The effect of CO_2 on Earth's Radiation Budget

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March 28, 2012

M.I. Budyko (1969)*

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - \boxed{(A+BT)} - C\left(T - \int_0^1 T\,dy\right)$$

Outgoing Longwave Radiation OLR

- $A, B > 0$
- $T \nearrow \quad \Rightarrow \quad \text{OLR} \nearrow$
- $\text{CO}_2 \ \leftrightarrow A \quad ?$

Ray Pierrehumbert

"Big ideas come from small models."

May 23, 2011, Snowbird, UT

*M.I. Budyko, The effect of solar radiation variations on the climate of the Earth, *Tellus* **21** (1969)

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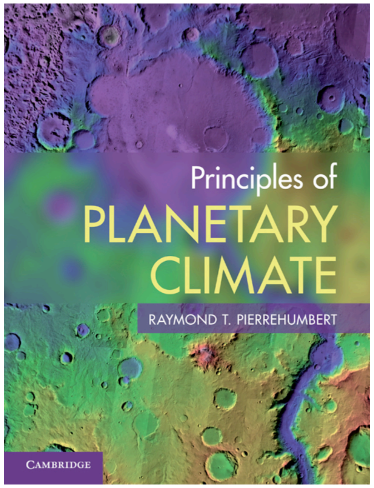
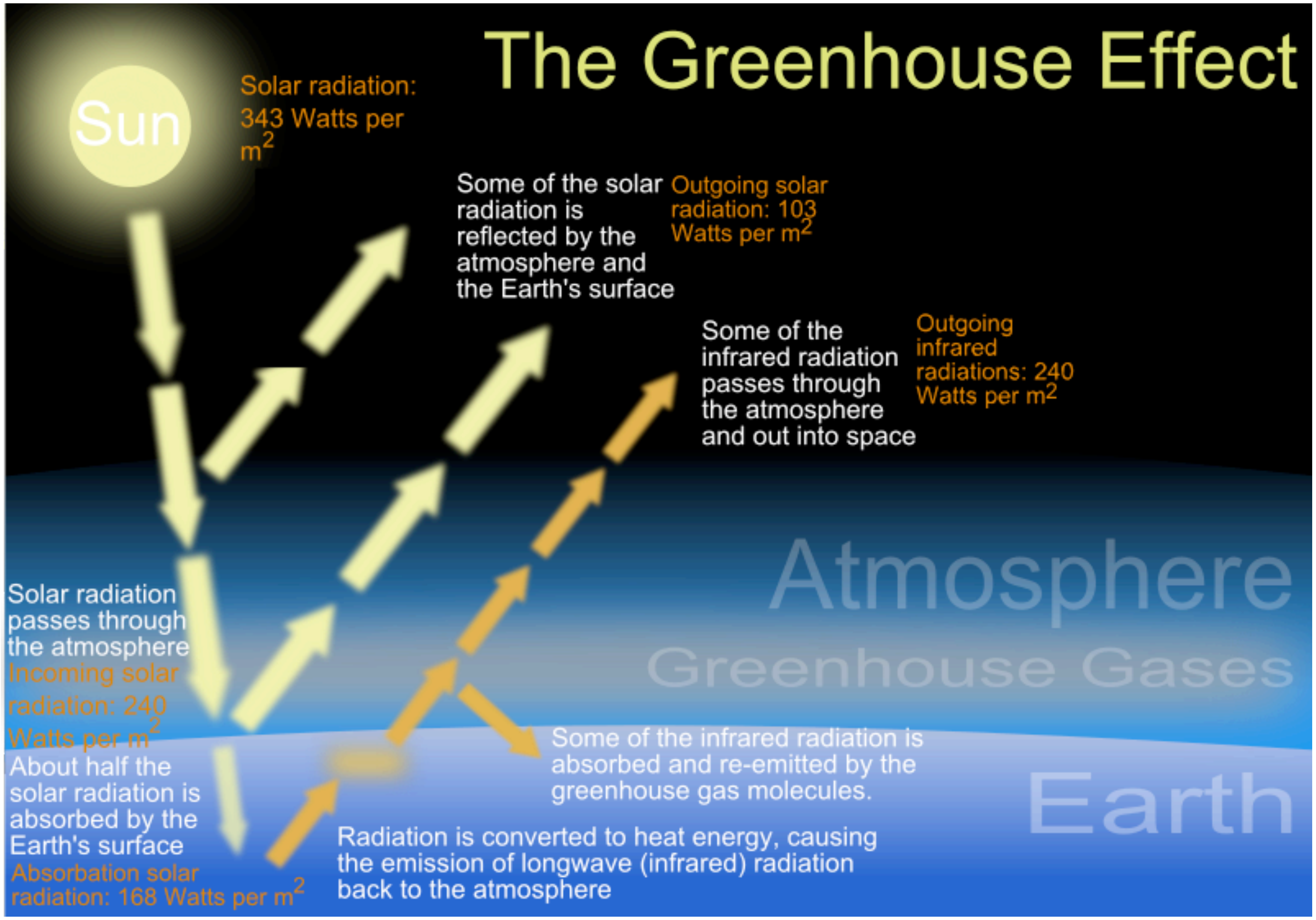
Ray Pierrehumbert

Principles of Planetary Climate
Cambridge, 2010
(PPC)





- Top-of-the-atmosphere considerations
- Role of greenhouse gasses in reducing OLR

Radiation: characterized by direction of propagation and frequency

frequency ν (Hz)

wavelength λ (m)

wavenumber $n = \nu / c \;\; \mathrm{m}^{-1}$

$$\nu\lambda = c = 3 \times 10^8 \text{ m/s}$$

Wavelength (m)	Wavenumber (m^{-1})	Frequency (Hz)	Band	Median emission temperature (K)	Peak-ν temperature (K)	Peak-λ temperature (K)
1000	0.001	3×10^{5}	Radio	4.1×10^{-6}	5.1×10^{-6}	2.9×10^{-6}
100	0.01	3×10^{6}		4.1×10^{-5}	5.1×10^{-5}	2.9×10^{-5}
10	0.1	3×10^{7}		4.1×10^{-4}	5.1×10^{-4}	2.9×10^{-4}
1	1	3×10^{8}		4.1×10^{-3}	5.1×10^{-3}	2.9×10^{-3}
0.1	10	3×10^{9}		0.041	0.051	0.029
0.01	100	3×10^{10}	Microwave	0.41	0.51	0.29
0.001	1000	3×10^{11}		4.1	5.1	2.9
10^{-4}	10^{4}	3×10^{12}	Infrared	41	51	29
10^{-5}	10^{5}	3×10^{13}		410	510	290
10^{-6}	10^{6}	3×10^{14}	Visible	4100	5100	2900
10^{-7}	10^{7}	3×10^{15}	Ultra violet	41 000	51 000	29 000
10^{-8}	10^{8}	3×10^{16}		4.1×10^{5}	5.1×10^{5}	2.9×10^{5}
10^{-9}	10^{9}	3×10^{17}	X-ray (soft)	4.1×10^{6}	5.1×10^{6}	2.9×10^{6}
10^{-10}	10^{10}	3×10^{18}	X-ray (hard)	4.1×10^{7}	5.1×10^{7}	2.9×10^{7}
10^{-11}	10^{11}	3×10^{19}	Gamma ray	4.1×10^{8}	5.1×10^{8}	2.9×10^{8}

