Math 5421
An Introduction to Mathematical Climate Models

Spring 2025
1:25 – 3:20 Tuesdays and Thursdays
Blegen Hall 155

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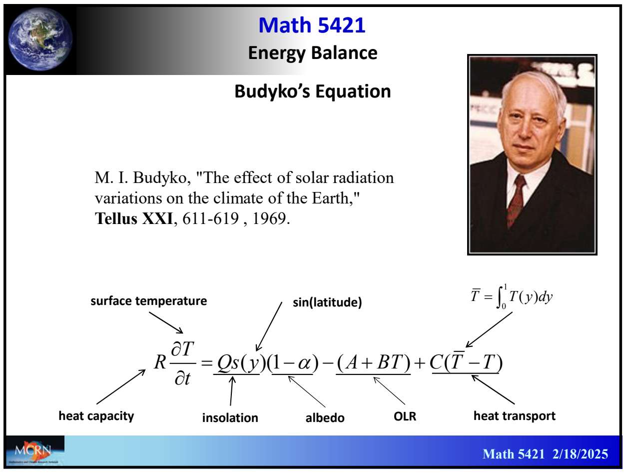
course website
`https://www-users.cse.umn.edu/~mcgehee/Course/Math5421/`



Math 5421
Energy Balance
Budyko's Equation

M. I. Budyko, "The effect of solar radiation variations on the climate of the Earth," **Tellus XXI**, 611-619 , 1969.

$$\bar{T} = \int_0^1 T(y)dy$$

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\bar{T} - T)$$

surface temperature
sin(latitude)
heat capacity
insolation
albedo
OLR
heat transport

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Energy Balance
Budyko's Equation

$$R\frac{\partial T}{\partial t}(y,t) = Qs(y)(1-\alpha) - \left(A + BT(y,t)\right) + C\left(\bar{T}(t) - T(y,t)\right)$$

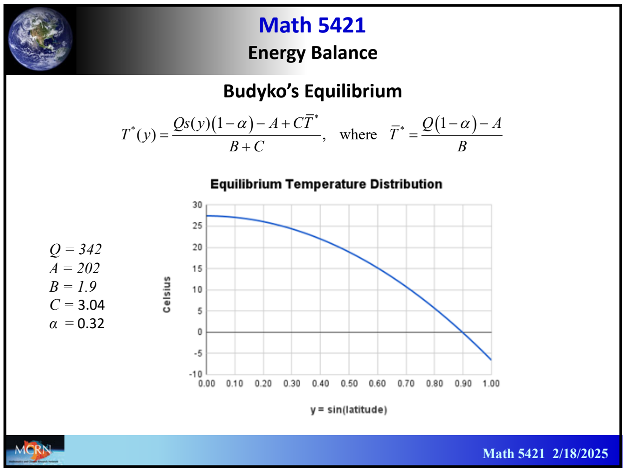
Equilibrium solution: temperature is a function of latitude.

$$T = T^*(y)$$

$$Qs(y)(1-\alpha) - \left(A + BT^*(y)\right) + C\left(\bar{T}^* - T^*(y)\right) = 0,$$

Equilibrium solution

$$\boxed{T^*(y) = \frac{Qs(y)(1-\alpha) - A + C\bar{T}^*}{B+C}, \quad \text{where} \quad \bar{T}^* = \frac{Q(1-\alpha) - A}{B}}$$



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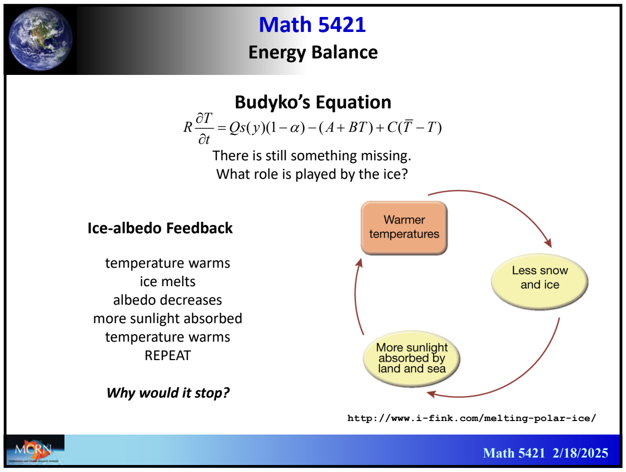
Energy Balance
Budyko's Equilibrium

$$T^*(y) = \frac{Qs(y)(1-\alpha) - A + C\bar{T}^*}{B+C}, \quad \text{where} \quad \bar{T}^* = \frac{Q(1-\alpha) - A}{B}$$

$Q = 342$
$A = 202$
$B = 1.9$
$C = 3.04$
$\alpha = 0.32$

Equilibrium Temperature Distribution



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Energy Balance
Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\bar{T} - T)$$

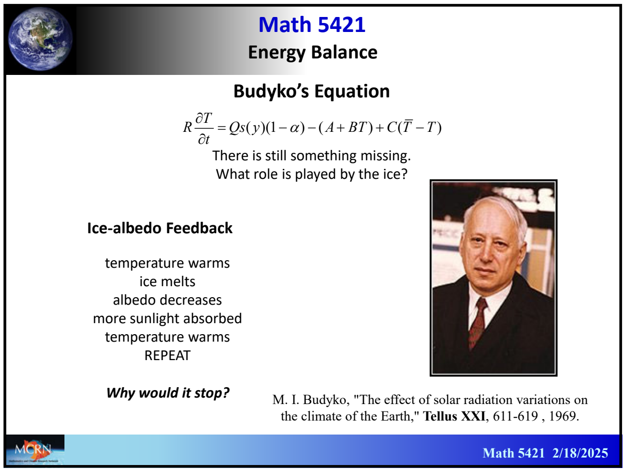
There is still something missing.
What role is played by the ice?

Ice-albedo Feedback

temperature warms
ice melts
albedo decreases
more sunlight absorbed
temperature warms
REPEAT

Why would it stop?

