

Math 5421

An Introduction to
Mathematical Climate Models

Spring 2025

1:25 – 3:20 Tuesdays and Thursdays

Blegen Hall 155

Richard McGehee, Instructor

458 Vincent Hall

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www-users.cse.umn.edu/~mcgehee/

course website

https://www-users.cse.umn.edu/~mcgehee/Course/Math5421/

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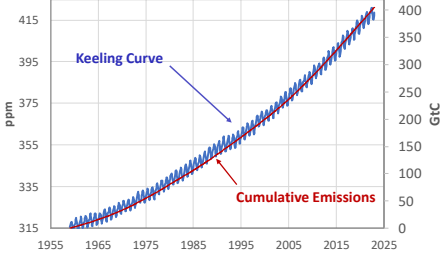
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Negative Emissions

Atmospheric CO2 and Cumulative Emissions

Since 1959 we have emitted more than 400 GtC into the atmosphere. During that time the carbon dioxide has increased by 105 ppm .



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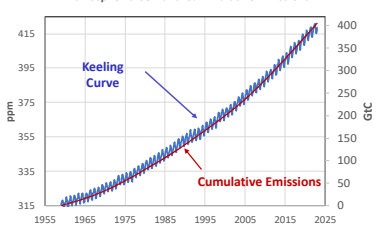
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Negative Emissions

James Hansen has proposed that we must hold the atmospheric carbon dioxide level below 350 ppm if we want a livable climate.



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Hansen et al. 2008

Hansen, J., M. Sato, P. Kharecha, D. Beerling, R. Berner, V. Masson-Delmotte, M. Pagani, M. Raymo, D.L. Royer, and J.C. Zachos. 2008. Target atmospheric CO₂: Where should humanity aim? *Open Atmos. Sci. J.* 2, 217-231. doi:10.2174/1874262300802010217.

Paleoclimate data show that climate sensitivity is ~3°C for doubled CO₂ including only fast feedback processes. Equilibrium sensitivity, including slower surface albedo feedbacks, is ~6°C for doubled CO₂ for the range of climate states between glacial conditions and ice-free Antarctica. Decreasing CO₂ was the main cause of a cooling trend that began 50 million years ago, the planet being nearly ice-free until CO₂ fell to 450±100 ppm; barring prompt policy changes, that critical level will be passed, in the opposite direction, within decades. If humanity wishes to preserve a planet similar to that on which civilization developed and to which life on Earth is adapted, paleoclimate evidence and ongoing climate change suggest that CO₂ will need to be reduced from its current 385 ppm to at most 350 ppm, but likely less than that. The largest uncertainty in the target arises from possible changes of non-CO₂ forcings. An initial 350 ppm CO₂ target may be achievable by phasing out coal use except where CO₂ is captured and adopting agricultural and forestry practices that sequester carbon. If the present overshoot of this target CO₂ is not brief, there is a possibility of seeding irreversible catastrophic effects.

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James Hansen, 2008

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James Hansen, 2008

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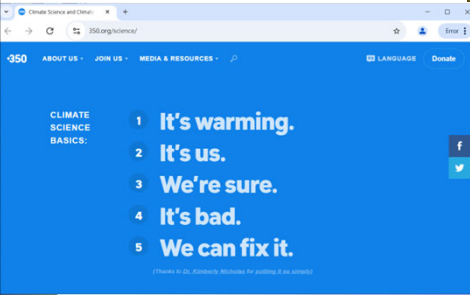


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350.org



Bill McKibben, Founder



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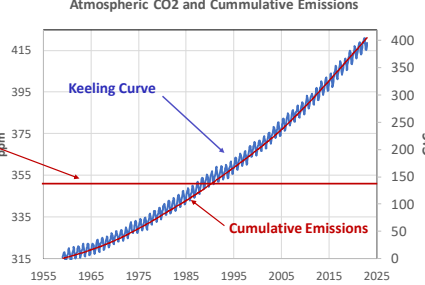
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James Hansen: CO₂ will need to be reduced ... to *at most 350 ppm*, but likely less than that.





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Negative Emissions

Can we remove carbon dioxide from the atmosphere?

Can we get back down to 350 ppm?



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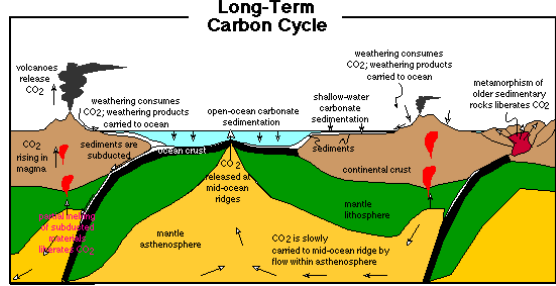
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Negative Emissions

Long-Term Carbon Cycle



http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm



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Long-Term Carbon Cycle

Volcanos emit CO₂.

