Milankovitch cycles as viewed through Budyko-Sellers' model during the Eocene

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UMN Climate Change Seminar

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Mathematics and Climate Research Network

Introduction

Goal:
To analyse the possible average mean temperature profiles for a non-symmetric planet with no ice.

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Process:
We will use the average mean temperature that can be calculated at equilibrium for a non-symmetric Budyko model.
We'll talk about non-symmetric first Budyko first.

Finding Equilibrium

As calculated by several smart folks before me we have the following equilibrium solution:

$$T^* = \frac{1}{B+C}(Q(e)s(y,\beta)(1-\alpha(y)) - A + C\overline{T}) \tag{1}$$

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We will be using the following definitions:

$$\begin{aligned} s(y,\beta) &= 1 + s_2(\beta)p_2(y) \\ s_2(\beta) &= \tfrac{15}{6} * (-2 + 3\sin^2\beta) \\ p_2(y) &= \tfrac{1}{2}(3y^2 - 1) \end{aligned}$$

Where e is eccentricity, β is obliquity, $Q(e)$ is the insolation as a function of eccentricity, α is the albedo function and $s(y)$ is the distribution of $Q(e)$ across all latitudes. Thus we need $\int s(y)dy = 1$. $A, B, C \in \mathbb{R}$ and found through modelling and data fitting.

Finding Equilibrium

Under the usual construction of Budyko,

$$\int_0^1 s(y)dy = 1.$$

We will be using the same $s(y)$ but in this case $y \in (-1, 1)$. Thus we can either think about redefining a measure $d\mu = \frac{1}{2}dy$ or we may think about redefining $s(y)$ to be half it's original value and $\int_{-1}^{1} T = \overline{T}$.

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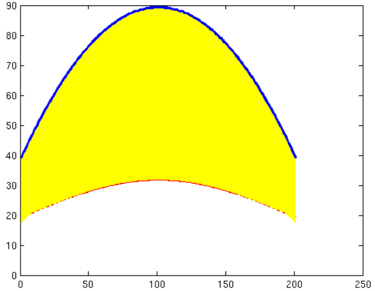
Thus when we consider equation 1, we see that we must integrate both sides against latitude and solve for $\overline{T}^*$. For now we will assume that α is constant. We arrive at:

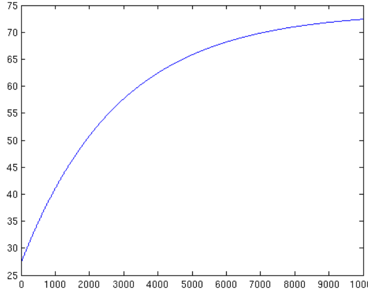
$$\overline{T}^* = \frac{Q(1 - \overline{\alpha}) - A}{B}$$

where $\overline{\alpha} = \frac{1}{2}\int_{-1}^{1} s(y)\alpha(y)dy$. This is the equation we will use to find the average equilibrium temperature means.

The $\alpha = 0$ case

We begin with $\alpha = 0$. So we have painted the Earth black and made it completely absorbing. For $Q = 343$, $A = 202$, and $B = 1.9$ which implies that $\overline{T}* \approx 74.2C$. If I apply a dynamic solver to the equations, then I can see the actual temperature profile over time. I can then take the average of each of the time steps to see what happens in the long run to the globally averaged temperature.

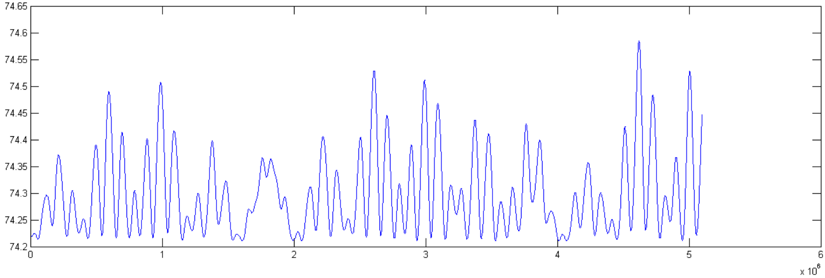




The $\alpha = 0$ case

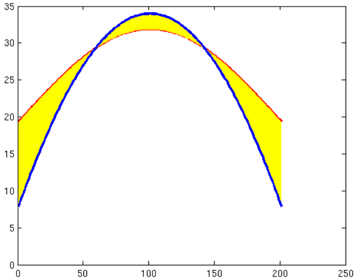
We can now view the equilibrium temperature as function of obliquity, β, and eccentricity, e.