Electromagnetic spectrum

(PPC, p. 137)

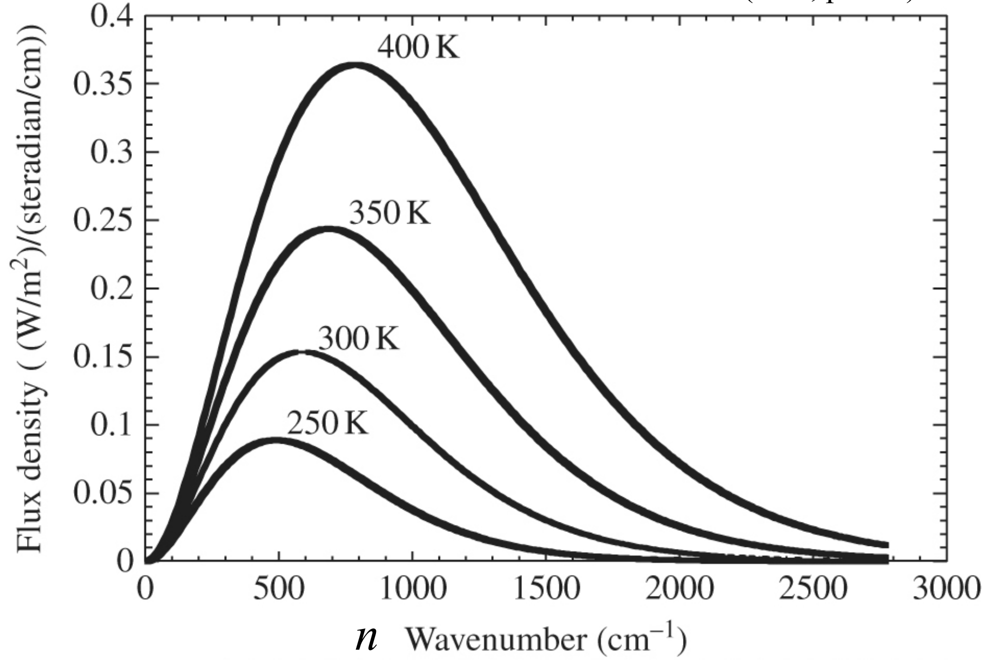
Blackbody radiation

• radiation reacts so strongly with matter that it achieves thermodynamic equilibrium at same temperature as the matter ("perfect absorbers and emitters").

Planck's Function: $$B(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

(PPC, p. 141)

k = Boltzman thermodynamic constant
h = Planck's constant
c = speed of light



• F.D. ↗ with T at all n
• F.D. small at large n for high T

• Treat Earth as blackbody, even though core $T \sim 6000$ K: sufficiently dense outer shell acts as blackbody

Blackbody radiation

Stefan-Boltzman Law

Total power exiting from each unit area of the surface of a blackbody:

$$F = \int_0^\infty \pi B(\nu, T)\, d\nu = \sigma T^4,$$

$$\sigma = 2\pi^5 k^4/(15c^2h^3) \approx 5.67 \cdot 10^{-8}\ \mathrm{W/m^2K^4}$$

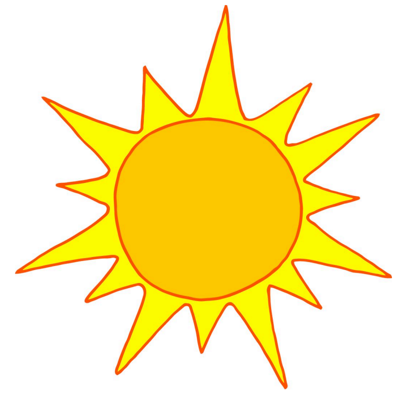
units of $\boldsymbol{F}$ = units of OLR = $\mathrm{W/m^2}$

Radiation balance of planets: An idealized example

- only source of energy heating the planet is absorption of light from planet's host star.
- the planetary albedo is spatially uniform.
- the planet is spherical and has a distinct liquid or solid surface which radiates like a perfect blackbody.
- the temperature is uniform over the surface of the planet.
- the planet's atmosphere is perfectly transparent to the electromagnetic energy emitted by the surface

