Glacial Cycles

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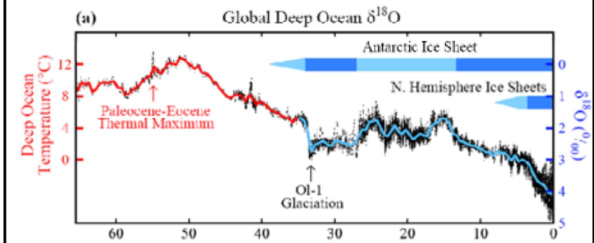
Seminar on the Mathematics of Climate Change
School of Mathematics
October 7, 2009

Glacial Cycles

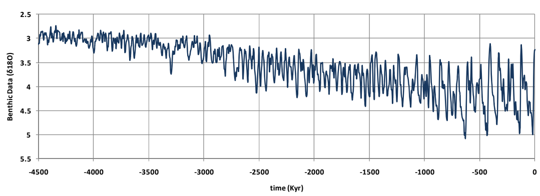
Temperatures in the Cenozoic Era



Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

Glacial Cycles

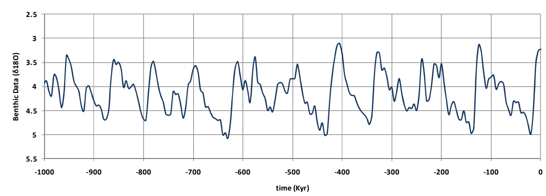
^{18}O in Foraminifera Fossils During the Past 4.5 Myr



Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

Glacial Cycles

^{18}O in Foraminifera Fossils During the Past 1.0 Myr

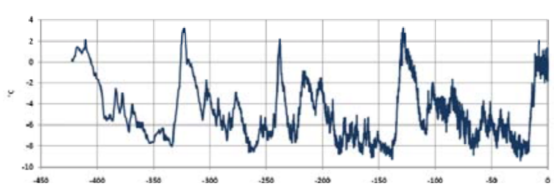


Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

Glacial Cycles

Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data



J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* **399**, 429-436.

Glacial Cycles

What Causes Glacial Cycles?

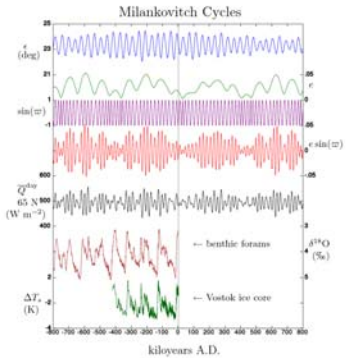
Widely Accepted Hypothesis

The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work is not fully understood.

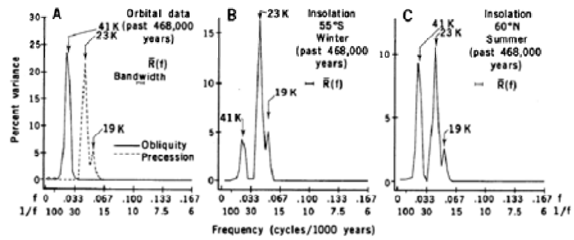
Glacial Cycles



http://en.wikipedia.org/wiki/Milankovitch_cycles

Glacial Cycles

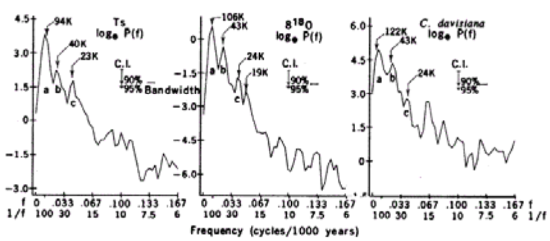
Solar Forcing (Hays, et al)



Hays, *et al*, *Science* **194** (1976), p. 1125

Glacial Cycles

Climate Response, Hays, et al

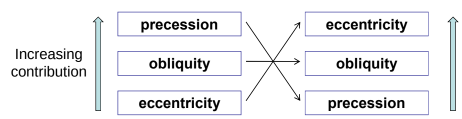


Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1127

Glacial Cycles

Hays Summary



The explanation is that there are nonlinear feedbacks.