`http://www.i-fink.com/melting-polar-ice/`



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Energy Balance
Budyko's Equation

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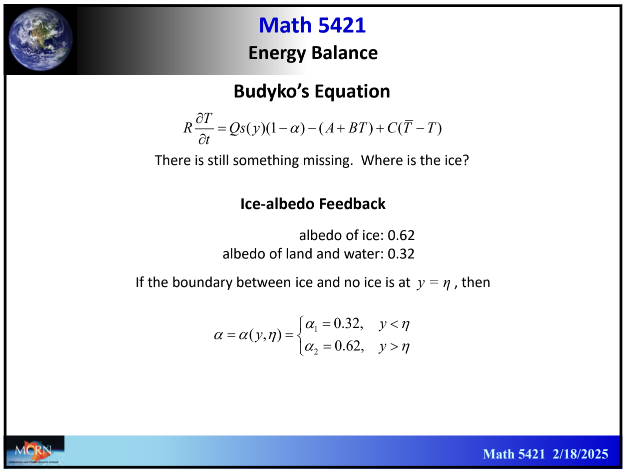

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Energy Balance

Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\overline{T} - T)$$

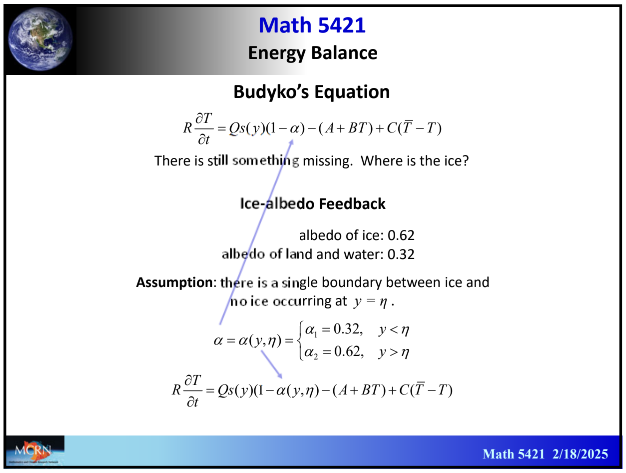
There is still something missing. Where is the ice?

Ice-albedo Feedback

albedo of ice: 0.62
albedo of land and water: 0.32

If the boundary between ice and no ice is at $y = \eta$, then

$$\alpha = \alpha(y,\eta) = \begin{cases} \alpha_1 = 0.32, & y < \eta \\ \alpha_2 = 0.62, & y > \eta \end{cases}$$



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Energy Balance

Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\overline{T} - T)$$

There is still something missing. Where is the ice?

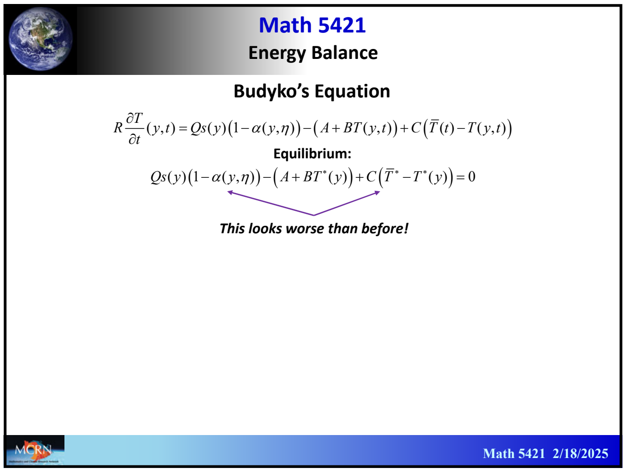
Ice-albedo Feedback

albedo of ice: 0.62
albedo of land and water: 0.32

Assumption: there is a single boundary between ice and no ice occurring at $y = \eta$.

$$\alpha = \alpha(y,\eta) = \begin{cases} \alpha_1 = 0.32, & y < \eta \\ \alpha_2 = 0.62, & y > \eta \end{cases}$$

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\overline{T} - T)$$



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Energy Balance

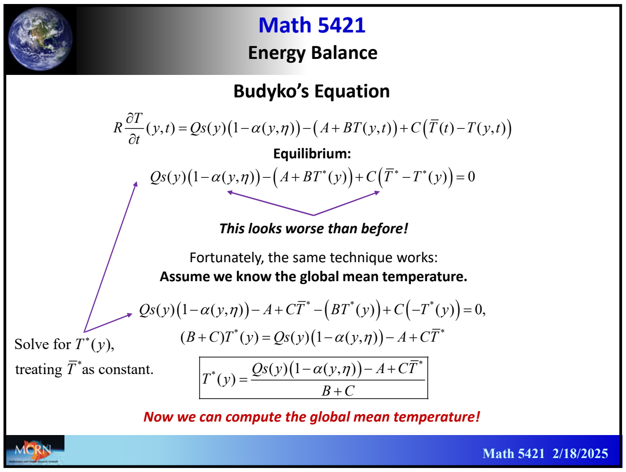
Budyko's Equation

$$R\frac{\partial T}{\partial t}(y,t) = Qs(y)\big(1-\alpha(y,\eta)\big) - \big(A + BT(y,t)\big) + C\big(\overline{T}(t) - T(y,t)\big)$$

Equilibrium:

$$Qs(y)\big(1-\alpha(y,\eta)\big) - \big(A + BT^*(y)\big) + C\big(\overline{T}^* - T^*(y)\big) = 0$$

This looks worse than before!



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Energy Balance

Budyko's Equation

$$R\frac{\partial T}{\partial t}(y,t) = Qs(y)\big(1-\alpha(y,\eta)\big) - \big(A + BT(y,t)\big) + C\big(\overline{T}(t) - T(y,t)\big)$$

Equilibrium:

$$Qs(y)\big(1-\alpha(y,\eta)\big) - \big(A + BT^*(y)\big) + C\big(\overline{T}^* - T^*(y)\big) = 0$$

This looks worse than before!

Fortunately, the same technique works:
Assume we know the global mean temperature.

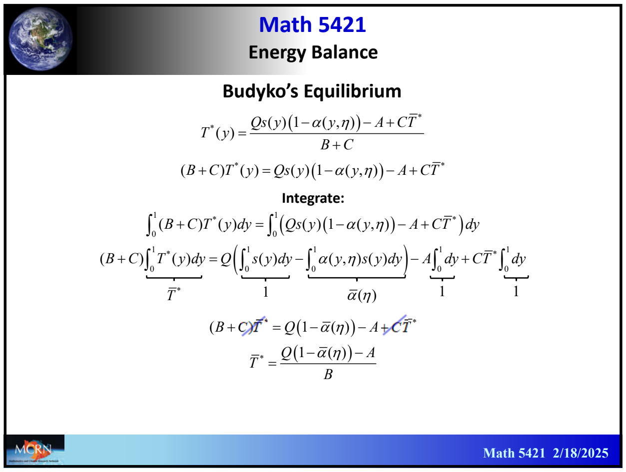
$$Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^* - \big(BT^*(y)\big) + C\big(-T^*(y)\big) = 0,$$

$$(B+C)T^*(y) = Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*$$

Solve for $T^*(y)$,
treating $\overline{T}^*$ as constant.

$$\boxed{T^*(y) = \frac{Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*}{B+C}}$$

Now we can compute the global mean temperature!



