Hall 155

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course website
https://www-users.cse.umn.edu/~mcgehee/Course/Math5421/

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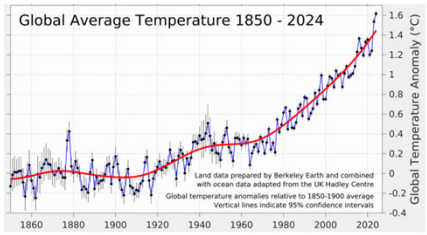
Global Mean Temperature

Is the climate changing?



It seems to be getting hotter.

Why?

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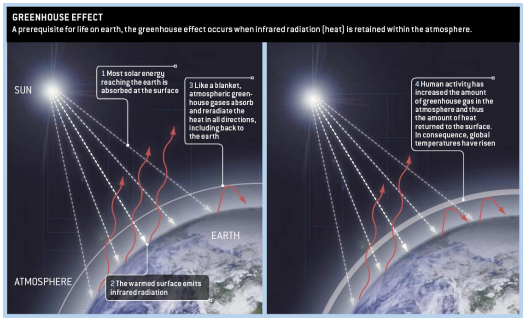
Greenhouse Effect



Gary Stix, *Scientific American* September 2006, pp.46-49

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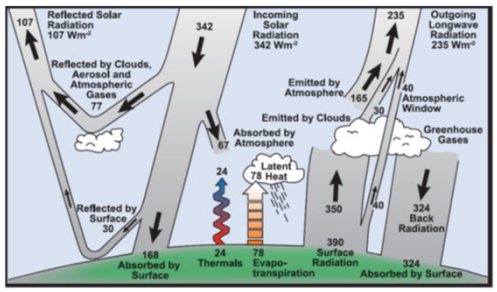
Greenhouse Effect



Historical Overview of Climate Change Science, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

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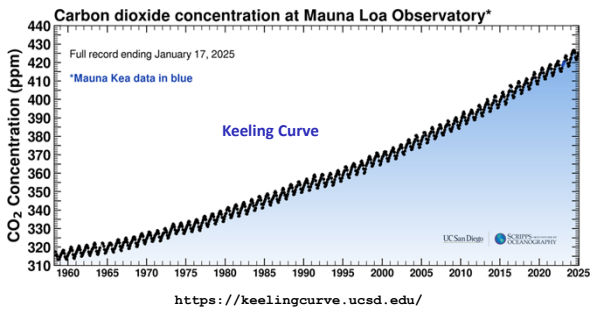
Atmospheric CO2

Can we measure greenhouse gasses?



https://keelingcurve.ucsd.edu/

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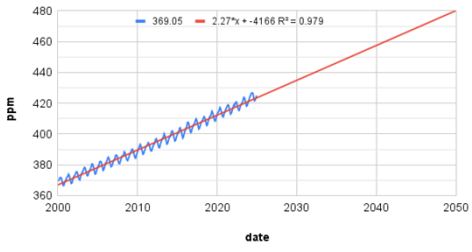
Mauna Loa Monthly CO2 Data



2030: 435 ppm
2050: 481 ppm

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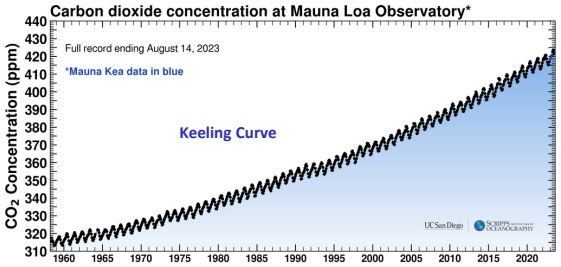
Why do we think that the Keeling curve has anything to do with human activity? It could just be an unknown natural source of carbon.



https://keelingcurve.ucsd.edu/

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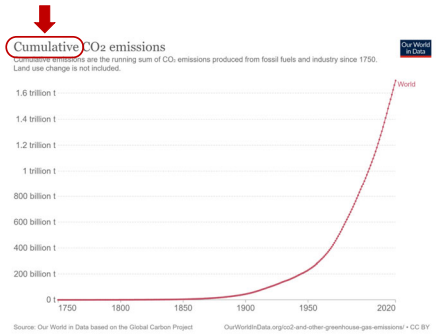