$$\overline{T}^* = \frac{Q(e)(1 - \overline{\alpha(\beta)}) - A}{B}$$



The $\alpha = 30\%$ case

For Budyko integration in Matlab with a constant albedo function of 0.3, we use an ice-free, symmetric Earth to arrive at the following dynamic solution.
Red line = initial.
Blue line= "equilibrium" solution.
Yellow = intermediary temperature profiles.



The $\alpha = 30\%$ case

For $\alpha = 0.3$, $Q = 343$, $A = 202$, and $B = 1.9$ we have that

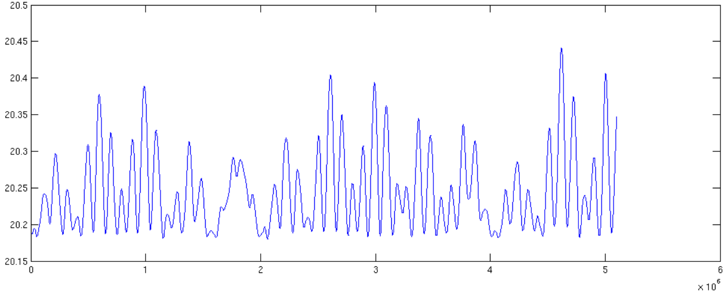
$$\overline{T}^* \approx 20.05C.$$

When we use matlab to compute $\overline{T}$ for the last computed temperature profile (at time step $100,000$) we have

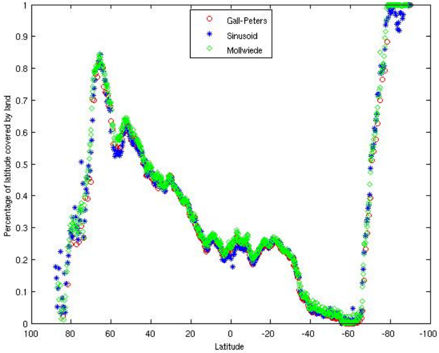
$$\overline{T}_{100,000} \approx 19.97C.$$

The $\alpha = 30\%$ case

Here are the $\overline{T}^*$ values over the last 5 mya.

