Math 5421

An Introduction to Mathematical Climate Models

Spring 2025
1:25 – 3:20 Tuesdays and Thursdays
Blegen Hall 155

Richard McGehee, Instructor
458 Vincent Hall
mcgehee@umn.edu
www-users.cse.umn.edu/~mcgehee/

course website
https://www-users.cse.umn.edu/~mcgehee/Course/Math5421/

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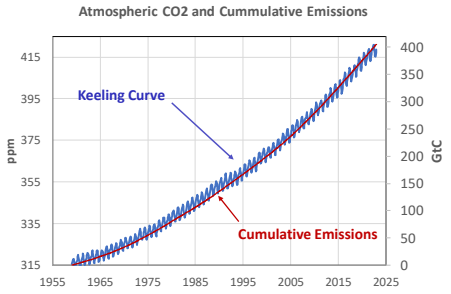
Negative Emissions

Since 1959 we have emitted more than 400 GtC into the atmosphere. During that time the carbon dioxide has increased by 105 ppm .



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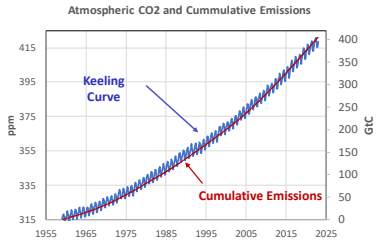
Negative Emissions

James Hansen has proposed that we must hold the atmospheric carbon dioxide level below 350 ppm if we want a livable climate.



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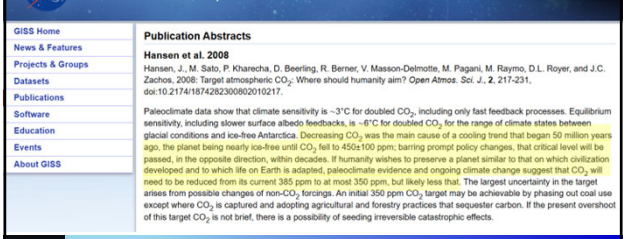
Negative Emissions



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Negative Emissions

James Hansen, 2008

Decreasing CO2 was the main cause of a cooling trend that began 50 million years ago, the planet being nearly ice-free until CO2 fell to 450 ± 100 ppm; barring prompt policy changes, that critical level will be passed, in the opposite direction, within decades. If humanity wishes to preserve a planet similar to that on which civilization developed and to which life on Earth is adapted, paleoclimate evidence and ongoing climate change suggest that CO2 will need to be reduced from its current 385 ppm to ***at most 350 ppm***, but likely less than that.

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Negative Emissions

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Negative Emissions

James Hansen, 2008

Paleoclimate data show that climate sensitivity is ~3°C for doubled CO2, including only fast feedback processes. Equilibrium sensitivity, including slower surface albedo feedbacks, is ~6°C for doubled CO2 for the range of climate states between glacial conditions and ice-free Antarctica.

If the present overshoot of this target CO2 is not brief, there is a possibility of seeding irreversible catastrophic effects.

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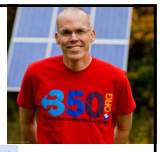
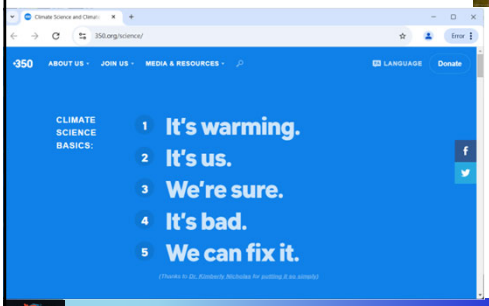


350.org



Bill McKibben, Founder

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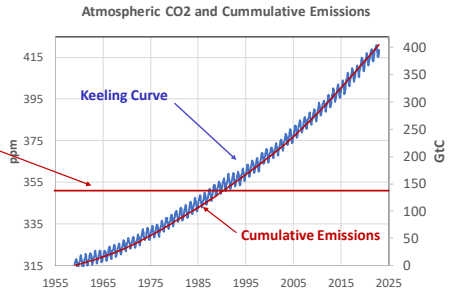
James Hansen: CO2 will need to be reduced … to ***at most 350 ppm***, but likely less than that.



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Can we remove carbon dioxide from the atmosphere?

Can we get back down to 350 ppm?

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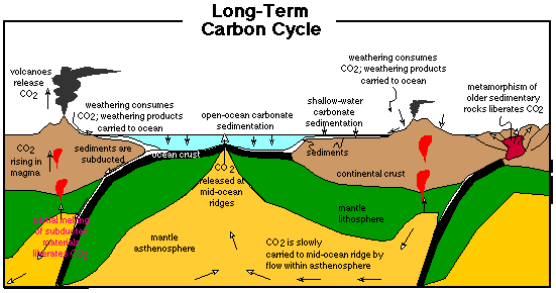
Negative Emissions



http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm

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Negative Emissions

Long-Term Carbon Cycle

Volcanos emit CO_2.