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Energy Balance

Budyko's Equilibrium

$$T^*(y) = \frac{Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*}{B+C}$$

$$(B+C)T^*(y) = Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*$$

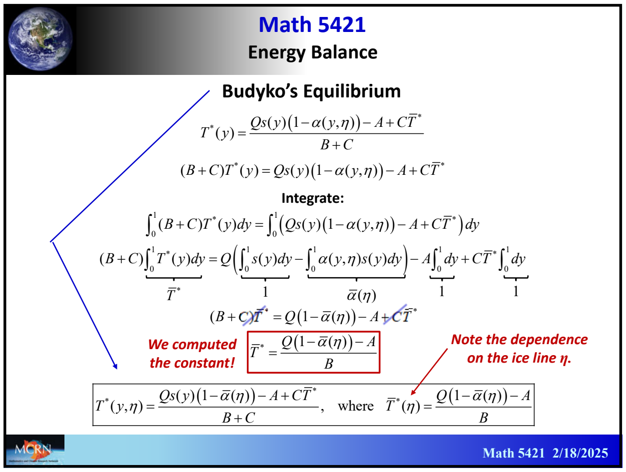
Integrate:

$$\int_0^1 (B+C)T^*(y)dy = \int_0^1 \Big(Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*\Big)dy$$

$$(B+C)\underbrace{\int_0^1 T^*(y)dy}_{\overline{T}^*} = Q\Big(\underbrace{\int_0^1 s(y)dy}_{1} - \underbrace{\int_0^1 \alpha(y,\eta)s(y)dy}_{\overline{\alpha}(\eta)}\Big) - A\underbrace{\int_0^1 dy}_{1} + C\overline{T}^*\underbrace{\int_0^1 dy}_{1}$$

$$(B+\cancel{C)\overline{T}^*} = Q\big(1-\overline{\alpha}(\eta)\big) - A + \cancel{C\overline{T}^*}$$

$$\overline{T}^* = \frac{Q\big(1-\overline{\alpha}(\eta)\big) - A}{B}$$



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Energy Balance

Budyko's Equilibrium

$$T^*(y) = \frac{Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*}{B+C}$$

$$(B+C)T^*(y) = Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*$$

Integrate:

$$\int_0^1 (B+C)T^*(y)dy = \int_0^1 \Big(Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*\Big)dy$$

$$(B+C)\underbrace{\int_0^1 T^*(y)dy}_{\overline{T}^*} = Q\Big(\underbrace{\int_0^1 s(y)dy}_{1} - \underbrace{\int_0^1 \alpha(y,\eta)s(y)dy}_{\overline{\alpha}(\eta)}\Big) - A\underbrace{\int_0^1 dy}_{1} + C\overline{T}^*\underbrace{\int_0^1 dy}_{1}$$

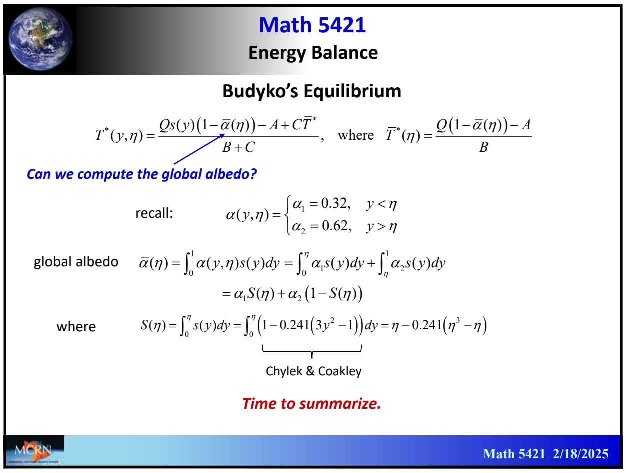
$$(B+\cancel{C)\overline{T}^*} = Q\big(1-\overline{\alpha}(\eta)\big) - A + \cancel{C\overline{T}^*}$$

We computed the constant!

$$\boxed{\overline{T}^* = \frac{Q\big(1-\overline{\alpha}(\eta)\big) - A}{B}}$$

Note the dependence on the ice line η.

$$\boxed{T^*(y,\eta) = \frac{Qs(y)\big(1-\overline{\alpha}(\eta)\big) - A + C\overline{T}^*}{B+C}, \quad \text{where} \quad \overline{T}^*(\eta) = \frac{Q\big(1-\overline{\alpha}(\eta)\big) - A}{B}}$$



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Energy Balance

Budyko's Equilibrium

$$T^*(y,\eta)=\frac{Qs(y)\left(1-\overline{\alpha}(\eta)\right)-A+C\overline{T}^*}{B+C},\quad \text{where}\quad \overline{T}^*(\eta)=\frac{Q\left(1-\overline{\alpha}(\eta)\right)-A}{B}$$

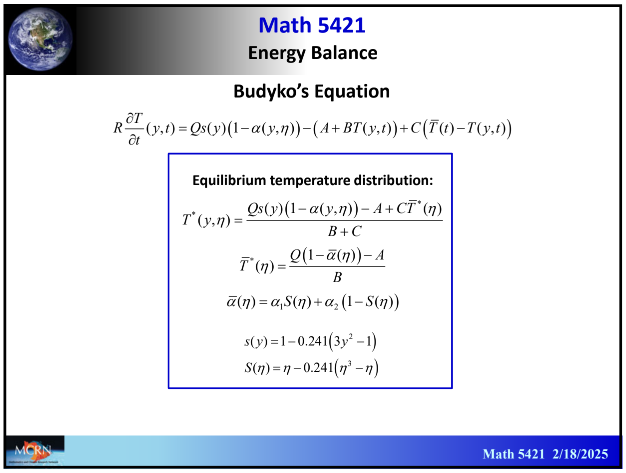
Can we compute the global albedo?

recall: $$\alpha(y,\eta)=\begin{cases}\alpha_1=0.32, & y<\eta\\ \alpha_2=0.62, & y>\eta\end{cases}$$

global albedo $$\overline{\alpha}(\eta)=\int_0^1\alpha(y,\eta)s(y)dy=\int_0^\eta\alpha_1 s(y)dy+\int_\eta^1\alpha_2 s(y)dy$$
$$=\alpha_1 S(\eta)+\alpha_2\left(1-S(\eta)\right)$$

where $$S(\eta)=\int_0^\eta s(y)dy=\int_0^\eta \underbrace{\left(1-0.241\left(3y^2-1\right)\right)}_{\text{Chylek \& Coakley}}dy=\eta-0.241\left(\eta^3-\eta\right)$$

Time to summarize.