$a =$ planet's radius $\quad T_* =$ star's temperature

$r_* =$ star's radius $\quad r =$ distance to star



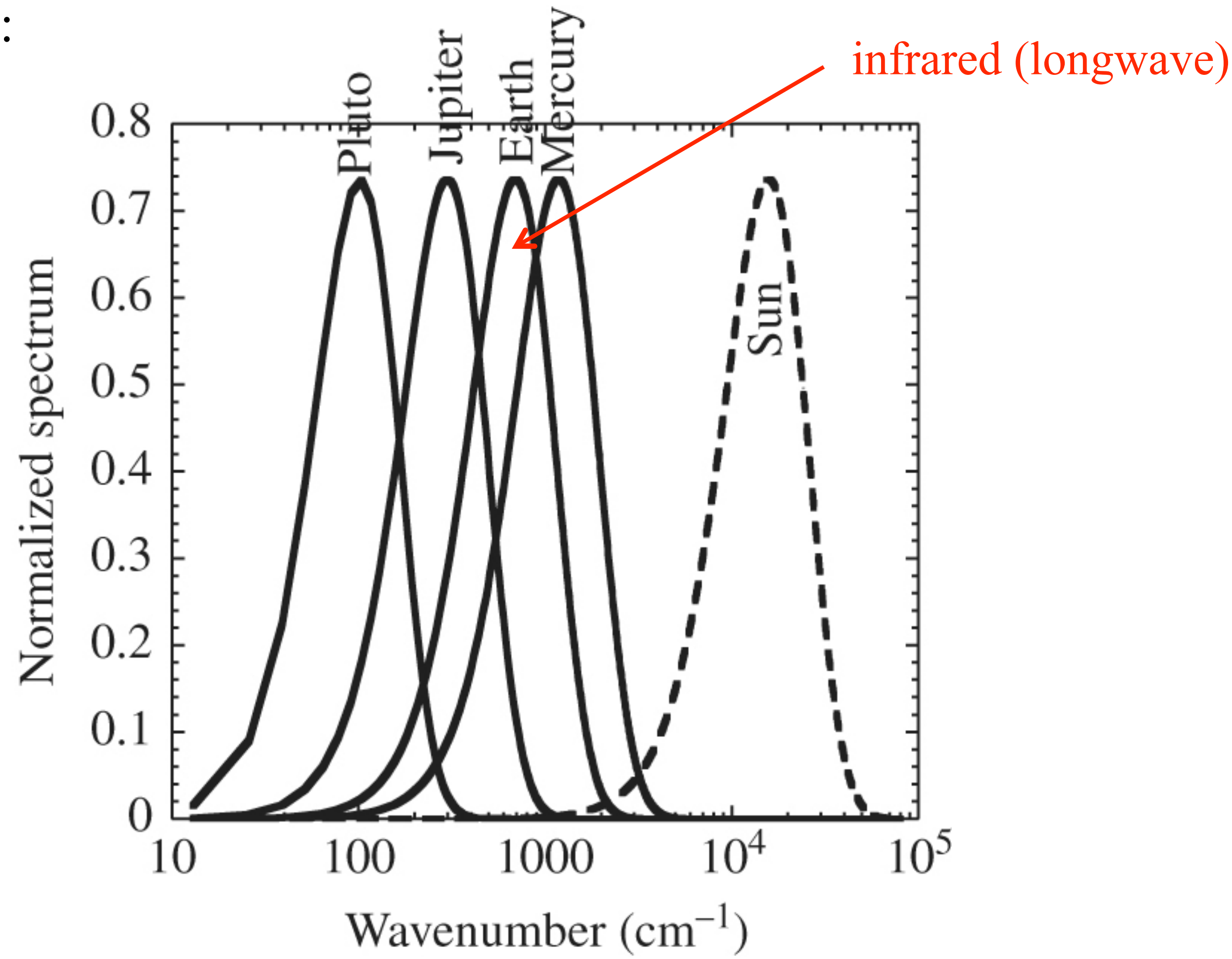
- Total flux impinging on planet $\sigma T_*^4 r_*^2 / r^2 =$ solar constant L_*
- Energy absorption: $\pi a^2 L_* (1 - \alpha)$
- Energy loss: $4\pi a^2 \sigma T^4$

Equilibrate: $\sigma T^4 = \frac{L_*}{4}(1 - \alpha)$ $\qquad$ $T = \frac{1}{\sqrt{2}}(1 - \alpha)^{1/4} \sqrt{\frac{r_*}{r}}\, T_*$

Planet loses energy through emission at a lower wavenumber than that at which it receives energy from the star.

$$T = \frac{1}{\sqrt{2}}(1-\alpha)^{1/4}\sqrt{\frac{r_*}{r}}\,T_*$$

Planck densities:



(PPC, p. 145)

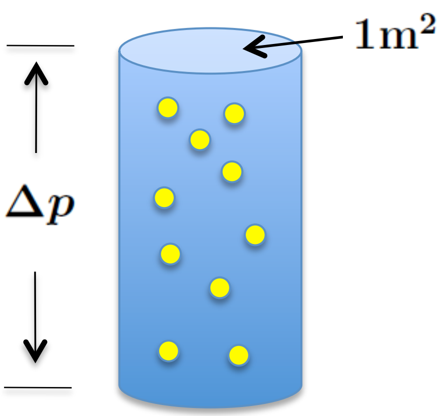
Greenhouse gas (G.G.): Basic ideas

$p_s =$ surface pressure
$T_s =$ surface temp
Assume $T_s = T(p_s)$

- Earth radiates in the infrared
- If the atmosphere was transparent to infrared: $\mathrm{OLR} = \sigma T_s^4$
- Mix in G.G. with unit mass concentration q

  Assume G.G. (i) transparent to solar radiation

  (ii) opaque to infrared at sufficiently high concentrations



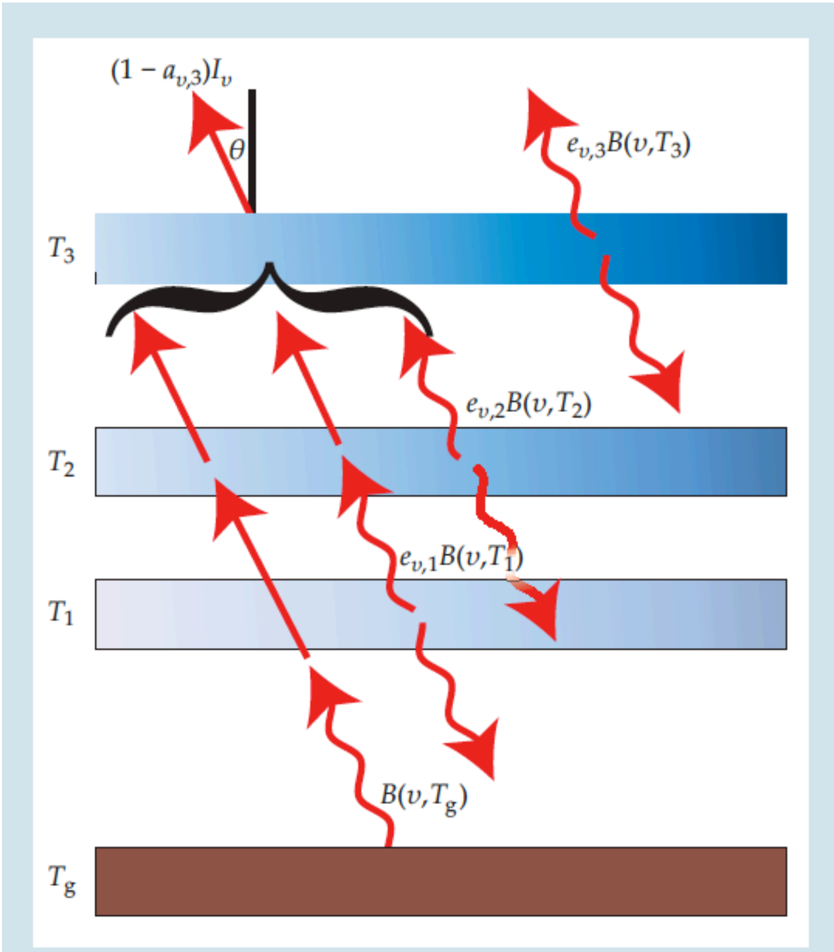
absorption coefficient $\kappa = \kappa(\nu, p, T)$ m^2/kg

$\kappa \Delta p q/g > 1 \Rightarrow$ column acts like blackbody

- $\kappa p_s q/g < 1 \Rightarrow$ atmosphere *optically thin*
- $\kappa p_s q/g \gg 1 \Rightarrow$ atmosphere *optically thick*