Silicate weathering carries carbon to the ocean.

Carbon sinks to the bottom of the ocean by the "biological pump" and by the precipitation of calcium carbonate.

The carbon is captured in the sediment.

The sediment is subducted beneath the continental crust by plate tectonics.

Volcanic activity released carbon from the carbonate rocks in the form of CO_2.

Repeat.

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Long-Term Carbon Cycle

What's this?

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Repeat.

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Silicate Weathering

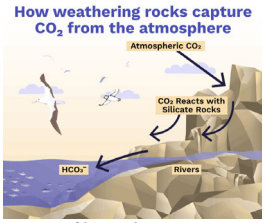
Rainwater containing dissolved CO_2 falling on silicate rocks replaces a silicon atom with a carbon atom, ultimately producing calcium carbonate (limestone) and silicon dioxide (quartz). For example, calcium silicate (Wollastonite):

$$CaSiO_3 + CO_2 \rightarrow CaCO_3 + SiO_2$$

Under volcanic conditions, the carbon atom is replaced by a silicon atom, completing the long-term carbon cycle.

$$CaCO_3 + SiO_2 \rightarrow CaSiO_3 + CO_2$$

Can we do this ourselves?



climatescience.org

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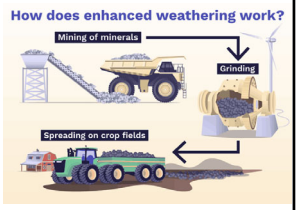
Enhanced Weathering

Thunberg, *The Climate Book*, p.237.

"Another drawdown technology is enhanced weathering. This approach tries to accelerate the rate at which rocks such as silicates react naturally with atmospheric CO_2."

"Imagine mining basalts, crushing them and exposing them to the air to react with carbon dioxide."

"You might even fertilize an agricultural field with them, enhancing plant growth … . Cost estimates … are $75 - $250 per tonne of CO_2 removed. … We know weathering works in nature over thousands of years; the question is whether we can speed it up to a policy-relevant timeframe of years to decades."



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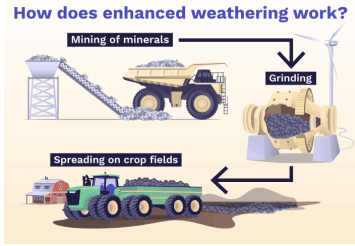
Enhanced Weathering

Thunberg, *The Climate Book*, p.237.

Cost estimates … are $75 - $250 per tonne of CO_2 removed.

How much would it cost to remove all the CO_2 emissions using this technology?



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Direct-air Capture

Thunberg, *The Climate Book*, p.238.

"… dozens of companies are working on direct-air capture of carbon dioxide using specialized chemicals."

science.org/content/article/cost-plunges-capturing-carbon-dioxide-air

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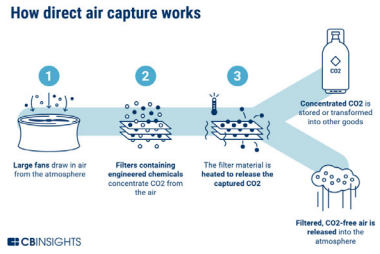
Direct-air Capture

Thunberg, *The Climate Book*, p.238.



cbinsights.com/research/direct-air-capture-corporate-carbon-reduction/

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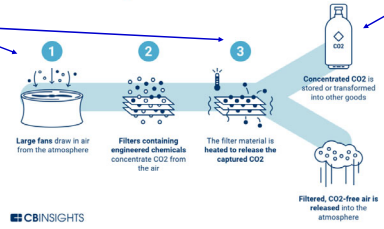
Where do we get the energy for these?

How direct air capture works

What do we do with this?



cbinsights.com/research/direct-air-capture-corporate-carbon-reduction/

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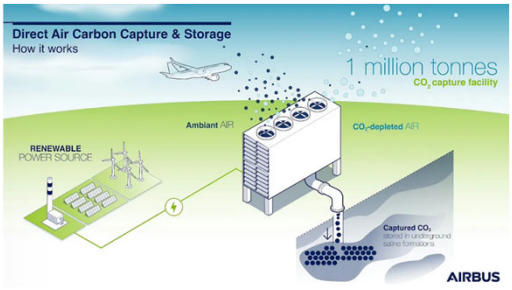
Direct-air Capture

Airbus Vision



airbus.com/en/newsroom/stories/2022-07-direct-air-carbon-capture-and-storage-for-aviation-explained

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Direct-air Capture

Thunberg, *The Climate Book*, p.238.

"The current cost range for direct-air capture is around $250 - $600 per tonne of CO2 removed."