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Energy Balance

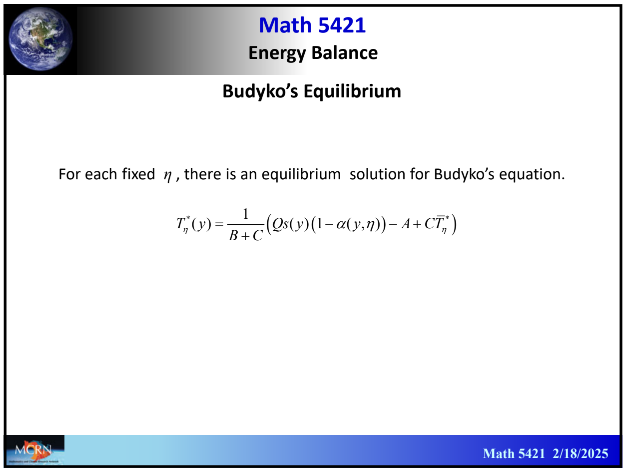
Budyko's Equation

$$R\frac{\partial T}{\partial t}(y,t)=Qs(y)\left(1-\alpha(y,\eta)\right)-\left(A+BT(y,t)\right)+C\left(\overline{T}(t)-T(y,t)\right)$$

Equilibrium temperature distribution:

$$T^*(y,\eta)=\frac{Qs(y)\left(1-\alpha(y,\eta)\right)-A+C\overline{T}^*(\eta)}{B+C}$$
$$\overline{T}^*(\eta)=\frac{Q\left(1-\overline{\alpha}(\eta)\right)-A}{B}$$
$$\overline{\alpha}(\eta)=\alpha_1 S(\eta)+\alpha_2\left(1-S(\eta)\right)$$

$$s(y)=1-0.241\left(3y^2-1\right)$$
$$S(\eta)=\eta-0.241\left(\eta^3-\eta\right)$$



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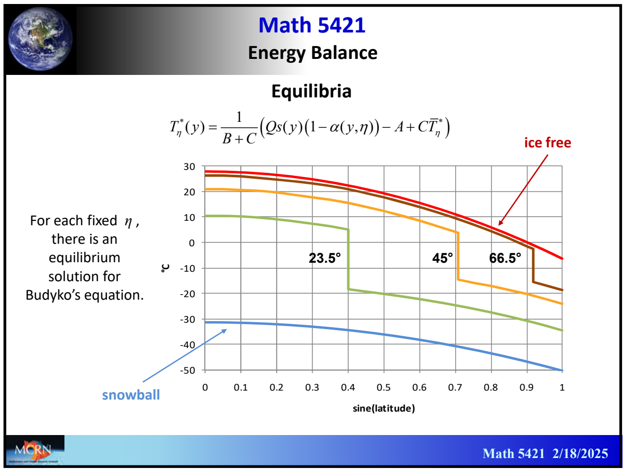
Energy Balance

Budyko's Equilibrium

For each fixed η, there is an equilibrium solution for Budyko's equation.

$$T_\eta^*(y)=\frac{1}{B+C}\left(Qs(y)\left(1-\alpha(y,\eta)\right)-A+C\overline{T}_\eta^*\right)$$



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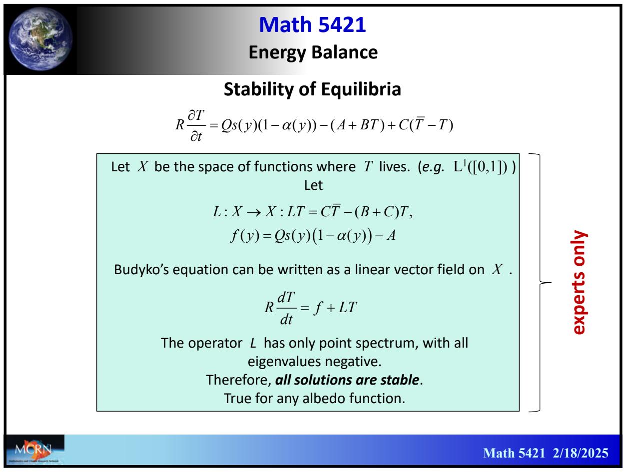
Energy Balance

Equilibria

$$T_\eta^*(y)=\frac{1}{B+C}\left(Qs(y)\left(1-\alpha(y,\eta)\right)-A+C\overline{T}_\eta^*\right)$$

For each fixed η, there is an equilibrium solution for Budyko's equation.



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Energy Balance

Stability of Equilibria

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y))-(A+BT)+C(\overline{T}-T)$$

Let X be the space of functions where T lives. (*e.g.* $\mathrm{L}^1([0,1])$)
Let

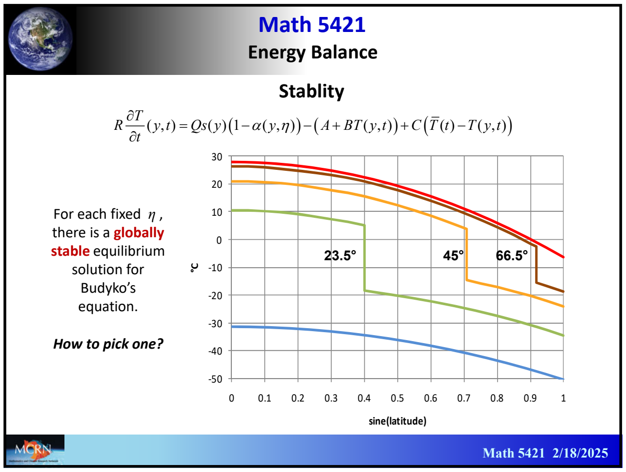
$$L:X\to X:LT=C\overline{T}-(B+C)T,$$
$$f(y)=Qs(y)\left(1-\alpha(y)\right)-A$$

Budyko's equation can be written as a linear vector field on X .

$$R\frac{dT}{dt}=f+LT$$

The operator L has only point spectrum, with all eigenvalues negative.
Therefore, ***all solutions are stable***.
True for any albedo function.

experts only



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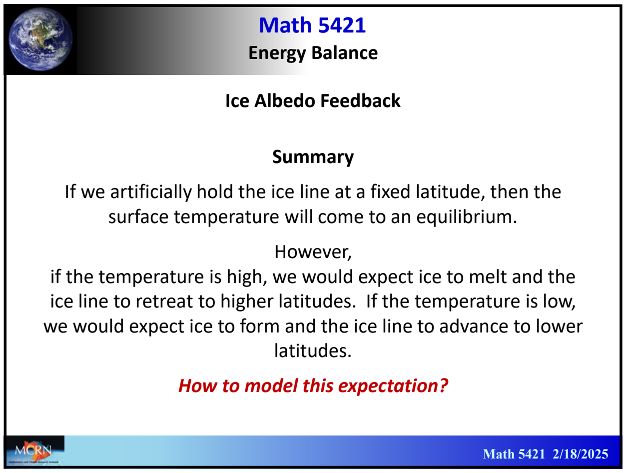
Energy Balance

Stablity

$$R\frac{\partial T}{\partial t}(y,t)=Qs(y)\left(1-\alpha(y,\eta)\right)-\left(A+BT(y,t)\right)+C\left(\overline{T}(t)-T(y,t)\right)$$

For each fixed η, there is a **globally stable** equilibrium solution for Budyko's equation.

How to pick one?



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Energy Balance

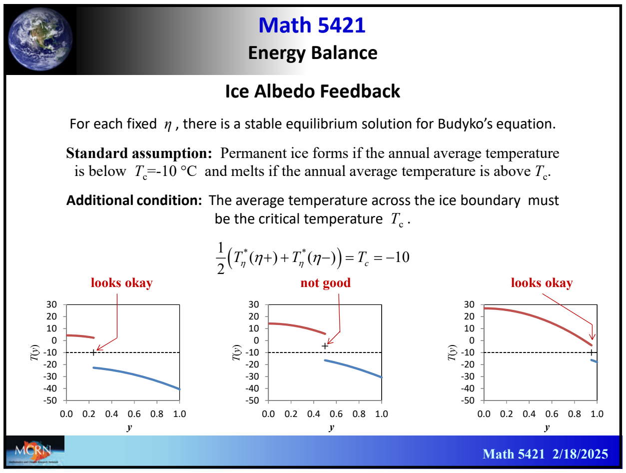
Ice Albedo Feedback

Summary

If we artificially hold the ice line at a fixed latitude, then the surface temperature will come to an equilibrium.