<u>Greenhouse gas (G.G.)</u>: Optically thick case $\kappa p_s q/g \gg 1$

Δp_1 {



$\kappa \Delta p_1 q/g > 1$

OLR escapes only from top slab

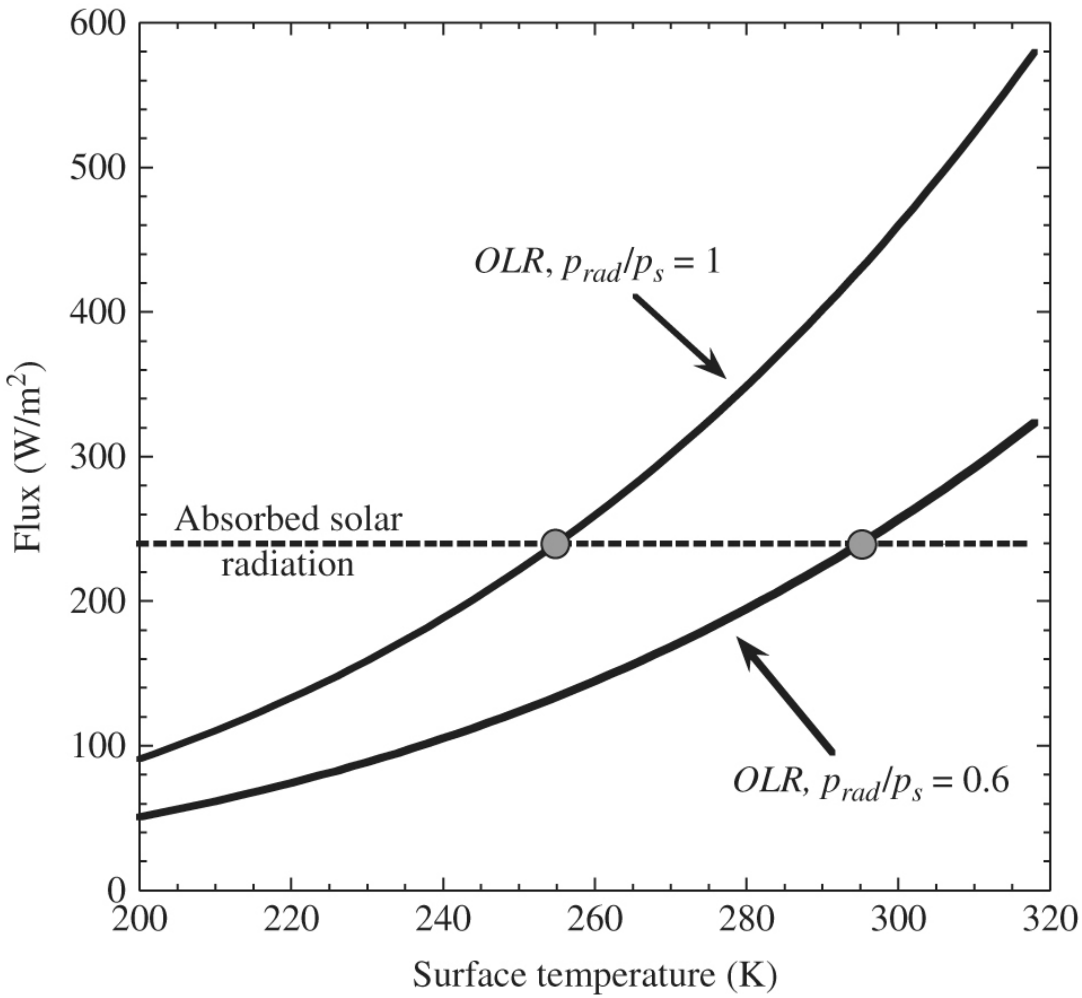
OLR determined by T_3

$\Delta p_1 = p_{\rm rad}$ characterizes the pressure at which OLR escapes

$$\mathrm{OLR} \approx \sigma T(p_{\rm rad})^4 < \sigma T_s^4$$

Figure from R. Pierrehumbert, Planetary radiation and planetary temperature, *Physics Today*, January 2011, 33-38.

<u>Greenhouse gas (G.G.)</u>: Basic ideas



(solar constant 1370 W/m^2, albedo 0.3)

(PPC, p. 147)

"In some sense, the whole subject of climate comes down to an ever-more sophisticated hierarchy of calculations of the curve *OLR*(T_s)."