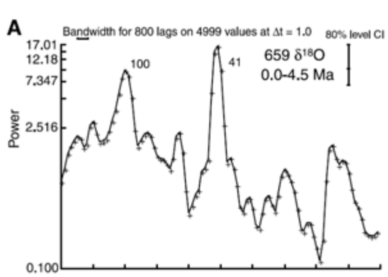
Is there another explanation?

Hays, *et al*, *Science* **194** (1976), p. 1127

Glacial Cycles

Climate Response (Zachos, et al)

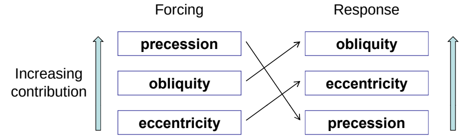
Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.



Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Zachos Summary



Nonlinear effects?

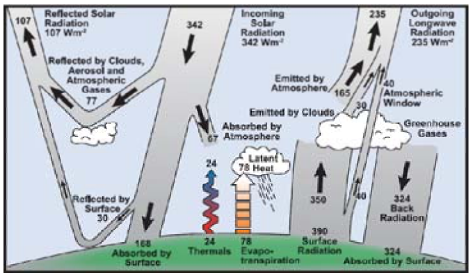
Forcing is defined as the maximum insolation at latitude 65° N.

Is there another definition of forcing?

Zachos, *et al*, *Science* **292** (2001), p. 689

Glacial Cycles

Heat Balance



Historical Overview of Climate Change Science, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

Glacial Cycles

Ice Albedo Feedback Model

$$R\frac{\partial T}{\partial t} = \underbrace{Qs(y)}_{\text{insolation}}\big(1-\underbrace{\alpha(y,\eta)}_{\text{albedo}}\big) - \underbrace{(A+BT)}_{\text{outward radiation}} + \underbrace{C(\overline{T}-T)}_{\text{heat transport}}$$

y = sine of latitude
$T(y)$ = annual mean temperature
$Qs(y)$ = annual mean insolation
Q = global annual mean insolation

This equation has a stable equilibrium consisting of polar ice caps.

The latitude of the equilibrium ice boundary and the equilibrium global annual mean temperature are functions of the parameters.

K.K. Tung, *Topics in Mathematical Modeling*, Princeton (2007), Chapt 8

Glacial Cycles

Ice Albedo Feedback Model

$$R\frac{\partial T}{\partial t} = \boxed{Qs(y)}\big(1-\alpha(y,\eta)\big) - (A+BT) + C(\overline{T}-T)$$

Idea

Instead of solar forcing (maximum insolation at 65 N latitude), use the global annual mean temperature predicted by the model.

Using Kepler's Laws, we can compute:

$$Q = \frac{Ka^2}{\sqrt{1-e^2}}$$

$$s(y) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\cos\gamma - y\cos\beta\right)^2}\,d\gamma$$

a = semimajor axis
e = eccentricity
β = obliquity

Glacial Cycles

Ice Albedo Feedback Model

$$Q = \frac{Ka^2}{\sqrt{1-e^2}}$$

$$s(y) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\cos\gamma - y\cos\beta\right)^2}\,d\gamma$$

Note that Q, the global annual mean insolation depends only on the semimajor axis and the eccentricity.

Note that $s(y)$, the insolation distribution by latitude, depends only on the obliquity.

Note that the effect due to precession disappears when averaged over a year.

Glacial Cycles

Global Annual Average Insolation

$$Q = \frac{Ka^2}{\sqrt{1-e^2}}$$

Laskar:

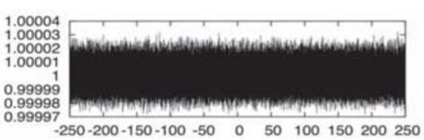


Fig. 11. Variation of the semi-major axis of the Earth–Moon barycenter (in AU) from −250 to +250 Myr.