However,
if the temperature is high, we would expect ice to melt and the ice line to retreat to higher latitudes. If the temperature is low, we would expect ice to form and the ice line to advance to lower latitudes.

How to model this expectation?



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Energy Balance

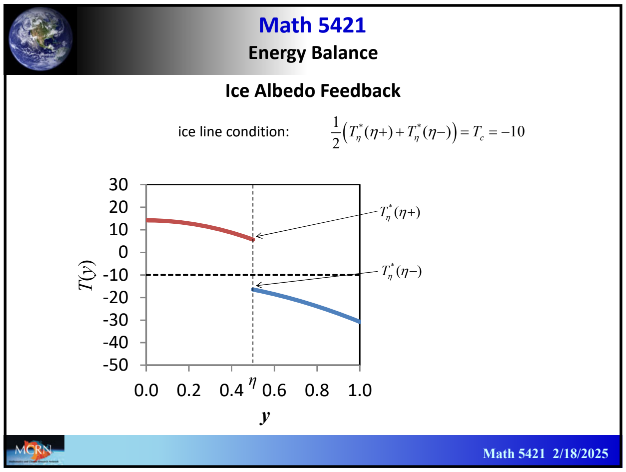
Ice Albedo Feedback

For each fixed η, there is a stable equilibrium solution for Budyko's equation.

Standard assumption: Permanent ice forms if the annual average temperature is below T_c=-10 °C and melts if the annual average temperature is above T_c.

Additional condition: The average temperature across the ice boundary must be the critical temperature T_c.

$$\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=T_c=-10$$



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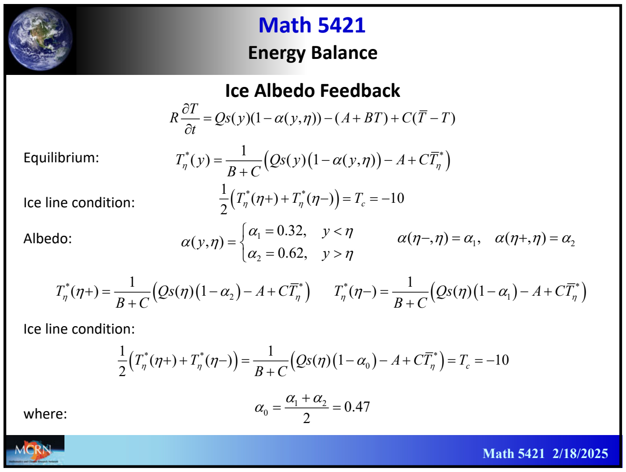
Energy Balance

Ice Albedo Feedback

ice line condition: $\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=T_c=-10$



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Energy Balance

Ice Albedo Feedback

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

Equilibrium: $T_\eta^*(y)=\frac{1}{B+C}\left(Qs(y)\left(1-\alpha(y,\eta)\right)-A+C\overline{T}_\eta^*\right)$

Ice line condition: $\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=T_c=-10$

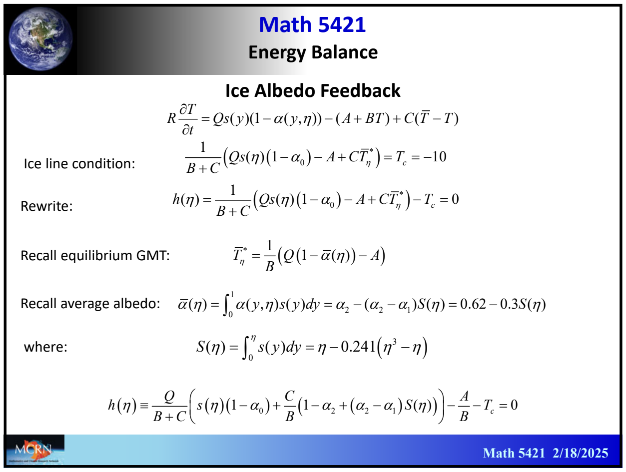
Albedo: $\alpha(y,\eta)=\begin{cases}\alpha_1=0.32, & y<\eta\\ \alpha_2=0.62, & y>\eta\end{cases}$ $\quad\alpha(\eta-,\eta)=\alpha_1,\quad \alpha(\eta+,\eta)=\alpha_2$

$$T_\eta^*(\eta+)=\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_2\right)-A+C\overline{T}_\eta^*\right)\qquad T_\eta^*(\eta-)=\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_1\right)-A+C\overline{T}_\eta^*\right)$$

Ice line condition:

$$\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_0\right)-A+C\overline{T}_\eta^*\right)=T_c=-10$$

where: $\alpha_0=\frac{\alpha_1+\alpha_2}{2}=0.47$



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Energy Balance

Ice Albedo Feedback

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

Ice line condition: $\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_0\right)-A+C\overline{T}_\eta^*\right)=T_c=-10$

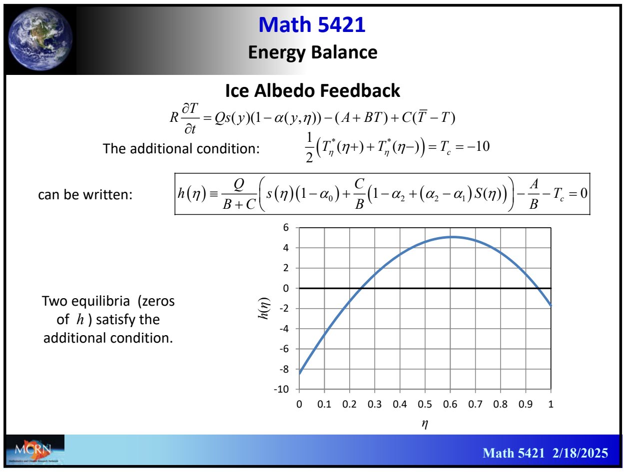
Rewrite: $h(\eta)=\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_0\right)-A+C\overline{T}_\eta^*\right)-T_c=0$

Recall equilibrium GMT: $\overline{T}_\eta^*=\frac{1}{B}\left(Q\left(1-\overline{\alpha}(\eta)\right)-A\right)$

Recall average albedo: $\overline{\alpha}(\eta)=\int_0^1\alpha(y,\eta)s(y)dy=\alpha_2-(\alpha_2-\alpha_1)S(\eta)=0.62-0.3S(\eta)$

where: $S(\eta)=\int_0^\eta s(y)dy=\eta-0.241\left(\eta^3-\eta\right)$

$$h\left(\eta\right)\equiv\frac{Q}{B+C}\left(s\left(\eta\right)\left(1-\alpha_0\right)+\frac{C}{B}\left(1-\alpha_2+\left(\alpha_2-\alpha_1\right)S(\eta)\right)\right)-\frac{A}{B}-T_c=0$$