--R. Pierrehumbert, PPC, p. 146

Greenhouse gas (G.G.): Basic ideas

$$I_s = \sigma T_0^4[1 - m\tanh(19\,T_0^6 \times 10^{-16})]$$

atmospheric transmission factor

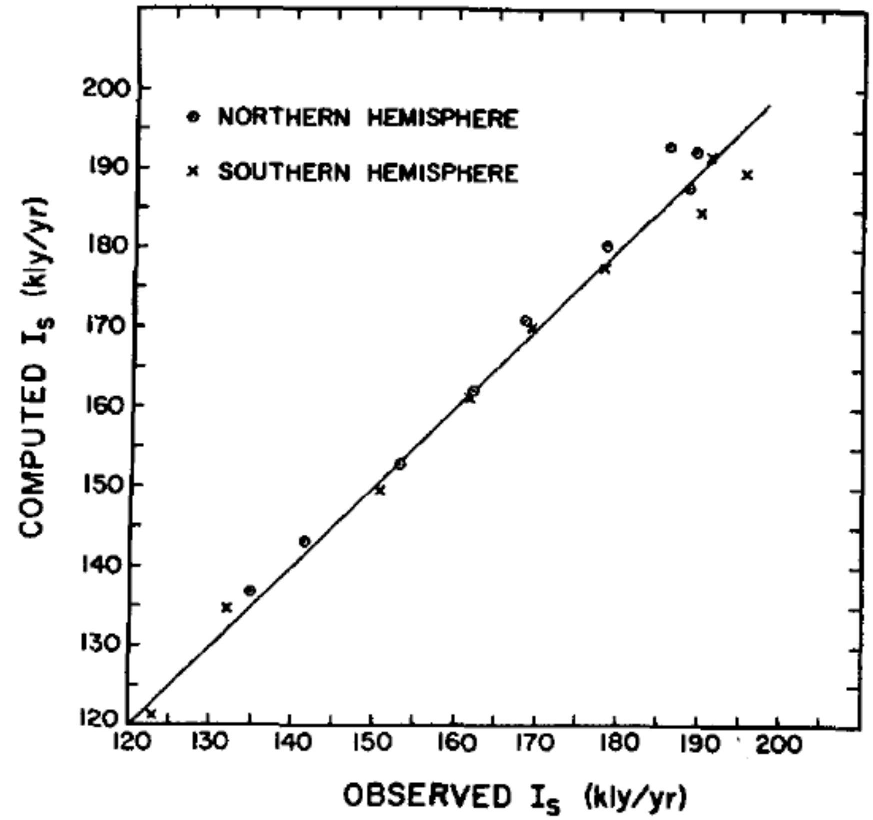
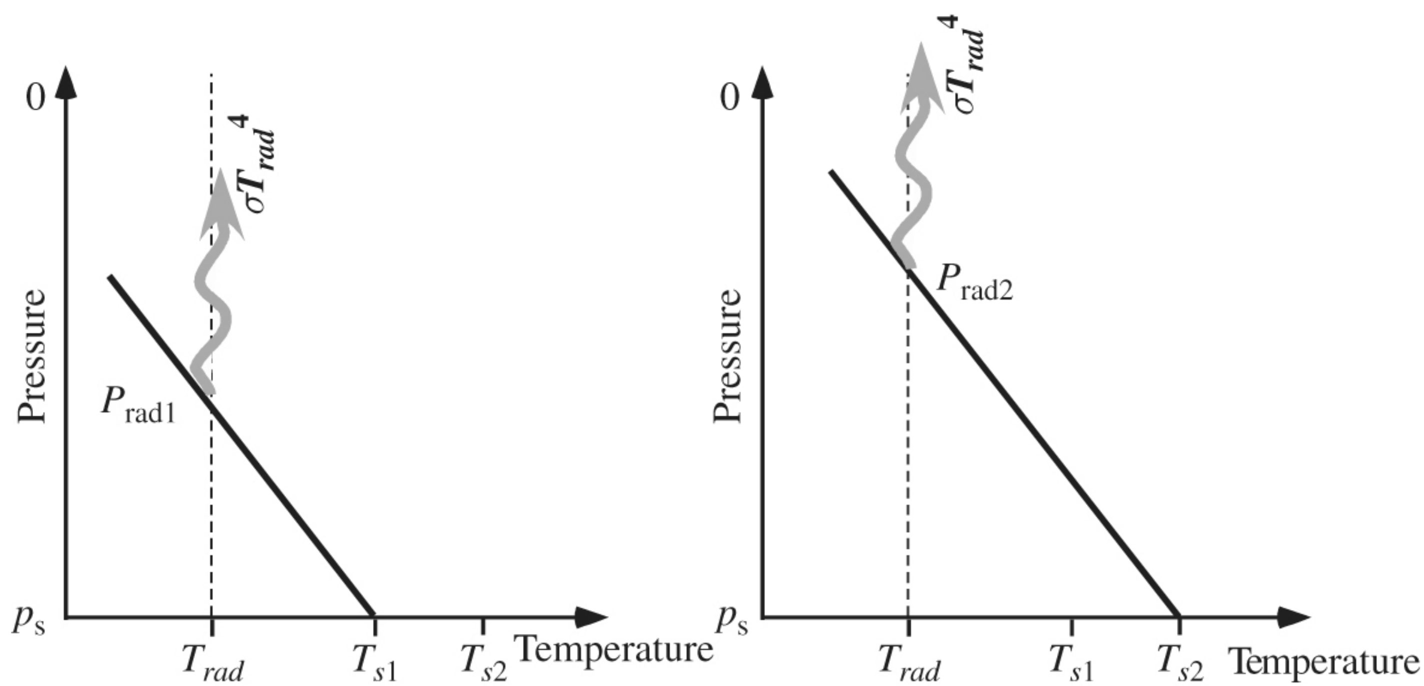


FIG. 1. A plot of the computed and observed values of the annual infrared emission to space in 10° latitude belts. The computed values were obtained from Eq. (6). The observed values are those given by Sellers (1965), modified slightly by recent satellite measurements.

W. Sellers, A global climate model based on the energy balance of the earth-atmosphere system, *J. Appl. Meteorology* **8** (1969), 392-400.

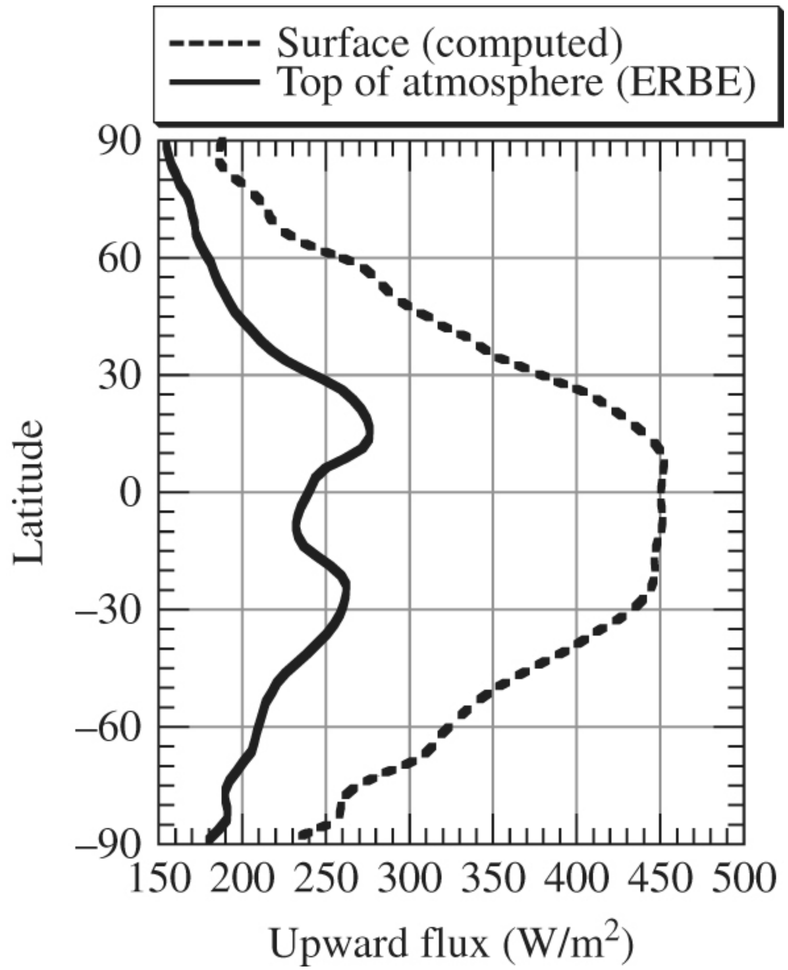
<u>Greenhouse gas (G.G.)</u>: Basic ideas

- $\sigma T_{\text{rad}}^4 = \frac{1}{4}(1-\alpha)L_*$
- $p_{\text{rad}} \searrow$ as G.G. $\nearrow$



(Figure from PPC, p. 147)

<u>Greenhouse gas (G.G.)</u>: Basic ideas



(Figure from PPC, p. 149)

The Earth's observed zonal-mean OLR for January, 1986 (solid curve).

σT_s^4 (dashed curve).