Semi major axis does not change much:
.005% corresponding to .01% change in global average insolation

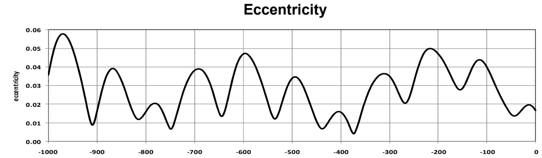
J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

Glacial Cycles

Global Annual Average Insolation

$$Q = \frac{Ka^2}{\sqrt{1-e^2}}$$

Eccentricity



Note periods of about 100 Kyr and 400 Kyr.

As e varies between 0 and 0.06, $(1-e^2)^{-1/2}$ varies between 1 and 0.0018, or about 0.2%. (Twenty times the effect due to a.)

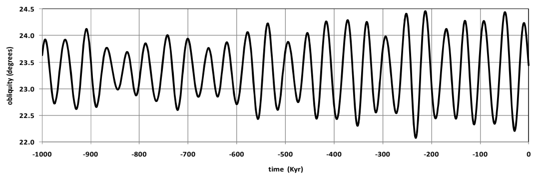
J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

Glacial Cycles

Relative Insolation Function

$$s(y)=\frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\cos\gamma-y\cos\beta\right)^2}\,d\gamma$$

Obliquity



Note period of about 41 Kyr.

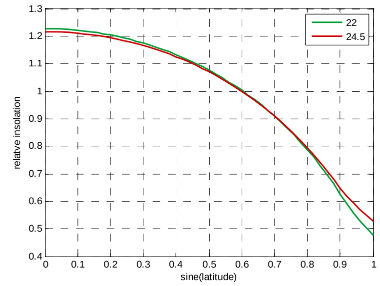
J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

Glacial Cycles

Relative Insolation Function

green = obliquity of 22.0°

red = obliquity of 24.5°



Glacial Cycles

Annual Mean Insolation
(as a function of latitude)

$$Qs(y)$$

y = sine of latitude

Q = global annual mean insolation
(depends primarily on eccentricity)

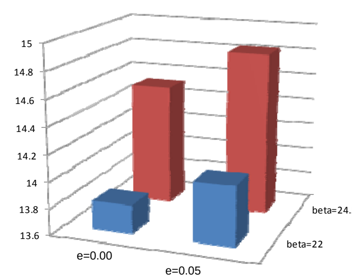
$s(y)$ = relative insolation as a function of latitude
(depends only on obliquity)

We can use the ice-albedo feedback model to compute the equilibrium ice line, the global mean temperature, and the polar mean temperature as functions of eccentricity and obliquity.

Glacial Cycles

Global Annual Mean Temperature

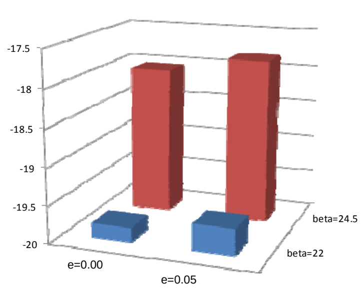
Computed global mean temperature for extremes in eccentricity and obliquity



Glacial Cycles

Polar Annual Mean Temperature

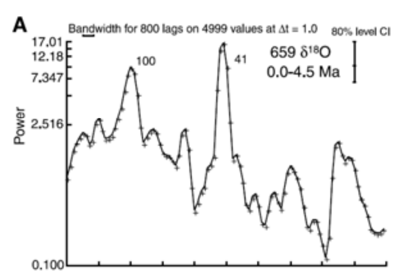
Computed polar mean temperature for extremes in eccentricity and obliquity



Glacial Cycles

Conclusions

1. Precession doesn't matter.
2. Obliquity is more important than eccentricity.
3. Polar temperatures vary twice as much as global temperatures.