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Energy Balance

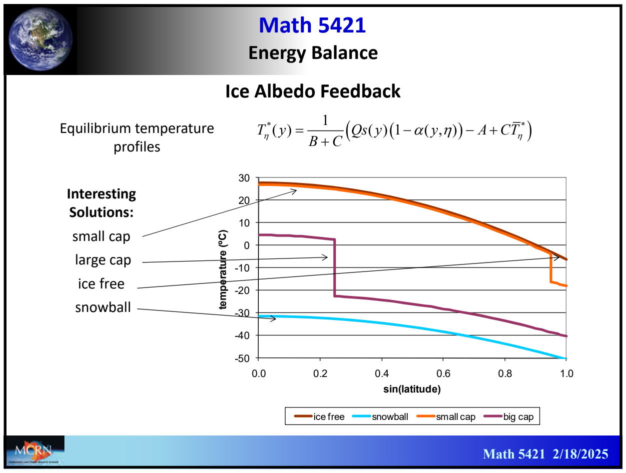
Ice Albedo Feedback

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

The additional condition: $\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=T_c=-10$

can be written: $\boxed{h\left(\eta\right)\equiv\frac{Q}{B+C}\left(s\left(\eta\right)\left(1-\alpha_0\right)+\frac{C}{B}\left(1-\alpha_2+\left(\alpha_2-\alpha_1\right)S(\eta)\right)\right)-\frac{A}{B}-T_c=0}$

Two equilibria (zeros of h) satisfy the additional condition.



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Energy Balance

Ice Albedo Feedback

Equilibrium temperature profiles $T_\eta^*(y)=\frac{1}{B+C}\left(Qs(y)\left(1-\alpha(y,\eta)\right)-A+C\overline{T}_\eta^*\right)$

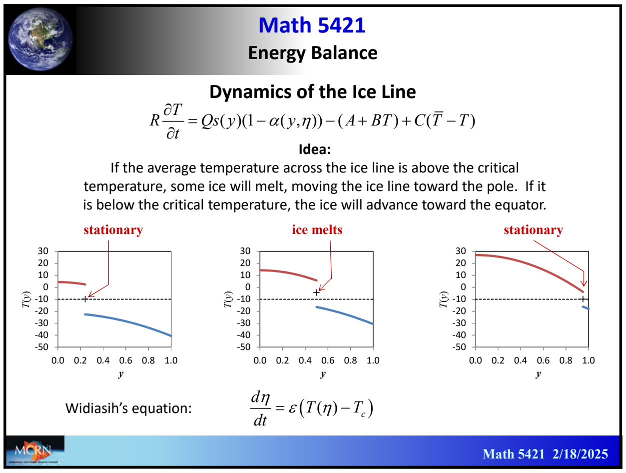
Interesting Solutions:

small cap

large cap

ice free

snowball



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Energy Balance

Dynamics of the Ice Line

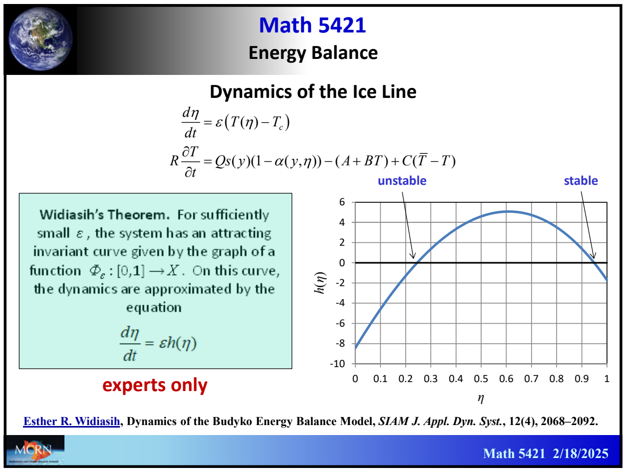
$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

Idea:

If the average temperature across the ice line is above the critical temperature, some ice will melt, moving the ice line toward the pole. If it is below the critical temperature, the ice will advance toward the equator.

stationary **ice melts** **stationary**

Widiasih's equation: $\frac{d\eta}{dt}=\varepsilon\left(T(\eta)-T_c\right)$



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Energy Balance

Dynamics of the Ice Line

$$\frac{d\eta}{dt}=\varepsilon\left(T(\eta)-T_c\right)$$

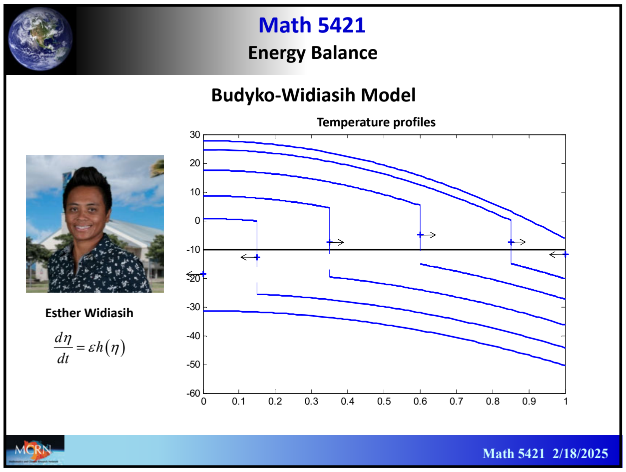
$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

Widiasih's Theorem. For sufficiently small ε, the system has an attracting invariant curve given by the graph of a function $\Phi_\varepsilon:[0,1]\to X$. On this curve, the dynamics are approximated by the equation

$$\frac{d\eta}{dt}=\varepsilon h(\eta)$$

experts only

Esther R. Widiasih, **Dynamics of the Budyko Energy Balance Model,** ***SIAM J. Appl. Dyn. Syst.*****, 12(4), 2068–2092.**



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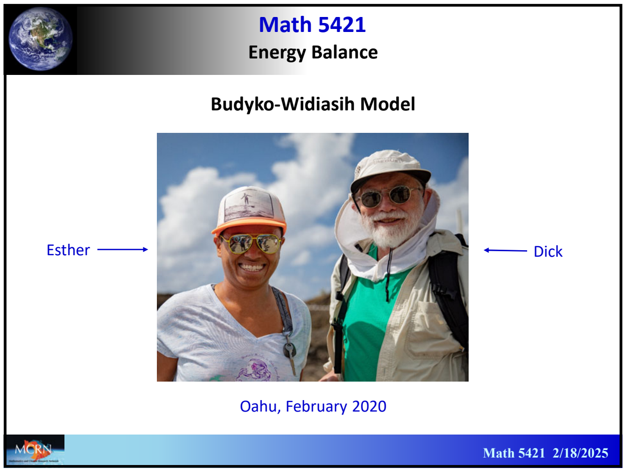
Energy Balance