Silicate weathering carries carbon to the ocean.

Carbon sinks to the bottom of the ocean by the "biological pump" and by the precipitation of calcium carbonate.

The carbon is captured in the sediment.

The sediment is subducted beneath the continental crust by plate tectonics.

Volcanic activity released carbon from the carbonate rocks in the form of CO₂.

Repeat.



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Negative Emissions

Long-Term Carbon Cycle

What's this?

Volcanos emit CO₂.

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Silicate Weathering

Rainwater containing dissolved CO₂ falling on silicate rocks replaces a silicon atom with a carbon atom, ultimately producing calcium carbonate (limestone) and silicon dioxide (quartz). For example, calcium silicate (Wollastonite):

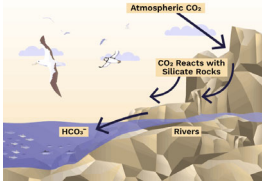
$$CaSiO_3 + CO_2 \rightarrow CaCO_3 + SiO_2$$

Under volcanic conditions, the carbon atom is replaced by a silicon atom, completing the long-term carbon cycle.

$$CaCO_3 + SiO_2 \rightarrow CaSiO_3 + CO_2$$

Can we do this ourselves?

How weathering rocks capture CO₂ from the atmosphere



climatescience.org



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Enhanced Weathering

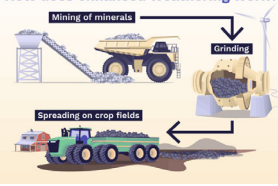
Thunberg, *The Climate Book*, p.237.

"Another drawdown technology is enhanced weathering. This approach tries to accelerate the rate at which rocks such as silicates react naturally with atmospheric CO₂."

"Imagine mining basalts, crushing them and exposing them to the air to react with carbon dioxide."

"You might even fertilize an agricultural field with them, enhancing plant growth ... Cost estimates ... are \$75 - \$250 per tonne of CO₂ removed. ... We know weathering works in nature over thousands of years; the question is whether we can speed it up to a policy-relevant timeframe of years to decades."

How does enhanced weathering work?



climatescience.org



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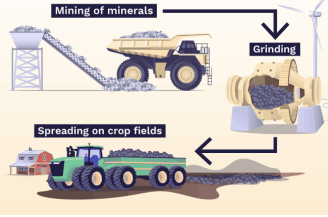
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Enhanced Weathering

Thunberg, *The Climate Book*, p.237.

Cost estimates ... are \$75 - \$250 per tonne of CO₂ removed.

How does enhanced weathering work?



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How much would it cost to remove all the CO₂ emissions using this technology?



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Direct-air Capture

Thunberg, *The Climate Book*, p.238.

"... dozens of companies are working on direct-air capture of carbon dioxide using specialized chemicals."



science.org/content/article/cost-plunges-capturing-carbon-dioxide-air



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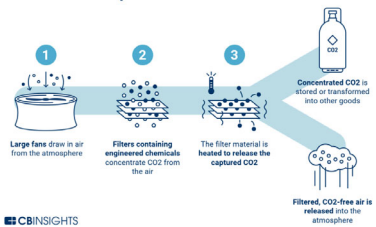
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Direct-air Capture

Thunberg, *The Climate Book*, p.238.

How direct air capture works



CBINSIGHTS

cbinsights.com/research/direct-air-capture-corporate-carbon-reduction/



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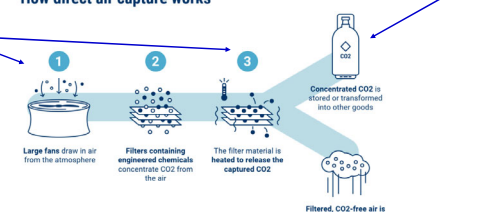
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Where do we get the energy for these?