How much would it cost to remove all the CO_2 emissions using this technology?

science.org/content/article/cost-plunges-capturing-carbon-dioxide-air

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Carbon Capture and Storage (CCS)

Thunberg, *The Climate Book*, p.236.

"The atmosphere contains about one molecule of carbon dioxide for every 2,500 molecules of other gases, which makes finding and 'removing' carbon dioxide for the atmosphere like pulling needles repeatedly from a haystack. About one in ten molecules emitted by a typical fossil power plant smokestack is carbon dioxide, so it makes no sense for these smokestacks to keep releasing concentrated carbon dioxide into the air and then for us to pay to remove it in diluted form later. Whenever we keep burning fossil fuels, we need to capture the carbon pollution form smokestacks now, before it pollutes the air."

"Currently, there are only thirty or so carbon capture and storage (**CCS**) plants running worldwide."

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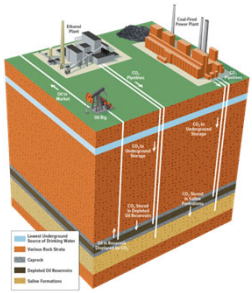
Carbon Capture and Storage (CCS)

Thunberg, *The Climate Book*, p.236.



en.wikipedia.org/wiki/Carbon_capture_and_storage

Downsides

1. Adds cost to electricity production.
2. CO_2 leaks cause inefficiencies.
3. CO_2 spills can kill people.

Upsides

1. Storage in silicate-rich strata can result in permanent conversion of CO_2 to carbonate rocks.
2. Cost is less than building machines to suck CO_2 out of the air.

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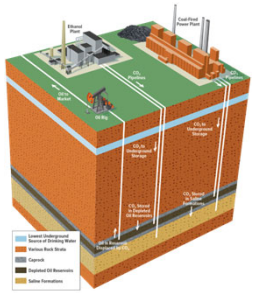
Carbon Capture and Storage (CCS)

Thunberg, *The Climate Book*, p.236.



en.wikipedia.org/wiki/Carbon_capture_and_storage

Major Downside

We can be lulled into thinking that using CCS means that we can go merrily on our way, continuing to use fossil fuels.

On the other hand,

increasing the cost of using fossil fuels may lead to accelerated adoption of solar and wind energy.

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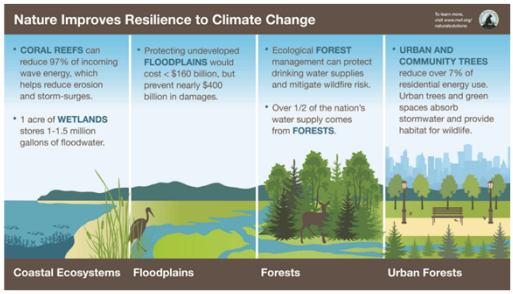
Natural Climate Solutions

Thunberg, *The Climate Book*, p.236.



blog.nwf.org/2020/08/natural-climate-solutions-can-store-carbon-and-enhance-resilience-to-climate-change/

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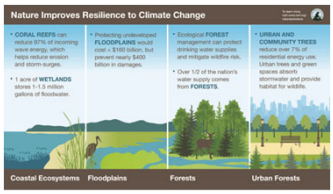
Natural Climate Solutions

Thunberg, *The Climate Book*, p.237.

Problems

1. These natural process were in equilibrium in the pre-industrial age. We have perturbed them far from equilibrium, so we cannot expect them to bring us back to a livable climate.

2. These systems cannot keep up with the rate at which we are adding carbon.



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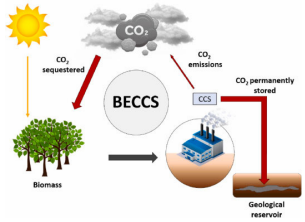
Bioenergy with Carbon Capture and Storage (BECCS)

Thunberg, *The Climate Book*, p.236.

"In BECCS, you gather or harvest plant biomass, burn it to produce electricity (or convert it into biofuels) and pump the carbon dioxide pollution underground to keep it out of the atmosphere."



sciencedirect.com/science/article/pii/S0961953422000678

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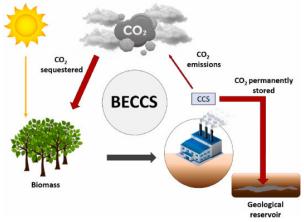

sciencedirect.com/science/article/pii/S0961953422000678

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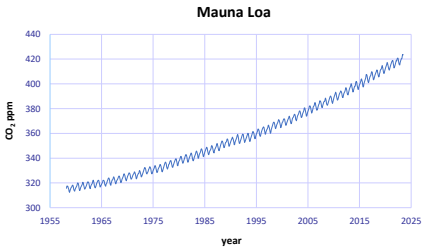
Negative Emissions

Help From the Trees

Each spring and summer the biological activity draws down more CO_2 than we add by burning fossil fuels.