Budyko-Widiasih Model

Esther Widiasih

$$\frac{d\eta}{dt}=\varepsilon h(\eta)$$



Math 5421

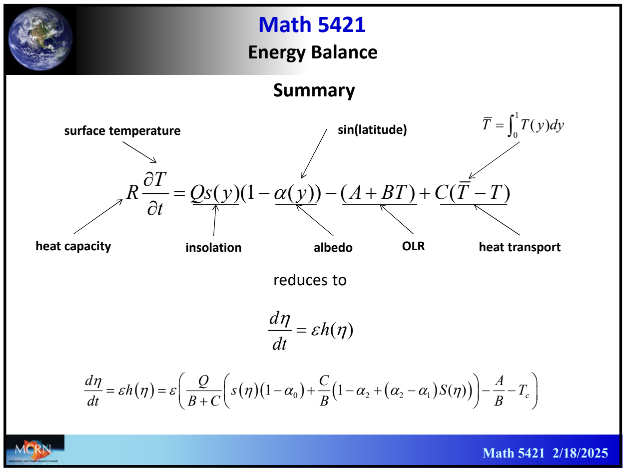
Energy Balance

Budyko-Widiasih Model

Esther →

← Dick

Oahu, February 2020



Math 5421

Energy Balance

Summary

surface temperature **sin(latitude)** $\overline{T}=\int_0^1 T(y)dy$

$$R\frac{\partial T}{\partial t}=\underline{Qs(y)}(1-\underline{\alpha(y)})-\underline{(A+BT)}+C\underline{(\overline{T}-T)}$$

heat capacity **insolation** **albedo** **OLR** **heat transport**

reduces to

$$\frac{d\eta}{dt}=\varepsilon h(\eta)$$

$$\frac{d\eta}{dt}=\varepsilon h(\eta)=\varepsilon\left(\frac{Q}{B+C}\left(s(\eta)(1-\alpha_0)+\frac{C}{B}\left(1-\alpha_2+(\alpha_2-\alpha_1)S(\eta)\right)\right)-\frac{A}{B}-T_c\right)$$

Math 5421
Energy Balance

Budyko-Widiasih Model

surface temperature

sin(latitude)

$\bar{T} = \int_0^1 T(y)dy$

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\bar{T} - T)$$

heat capacity

insolation

albedo

OLR

heat transport

What about the greenhouse effect?

$A+BT$ is the outgoing long wave radiation. This term decreases if the greenhouse gases increase.

We view A as a parameter.

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A)$$

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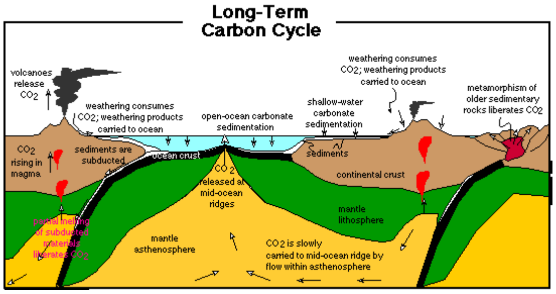
Earth's Carbon Cycle



http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm

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Math 5421
Earth's Carbon Cycle

Long-Term Carbon Cycle

Volcanos emit CO_2.

Silicate weathering carries carbon to the ocean.

Carbon sinks to the bottom of the ocean by the "biological pump" and by the precipitation of calcium carbonate.

The carbon is captured in the sediment.

The sediment is subducted beneath the continental crust by plate tectonics.

Volcanic activity released carbon from the carbonate rocks in the form of CO_2.

Repeat.

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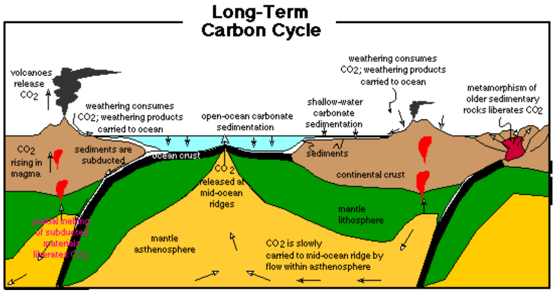
Earth's Carbon Cycle



http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm

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Earth's Carbon Cycle

Silicate Weathering

Rainwater containing dissolved CO_2 falling on silicate rocks replaces a silicon atom with a carbon atom, ultimately producing calcium carbonate (limestone) and silicon dioxide (quartz). For example, calcium silicate (Wollastonite)**:**

$$CaSiO_3 + CO_2 \to CaCO_3 + SiO_2$$

Under volcanic conditions, the carbon atom is replaced by a silicon atom, completing the long term carbon cycle.

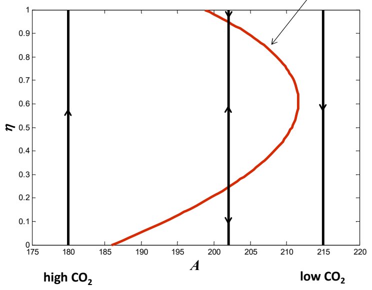
$$CaCO_3 + SiO_2 \to CaSiO_3 + CO_2$$

Math 5421
Energy Balance

Budyko-Widiasih Model

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A)$$

isocline $h(\eta, A) = 0$



high CO_2

low CO_2

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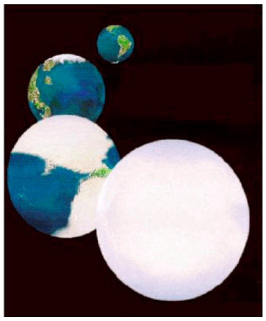
Snowball Earth

Is it possible for Earth to become completely covered in ice?
(**Snowball Earth**)

Did it ever happen?



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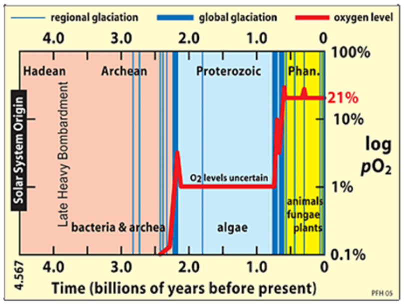
Snowball Earth

There is evidence that Snowball Earth has occurred, the last time about 600 million years ago.



`http://www.snowballearth.org/when.html`

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Snowball Earth

The continents were clustered near the equator.

Hoffman & Schrag, *Snowball Earth*, Scientific American, January 2000, 68-75

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Snowball Earth

“Ice-rafted debris” occurred in ocean sediments near the equator, indicating large equatorial glaciers calving icebergs.

Hoffman & Schrag, *Snowball Earth*, Scientific American, January 2000, 68-75

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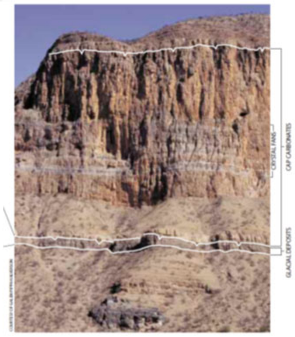
Snowball Earth

Large limestone deposits
“cap carbonates”
are found immediately above the glacial debris, indicating a rapid warming period following the snowball.