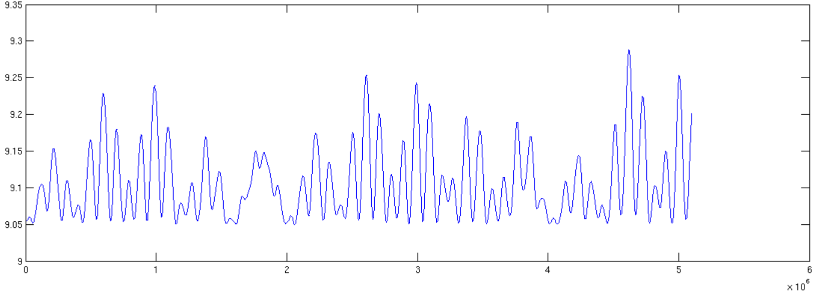


Lat vs Land Albedo

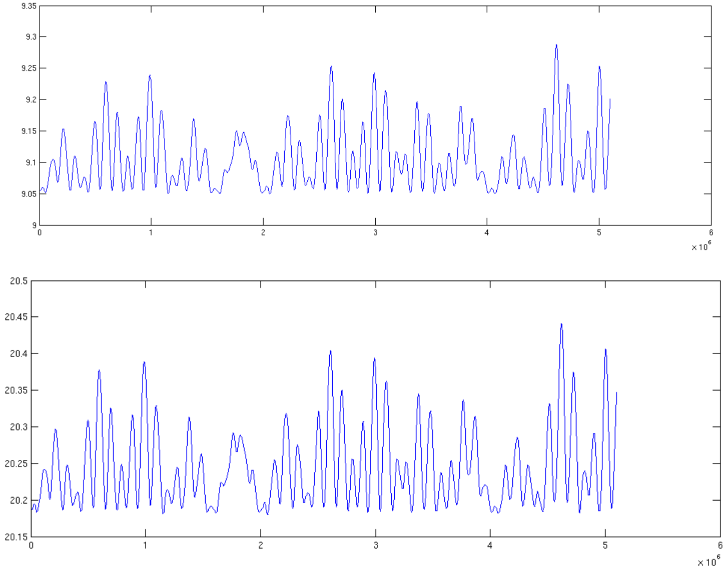


We can view $\alpha = \alpha(y)$. Then $\overline{\alpha} = \frac{1}{2}\int_{-1}^{1} s(y)\alpha(y)dy$. integrates the y out of the equation, leaving $\overline{\alpha} = \alpha(\beta)$ because $s(y)$ also depends on β. Using an approximation of this graph of α in the $\overline{T}^*$ equation we find...

Lat vs Land Albedo

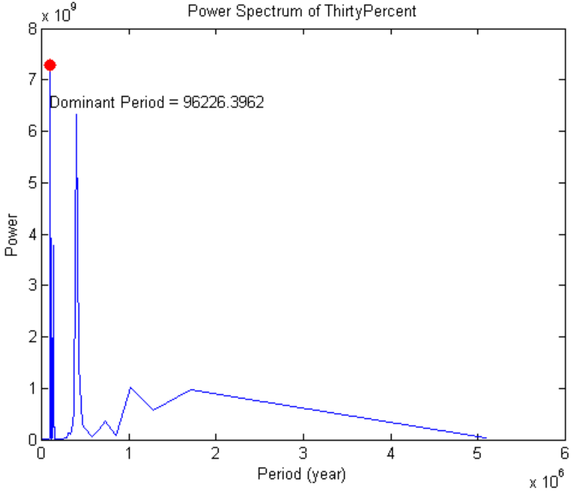


Top: Land vs Lat Albedo Function.
Bottom: Constant 30% Albedo Function.



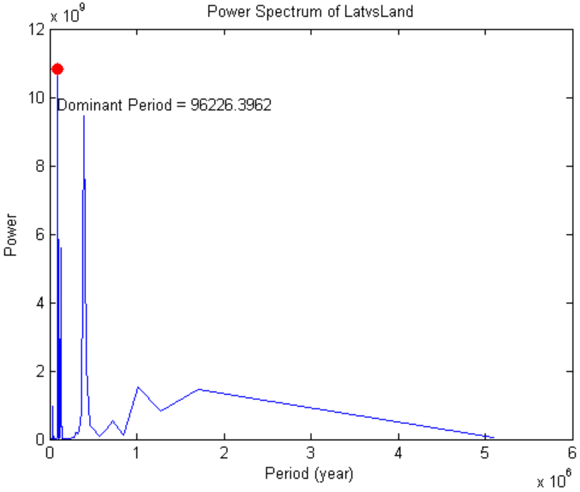
Power Spectrums

This is the constant 30% albedo ($\alpha = 0.3$) power spectrum.

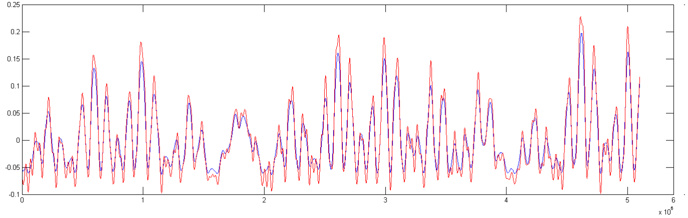


Power Spectrums

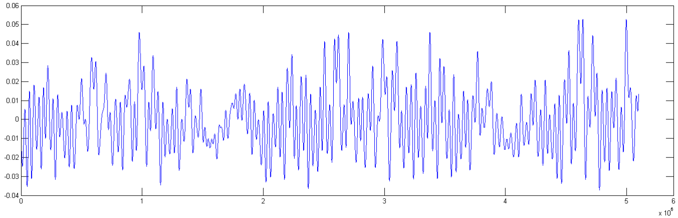
This is the Lat vs Land power spectrum.