The cumulative carbon emission are going up. This graph resembles the Keeling curve.

https://www.weforum.org/agenda/2022/11/visualizing-changes-carbon-dioxide-emissions-since-1900/

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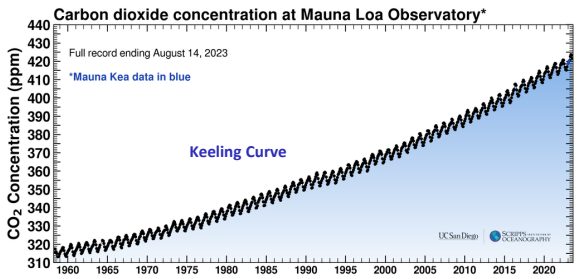
Looks like the cumulative emissions curve!



https://keelingcurve.ucsd.edu/

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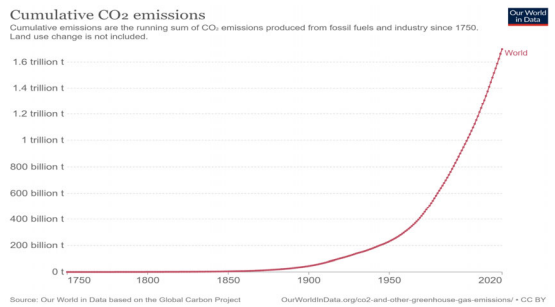

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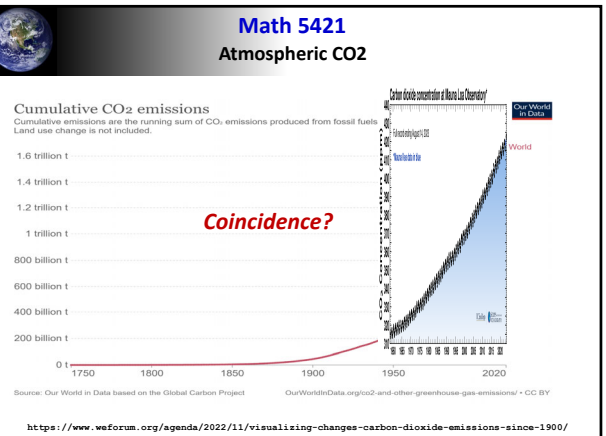

https://www.weforum.org/agenda/2022/11/visualizing-changes-carbon-dioxide-emissions-since-1900/

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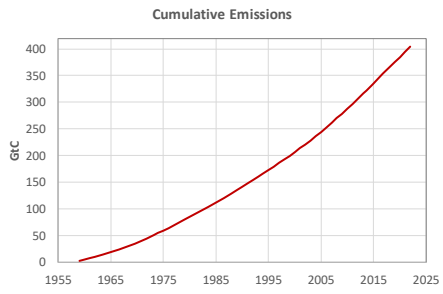

data source: https://globalcarbonbudgetdata.org/latest-data.html

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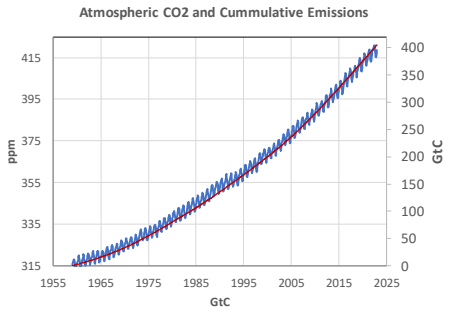

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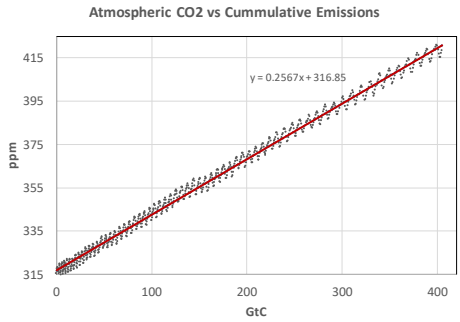

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No big deal. Both the accumulated emissions and the atmospheric CO2 are increasing. Circumstantial evidence.

Is that all you got?

Nope.

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Energy production using fossil fuels burns carbon to produce carbon dioxide. The oxygen comes from the atmosphere.

$$C + O_2 \rightarrow CO_2$$

Every atom of carbon that is burned consumes two atoms of oxygen.