How direct air capture works

What do we do with this?



cbinsights.com/research/direct-air-capture-corporate-carbon-reduction/

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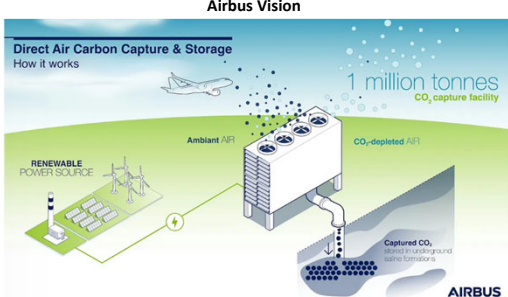


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Direct-air Capture

Airbus Vision



airbus.com/en/newsroom/stories/2022-07-direct-air-carbon-capture-and-storage-for-aviation-explained

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Negative Emissions

Direct-air Capture

Thunberg, *The Climate Book*, p.238.

"The current cost range for direct-air capture is around \$250 - \$600 per tonne of CO2 removed."

How much would it cost to remove all the CO2 emissions using this technology?



science.org/content/article/cost-plunges-capturing-carbon-dioxide-air

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Negative Emissions

Carbon Capture and Storage (CCS)

Thunberg, *The Climate Book*, p.236.

"The atmosphere contains about one molecule of carbon dioxide for every 2,500 molecules of other gases, which makes finding and 'removing' carbon dioxide for the atmosphere like pulling needles repeatedly from a haystack. About one in ten molecules emitted by a typical fossil power plant smokestack is carbon dioxide, so it makes no sense for these smokestacks to keep releasing concentrated carbon dioxide into the air and then for us to pay to remove it in diluted form later. Whenever we keep burning fossil fuels, we need to capture the carbon pollution from smokestacks now, before it pollutes the air."

"Currently, there are only thirty or so carbon capture and storage (CCS) plants running worldwide."

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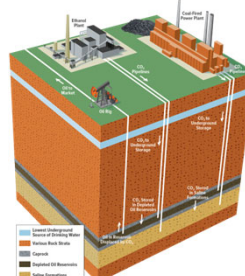


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Carbon Capture and Storage (CCS)

Thunberg, *The Climate Book*, p.236.



Downsides

1. Adds cost to electricity production.
2. CO2 leaks cause inefficiencies.
3. CO2 spills can kill people.

Upsides

1. Storage in silicate-rich strata can result in permanent conversion of CO2 to carbonate rocks.
2. Cost is less than building machines to suck CO2 out of the air.

en.wikipedia.org/wiki/Carbon_capture_and_storage

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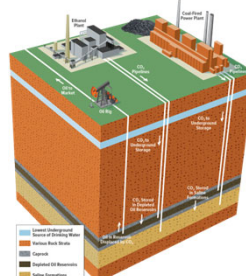


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Negative Emissions

Carbon Capture and Storage (CCS)

Thunberg, *The Climate Book*, p.236.



Major Downside

We can be lulled into thinking that using CCS means that we can go merrily on our way, continuing to use fossil fuels.

On the other hand,

increasing the cost of using fossil fuels may lead to accelerated adoption of solar and wind energy.

en.wikipedia.org/wiki/Carbon_capture_and_storage

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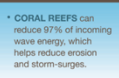
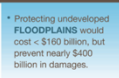
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Natural Climate Solutions

Thunberg, *The Climate Book*, p.236.

Nature Improves Resilience to Climate Change



• CORAL REEFS can reduce 97% of incoming wave energy, which helps reduce erosion and storm-surges. • 1 acre of WETLANDS stores 1-1.5 million gallons of floodwater.	• Protecting undeveloped FLOODPLAINS would cost < \$160 billion, but prevent nearly \$400 billion in damages.	• Ecological FOREST management can protect drinking water supplies and mitigate wildfire risk. • Over 1/2 of the nation's water supply comes from FORESTS.	• URBAN AND COMMUNITY TREES reduce over 7% of residential energy use. Urban trees and green spaces absorb stormwater and provide habitat for wildlife.
 Coastal Ecosystems	 Floodplains	 Forests	 Urban Forests

blog.wur.org/2020/08/natural-climate-solutions-can-store-carbon-and-enhance-resilience-to-climate-change/



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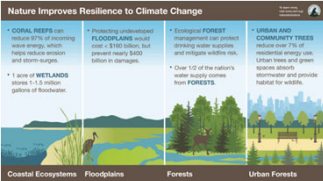
Negative Emissions

Natural Climate Solutions

Thunberg, *The Climate Book*, p.237.

Problems

1. These natural process were in equilibrium in the pre-industrial age. We have perturbed them far from equilibrium, so we cannot expect them to bring us back to a livable climate.
2. These systems cannot keep up with the rate at which we are adding carbon.



Nature Improves Resilience to Climate Change

COASTAL ECOSYSTEMS
CORAL REEFS can reduce 87% of incoming wave energy, offset erosion, and storm surges.
* 1 acre of mangroves stores 1.5 million gallons of floodwater.