Glacial Cycles

Conclusions

	Model Prediction		Response	
Increasing contribution ↑	**obliquity**	→	**obliquity**	↑
	eccentricity	→	**eccentricity**	
	precession	→	**precession**	

When the usual definition of forcing is replaced by the predictions of the ice-albedo feedback model, the relative effects due to the Milankovitch cycles agree with the observed data.

Glacial Cycles

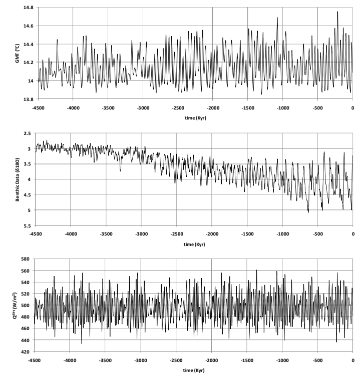
Annual Global Mean Temperature

The data are not duplicated by either method, but note the spectral similarities.

ice-albedo model

data

Insolation at 65° N at summer solstice



Glacial Cycles

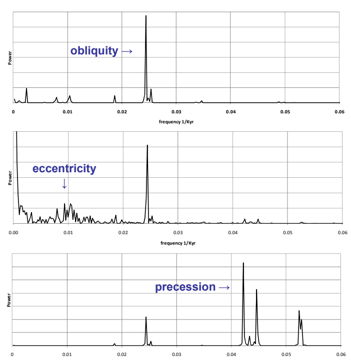
Power Spectra

Note dominance of the obliquity signal.

ice-albedo model

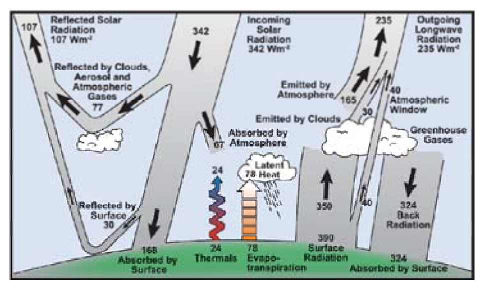
data

Insolation at 65° N at summer solstice



Climate and the Earth's Glacial Cycles

Heat Balance



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`http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf`

Climate and the Earth's Glacial Cycles

Not Explained By Ice Albedo Feedback

$$R\frac{\partial T}{\partial t} = \underbrace{Qs(y)}_{\text{insolation}}\underbrace{\left(1-\alpha(y,\eta)\right)}_{\text{albedo}} - \underbrace{(A+BT)}_{\text{outward radiation}} + \underbrace{C\left(\bar{T}-T\right)}_{\text{heat transport}}$$

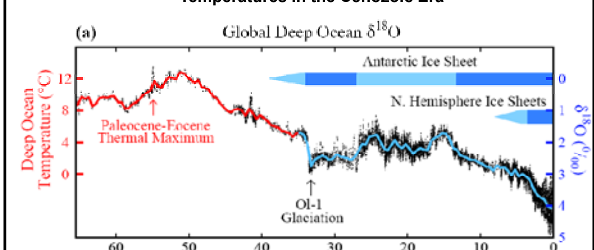
The observed *amplitude* of temperature variation is about 5 times higher than that predicted by the model.

Other feedback mechanisms (e.g. greenhouse gases) clearly matter.

The long-term trends are also not in the model and are not explained by Milankovitch cycles.

Glacial Cycles

Temperatures in the Cenozoic Era



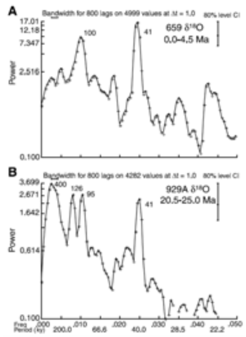
Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

Glacial Cycles

More from Zachos

A. Power spectrum of climate for the last 4.5 Myr. Note the peaks at 41Kyr and 100 Kyr.

B. Power spectrum of climate for the period 25 Myr bp to 20.5 Myr bp. Note the new peak at 400 Kyr and the "split" peaks at 126Kyr and 95 Kyr.



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