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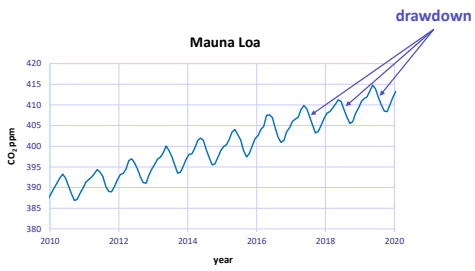

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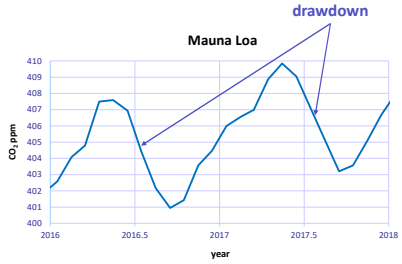

Drawdown looks to be 6 or 7 ppm/yr

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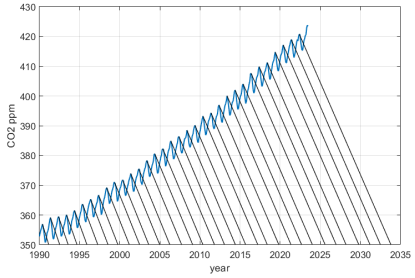
Negative Emissions

Help From the Trees

Suppose we were able to capture the full capacity of the biological activity at 6 ppm/yr.



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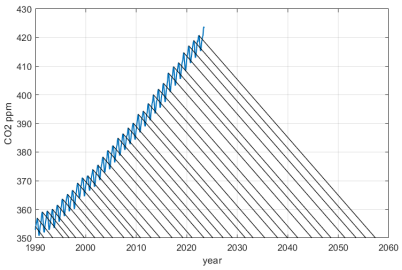
Help From the Trees

Suppose we were able to capture one third of the capacity of the biological activity at 6 ppm/yr.



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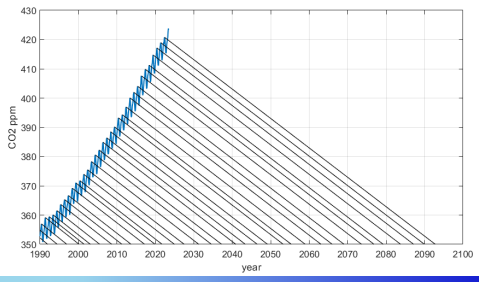
Suppose we were able to capture one third of the capacity of the biological activity at 6 ppm/yr, but the ocean released what it had absorbed.



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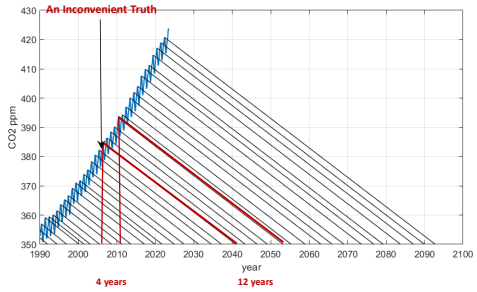
Help From the Trees



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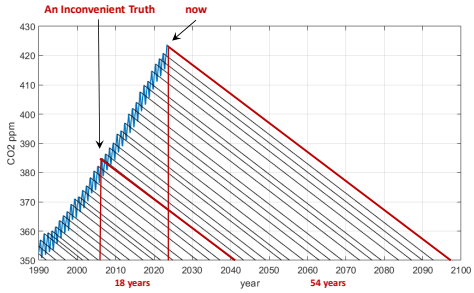

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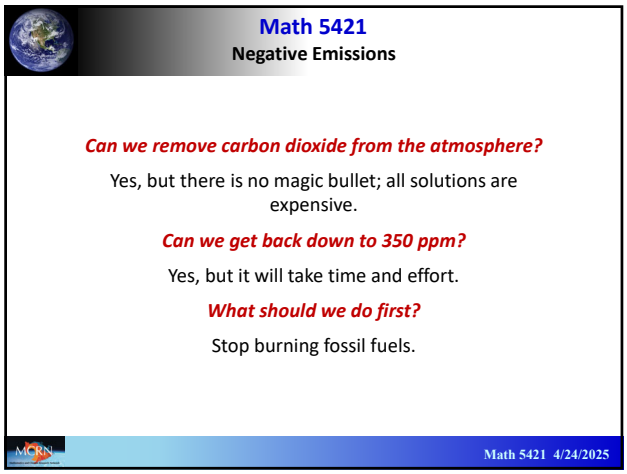
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Negative Emissions
Can we remove carbon dioxide from the atmosphere?
Yes, but there is no magic bullet; all solutions are expensive.
Can we get back down to 350 ppm?
Yes, but it will take time and effort.
What should we do first?
Stop burning fossil fuels.
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