FLOODPLAINS
Preserving undisturbed FLOODPLAINS would prevent 12.5 billion dollars in damages.

FORESTS
Ecosystem FOREST management can prevent water scarcity, reduce drought, and wildfire.
Over 1/3 of the nation's water supply comes from FORESTS.

URBAN AND COMMUNITY TREES
Indicate 75% of urban forest canopy. Urban trees and green spaces directly store carbon, improve air quality, and provide habitat for wildlife.

Coastal Ecosystems **Floodplains** **Forests** **Urban Forests**

blog.nwf.org/2020/08/natural-climate-solutions-can-store-carbon-and-enhance-resilience-to-climate-change/



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The diagram illustrates the BECCS (Biomass Energy with Carbon Capture and Storage) process. It starts with a sun icon and a tree icon labeled 'Biomass'. A red arrow labeled 'CO₂ sequestered' points from the sun to the biomass. A black arrow points from the biomass to a central circle labeled 'BECCS'. From the 'BECCS' circle, a red arrow labeled 'CO₂ emissions' points to a cloud icon labeled 'CO₂'. Another red arrow labeled 'CO₂ permanently stored' points from the 'BECCS' circle to a 'CCS' box, which then points to a 'Geological reservoir' icon. A black arrow points from the 'BECCS' circle to a power plant icon.

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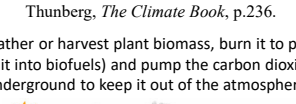
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Bioenergy with Carbon Capture and Storage (BECCS)

Thunberg, *The Climate Book*, p.236.

“In BECCS, you gather or harvest plant biomass, burn it to produce electricity (or convert it into biofuels) and pump the carbon dioxide pollution underground to keep it out of the atmosphere.”



[sciencedirect.com/science/article/pii/S0961953422000678](https://www.sciencedirect.com/science/article/pii/S0961953422000678)


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Negative Emissions

Help From the Trees

Each spring and summer the biological activity draws down more CO_2 than we add by burning fossil fuels.

Mauna Loa

CO_2 ppm

year

Year	CO_2 ppm
1955	315
1965	320
1975	330
1985	345
1995	360
2005	375
2015	395
2025	420

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Negative Emissions

Help From the Trees

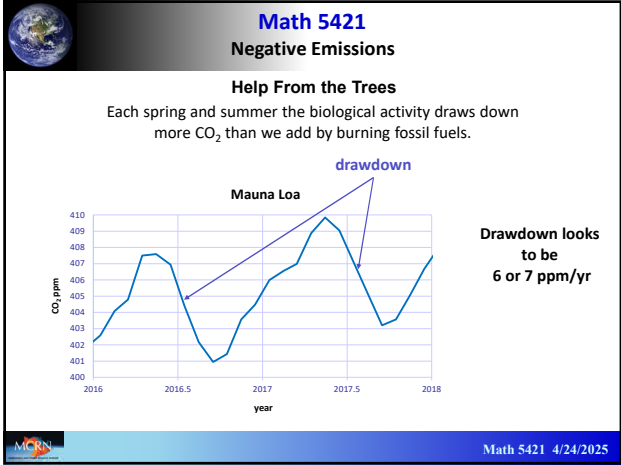
Each spring and summer the biological activity draws down more CO_2 than we add by burning fossil fuels.

The graph illustrates the increasing concentration of CO_2 in the atmosphere over time. The y-axis represents CO_2 in ppm, ranging from 380 to 420. The x-axis represents the year, from 2010 to 2020. The data shows a consistent upward trend, with seasonal fluctuations visible in the annual average line. A red line labeled 'drawdown' points to the end of the graph in 2020.

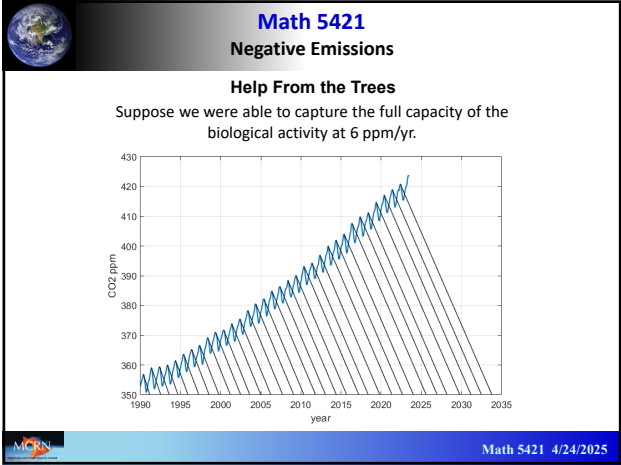
Year	CO_2 (ppm)
2010	388
2011	392
2012	390
2013	395
2014	390
2015	398
2016	402
2017	405
2018	408
2019	412
2020	415