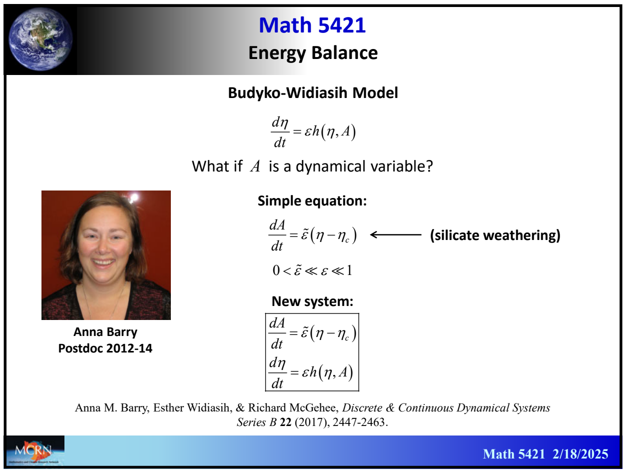
Hoffman & Schrag, *Snowball Earth*, Scientific American, January 2000, 68-75

Math 5421
Snowball Earth

Idea:

When the Earth is mostly ice-covered, silicate weathering slows down, but volcanic activity stays the same, allowing for a build-up of CO_2 in the atmosphere.

When the Earth is mostly ice-free, silicate weathering speeds up, drawing down the CO_2 in the atmosphere.



Math 5421

Energy Balance

Budyko-Widiasih Model

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A)$$

What if A is a dynamical variable?

Simple equation:

$$\frac{dA}{dt} = \tilde{\varepsilon}(\eta - \eta_c) \longleftarrow \textbf{(silicate weathering)}$$

$$0 < \tilde{\varepsilon} \ll \varepsilon \ll 1$$

New system:

$$\boxed{\begin{aligned} \frac{dA}{dt} &= \tilde{\varepsilon}(\eta - \eta_c) \\ \frac{d\eta}{dt} &= \varepsilon h(\eta, A) \end{aligned}}$$

Anna Barry
Postdoc 2012-14

Anna M. Barry, Esther Widiasih, & Richard McGehee, *Discrete & Continuous Dynamical Systems Series B* **22** (2017), 2447-2463.

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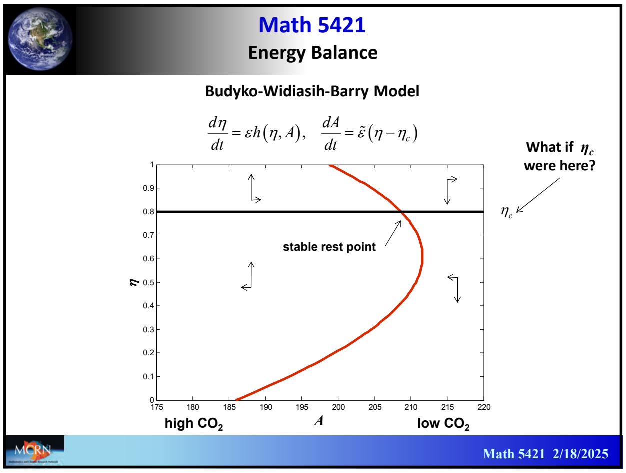

Math 5421

Energy Balance

Budyko-Widiasih-Barry Model

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A), \quad \frac{dA}{dt} = \tilde{\varepsilon}(\eta - \eta_c)$$

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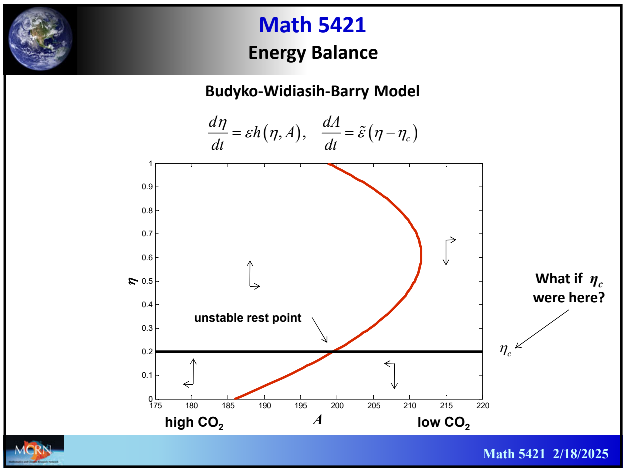

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Energy Balance

Budyko-Widiasih-Barry Model

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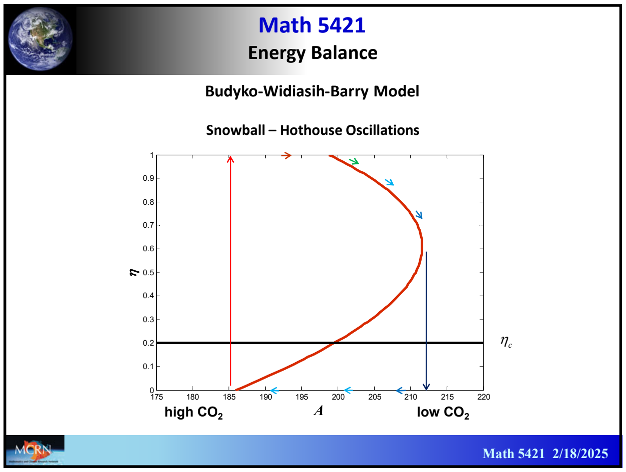

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Energy Balance

Budyko-Widiasih-Barry Model

Snowball – Hothouse Oscillations

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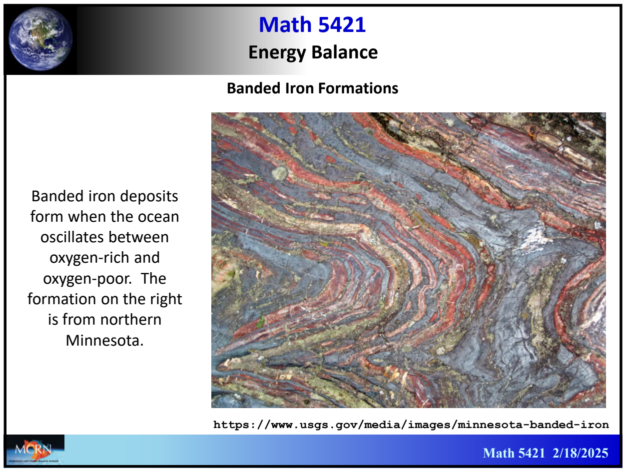

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Energy Balance

Banded Iron Formations

Banded iron deposits form when the ocean oscillates between oxygen-rich and oxygen-poor. The formation on the right is from northern Minnesota.

`https://www.usgs.gov/media/images/minnesota-banded-iron`

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