<u>Greenhouse gas (G.G.)</u>: Basic ideas

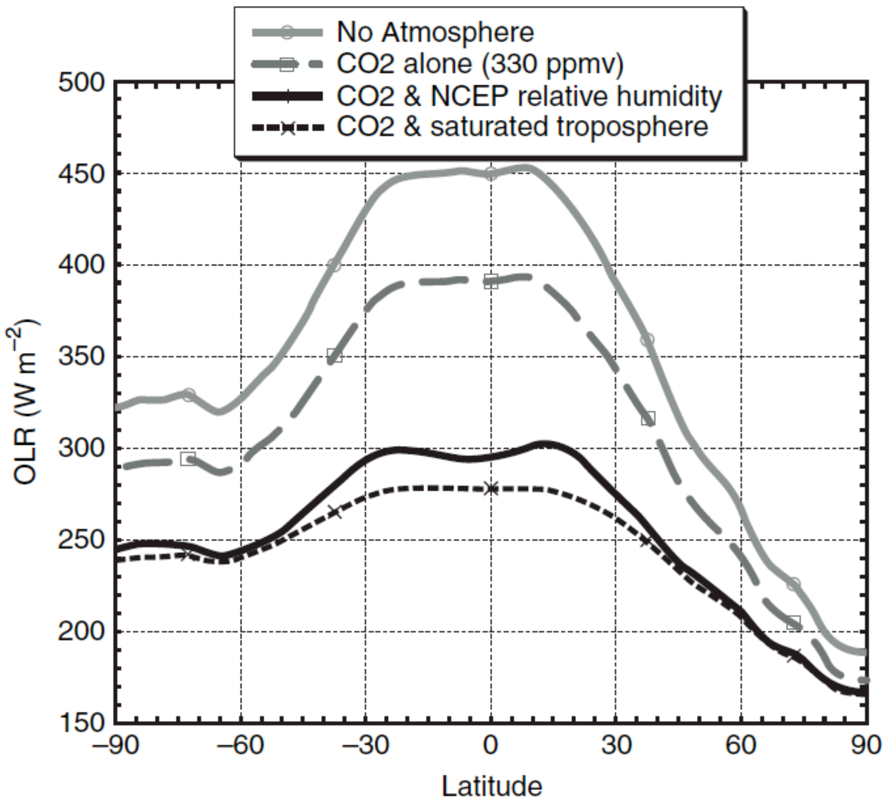
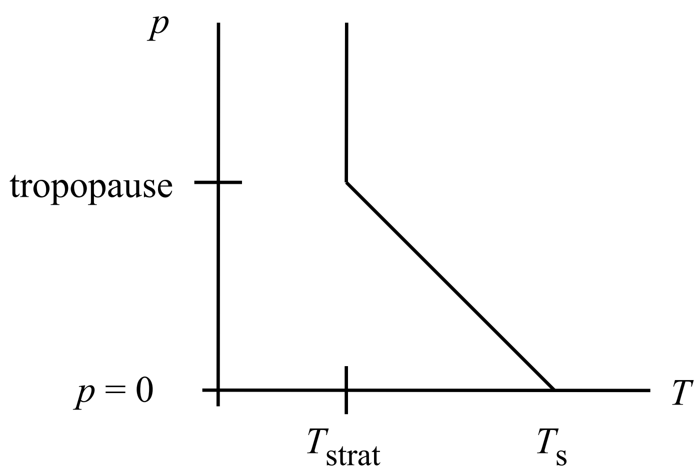
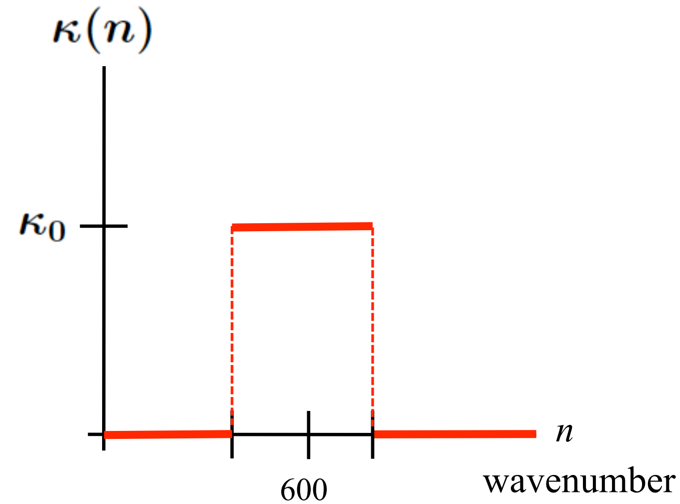


FIGURE 6.1. The effect of atmospheric composition on OLR, with atmospheric and surface temperature held fixed at January climatological values, 1960–1980.

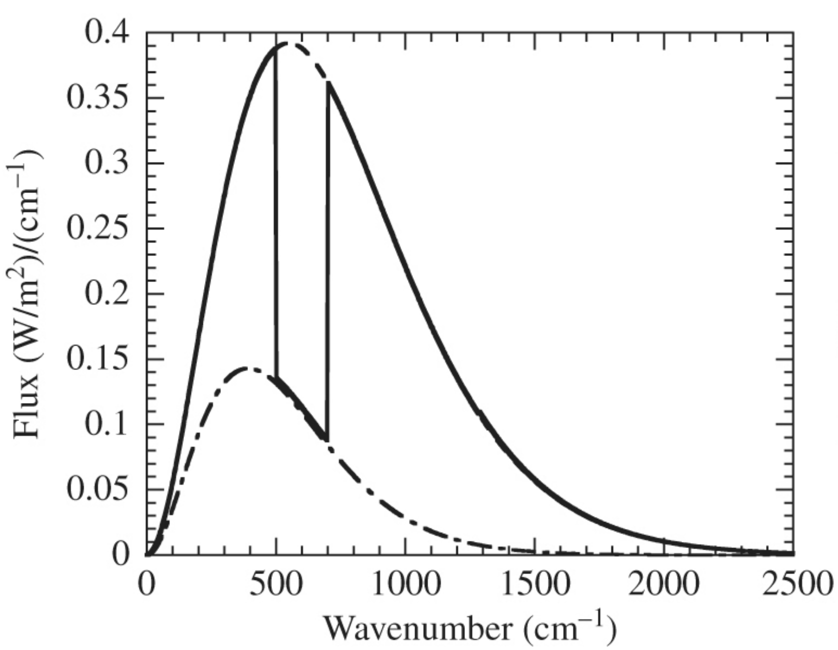
R. Pierrehumbert et al, On the relative humidity of the atmosphere, *The Global Circulation of the Atmosphere*, T. Schneider and A Sobel, eds. Princeton University Press, 2007, p. 143.