Fun Comparison

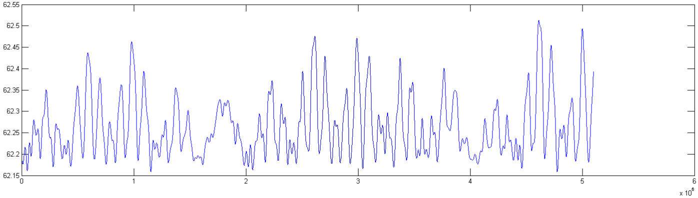


Top: Red = Land vs Lat. Blue = 30%. **Bottom:** Land vs Lat - 30%.



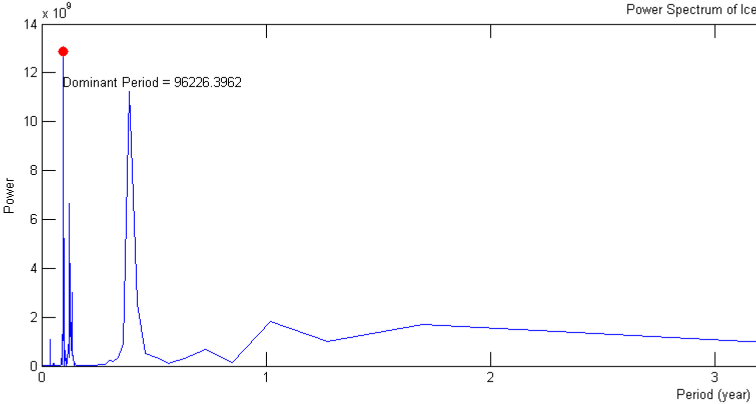
Bringing out Obliquity

First consider an "ICEY" scenario.
Northern most and southern most 15% of the world is covered in ice (50% albedo) and the rest is water (10% albedo).



Bringing out Obliquity

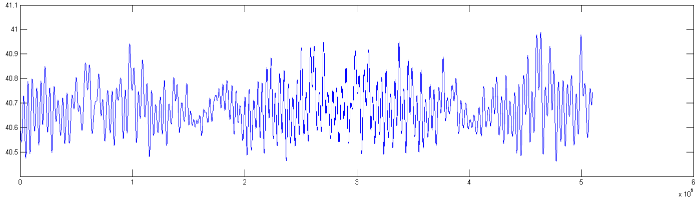
Here is the power signal for ICEY poles scenario.



The ICEY scenario sees eccentricity signal most prominently.

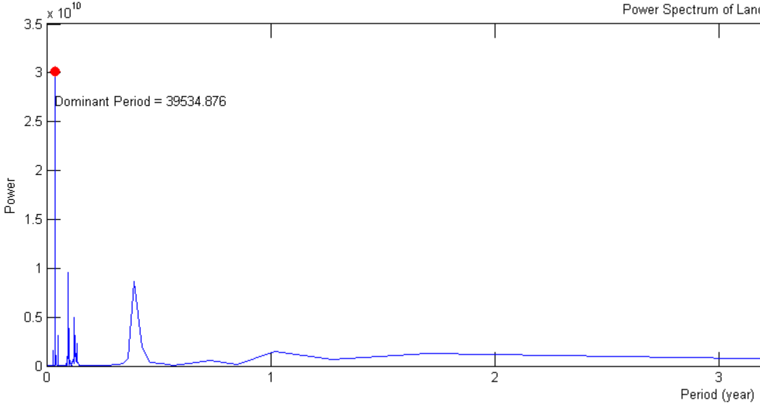
Beginning out Obliquity

Next consider the “DONUT” scenario.
90N to 60N at 10% albedo.
60N to 30N at 40% albedo.
30N to 30S at 50% albedo.
30S to 60S at 40% albedo.
60S to 90S at 10% albedo.



Bringing out Obliquity

Here is the power spectrum for the DONUT scenario.



The DONUS scenario sees obliquity most prominently.

Conclusions

The power spectrum of the global average temperature of an ice-free planet can follow either the eccentricity or the obliquity signal.

The power spectrum for the Eocene (with the land masses approximately where they were at that time) follows the obliquity.

It is possible for ancient times, when land was clustered near the equator, to imagine a strong obliquity signal due to land mass albedo.

The End

Thanks!