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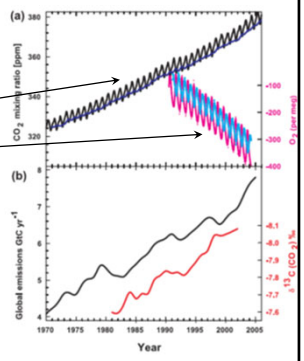
Atmospheric CO2

Every atom of carbon that is burned consumes two atoms of oxygen. Atmospheric carbon dioxide is increasing as atmospheric oxygen is decreasing, indicating that the carbon dioxide in the atmosphere is produced by burning carbon and is not coming from an unknown source of carbon dioxide.



Changes in Atmospheric Constituents and in Radiative Forcing, IPCC AR4, Chap. 2, p.138
https://www.ipcc.ch/site/assets/uploads/2018/02/ar4-wg1-chapter2-1.pdf

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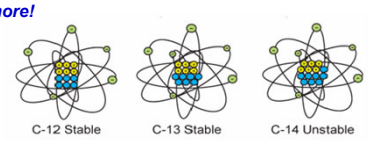
But there's more!



Photosynthesis prefers ^{12}C to ^{13}C.

$$6CO_2 + 6H_2O \rightarrow C_6H_{12}O_6 + 6O_2$$

Result: Any carbon that traces its ancestry to photosynthesis is lighter in ^{13}C than carbon from inorganic sources.

Examples: plants, animals, coal, oil.

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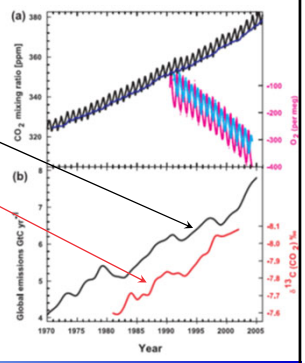
As the carbon dioxide is emitted by fossil fuel burning, the proportion of ^{13}C in the atmosphere is decreasing, indicating that the addition carbon dioxide is coming from combustion with atmospheric oxygen.



Changes in Atmospheric Constituents and in Radiative Forcing, IPCC AR4, Chap. 2, p.138
https://www.ipcc.ch/site/assets/uploads/2018/02/ar4-wg1-chapter2-1.pdf

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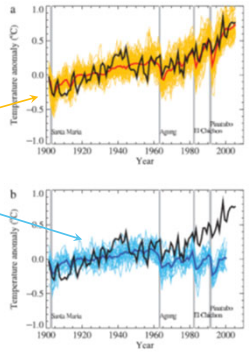
But there's more!

Models with and without human activity.



Understanding and Attributing Climate Change, IPCC AR4, Chap. 9, p.684
https://www.ipcc.ch/site/assets/uploads/2018/02/ar4-wg1-chapter9-1.pdf

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Scientific Conclusion

The scientific data are consistent with the hypothesis that the burning of fossil fuels is causing changes in the Earth's climate.

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Scientific Conclusion

The scientific data are consistent with the hypothesis that the burning of fossil fuels is causing changes in the Earth's climate.

Reasonable Human Reaction

We gotta stop burning fossil fuels!

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https://www.lemonde.fr/en/environment/article/2023/01/01/uk-climate-group-extinction-rebellion-suspends-public-disruption-tactics_6009966_114.html

https://newrepublic.com/article/166821/climate-delay-discourse-denial

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Measuring Atmospheric Carbon

ppm: parts per million
GtC: gigatonnes carbon
GtCO2: gigatonnes carbon dioxide

tonne: metric ton = 1000 kilograms = 10^6 grams
≈ 1.10 tons

gigatonne = 10^9 tonnes = 10^{15} grams = petagram

atmospheric carbon:
1 ppm ≈ 2.13 GtC

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What is the mass of the carbon in the atmosphere at 420 ppm?

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atmospheric carbon:
1 ppm ≈ 2.13 GtC

What is the mass of the carbon in the atmosphere at 420 ppm?

2.13 × 420 ≈ 895 Gt

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Measuring Atmospheric Carbon

carbon atomic weight ≈ 12
oxygen atomic weight ≈ 16
carbon dioxide molecular weight ≈ 12 + 2×16 = 44

1 GtC = (44/12) GtCO2 ≈ 3.67 GtC02

1 GtCO2 = (12/44) GtC ≈ 0.273 GtC

How much carbon is in 1.5 trillion tons of carbon dioxide?