OLR spectrum: Toy example with one (fictitious) G.G. *oobleck*



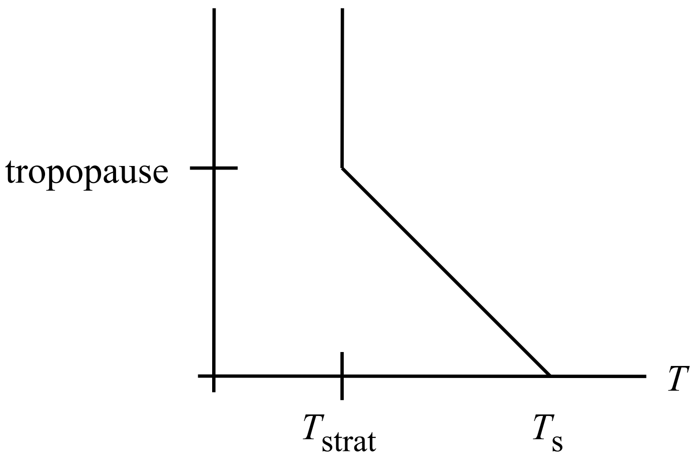


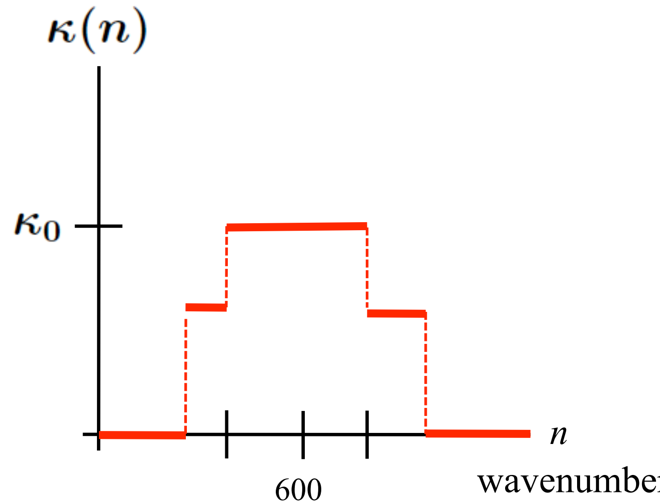
(T_s = 280 K, T_{strat} = 200 K)



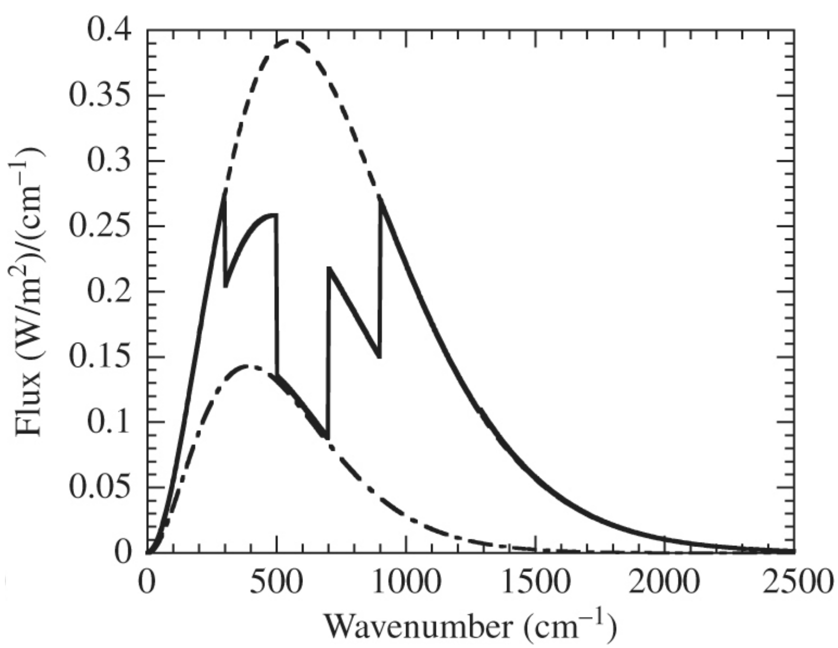
(Figure from PPC, p. 218)