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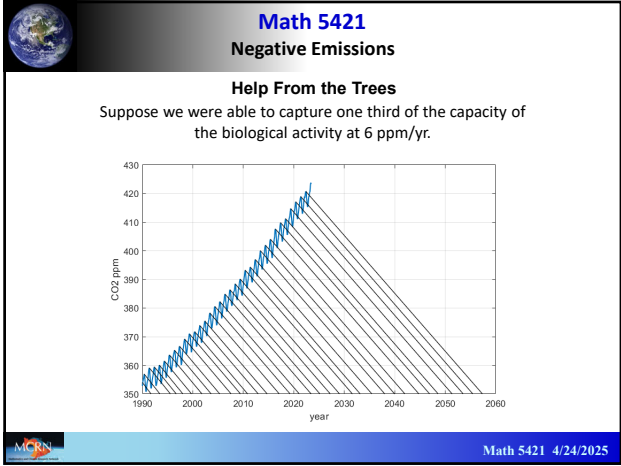
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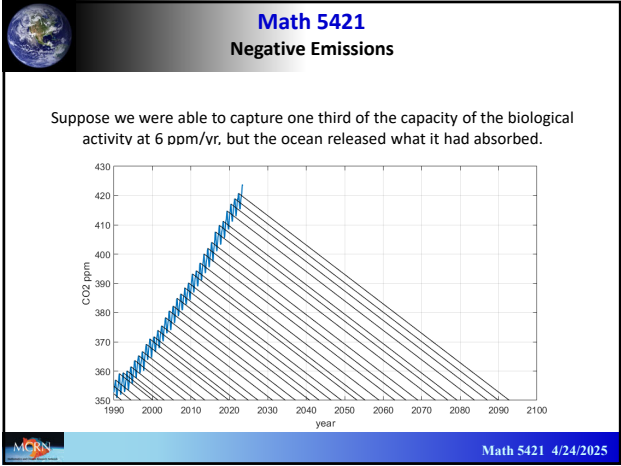
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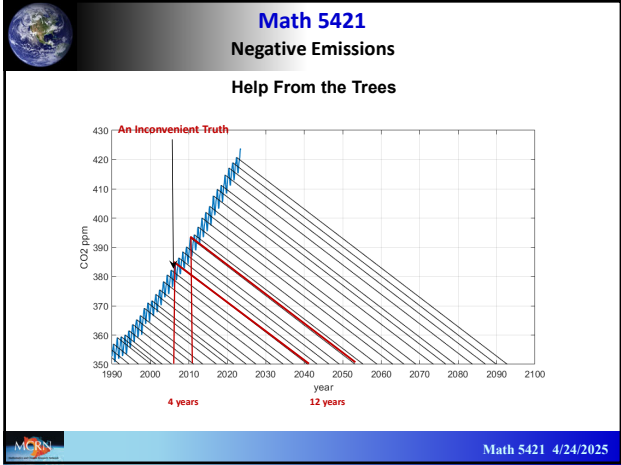
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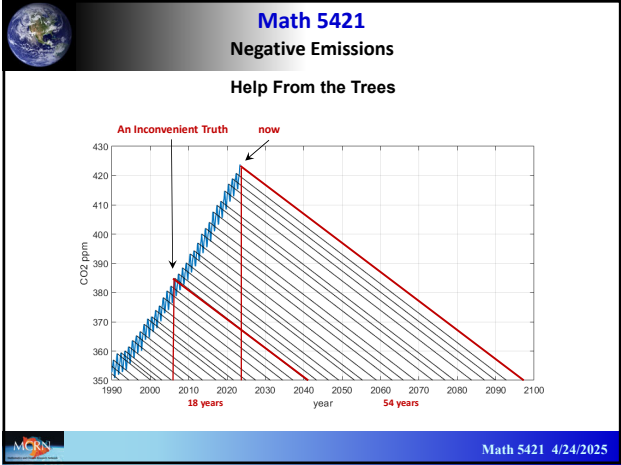
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Negative Emissions

Can we remove carbon dioxide from the atmosphere?
Yes, but there is no magic bullet; all solutions are expensive.

Can we get back down to 350 ppm?
Yes, but it will take time and effort.

What should we do first?
Stop burning fossil fuels.



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