1.5 trillion tons of CO2 = 1500 GtCO2 = 0.273 × 1500 GtC ≈ 410 GtC

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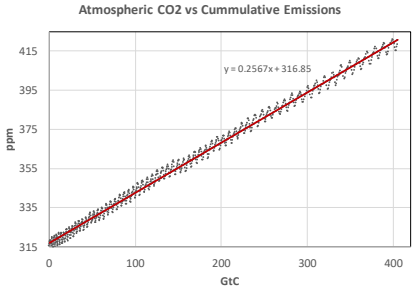
Since 1959 we have added more than 400 GtC to the atmosphere. During that time the carbon dioxide has increased by 105 ppm
≈ 2.13×105 GtC
≈ 224 GtC.
Almost half of the added CO_2 went elsewhere.



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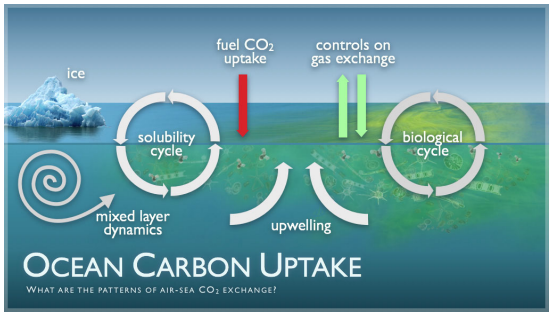

https://www.pmel.noaa.gov/co2/file/Ocean Carbon Uptake Image

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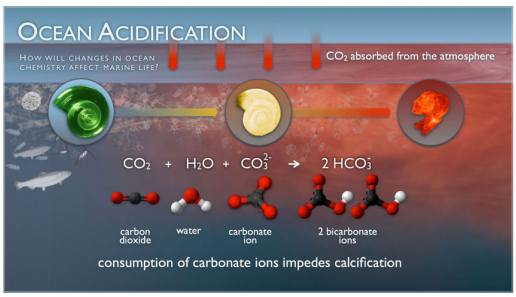

https://www.noaa.gov/education/resource-collections/ocean-coasts/ocean-acidification

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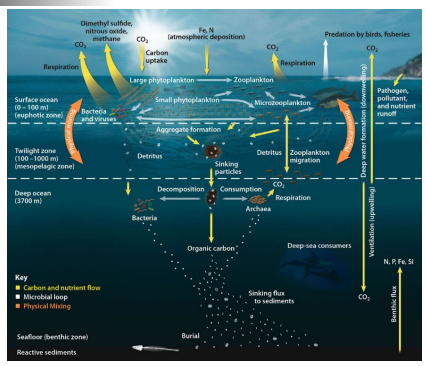
Biological Pump



https://en.wikipedia.org/wiki/Biological_pump

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Paris Agreement 2015

https://www.npr.org/sections/thetwo-way/2015/12/12/459502597/2-degrees-100-billion-the-world-climate-agreement-by-the-numbers

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Paris Agreement 2015

Article 2

1. This Agreement, in enhancing the implementation of the Convention, including its objective, aims to strengthen the global response to the threat of climate change, in the context of sustainable development and efforts to eradicate poverty, including by:

(a) Holding the increase in the global average temperature to well below 2°C above pre-industrial levels and pursuing efforts to limit the temperature increase to 1.5°C above pre-industrial levels, recognizing that this would significantly reduce the risks and impacts of climate change;

https://www.youtube.com/watch?v=I-4F5MJEeqs

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The Carbon Budget

The Carbon Budget

We have seen that, for the twenty-first century, the temperature is going up linearly with the amount of carbon in the atmosphere. We have also seen that the amount of atmospheric carbon goes up linearly with the emissions. Therefore, we expect that the temperature will increase linearly with the emissions. We can compute how much more carbon we can emit before the temperature anomaly reaches 1.5. That amount is called the carbon budget allowing us to stay below 1.5.

We can compute a similar budget for any temperature anomaly.

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The Carbon Budget

The Carbon Budget

https://www-users.cse.umn.edu/~mcgehee/Course/Math5421/assignments/A03.html

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