<u>OLR spectrum</u>: Toy example with one (fictitious) G.G. *em*



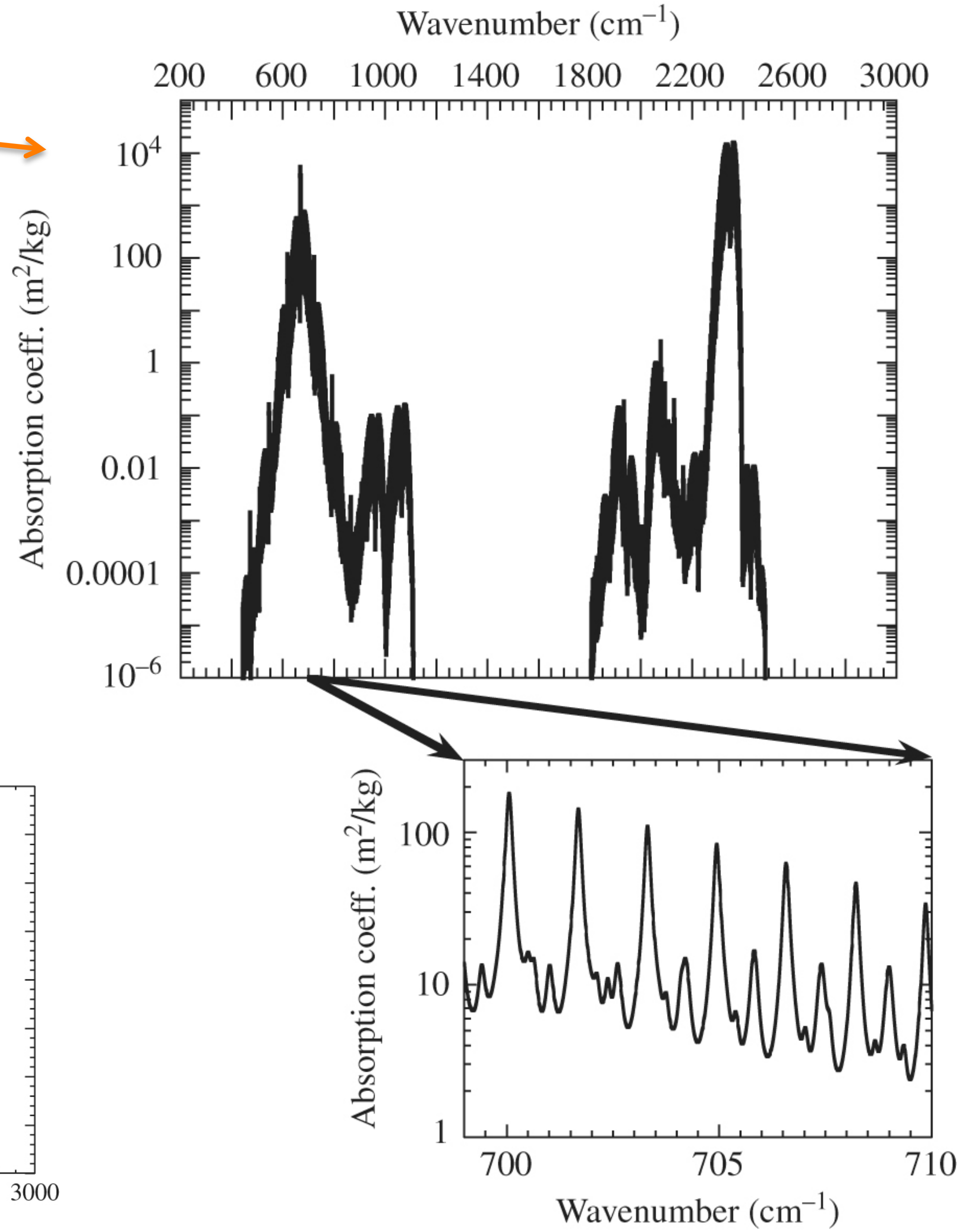


(T_s = 280 K, T_{strat} = 200 K)

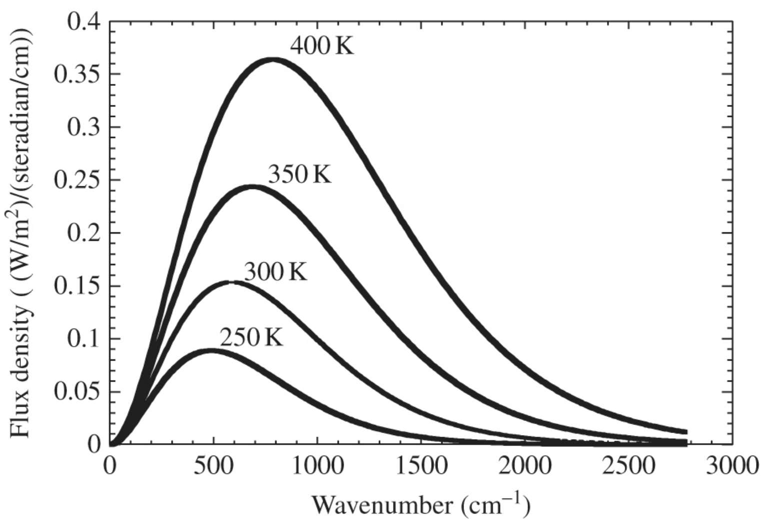


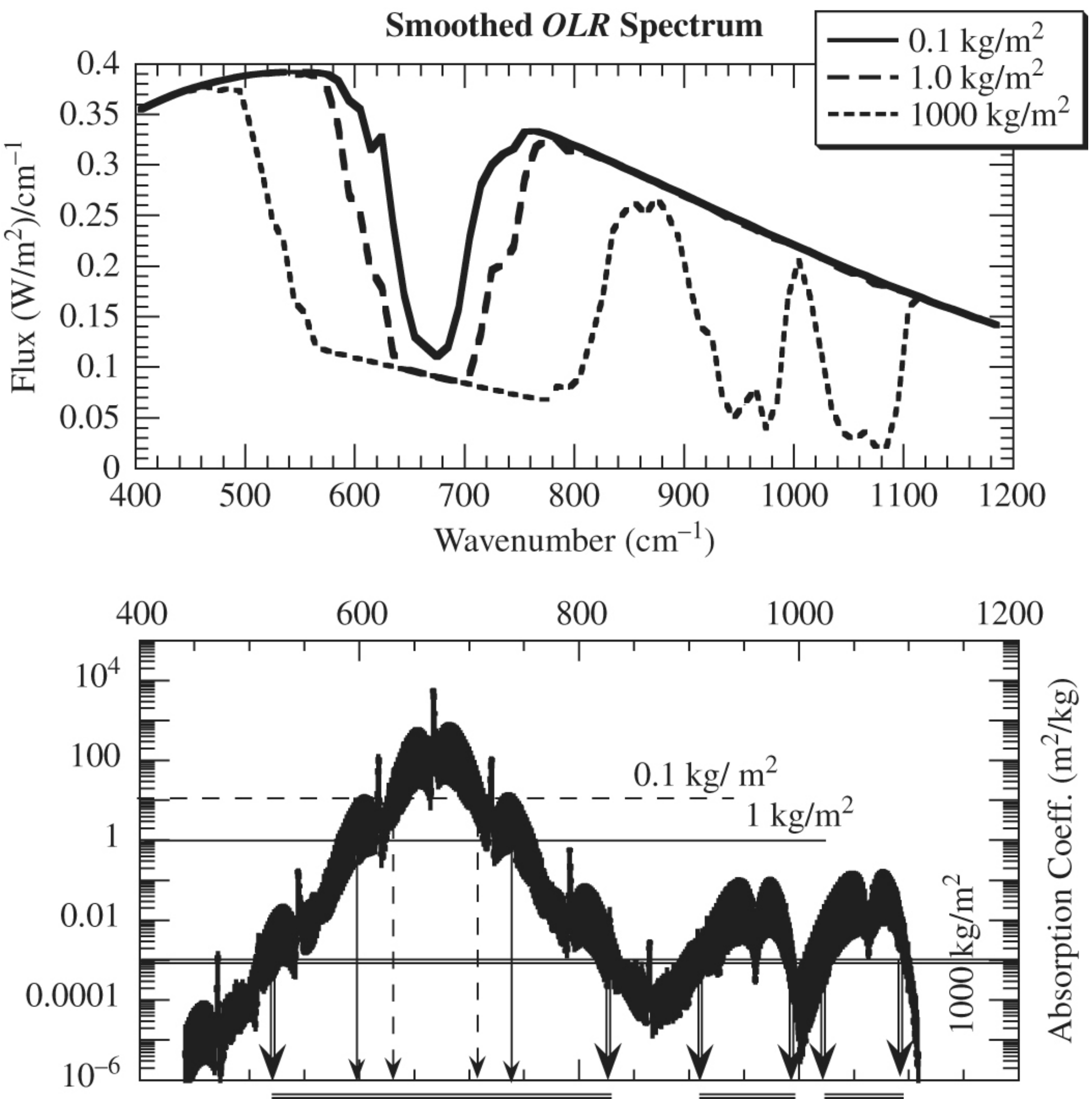
(Figure from PPC, p. 218)

Absorption coefficient for pure CO_2 at $T = 293$ K, $p = 1000$ mb



Infrared emission peak for Earth centered on ~ $n = 670$ cm^{-1}

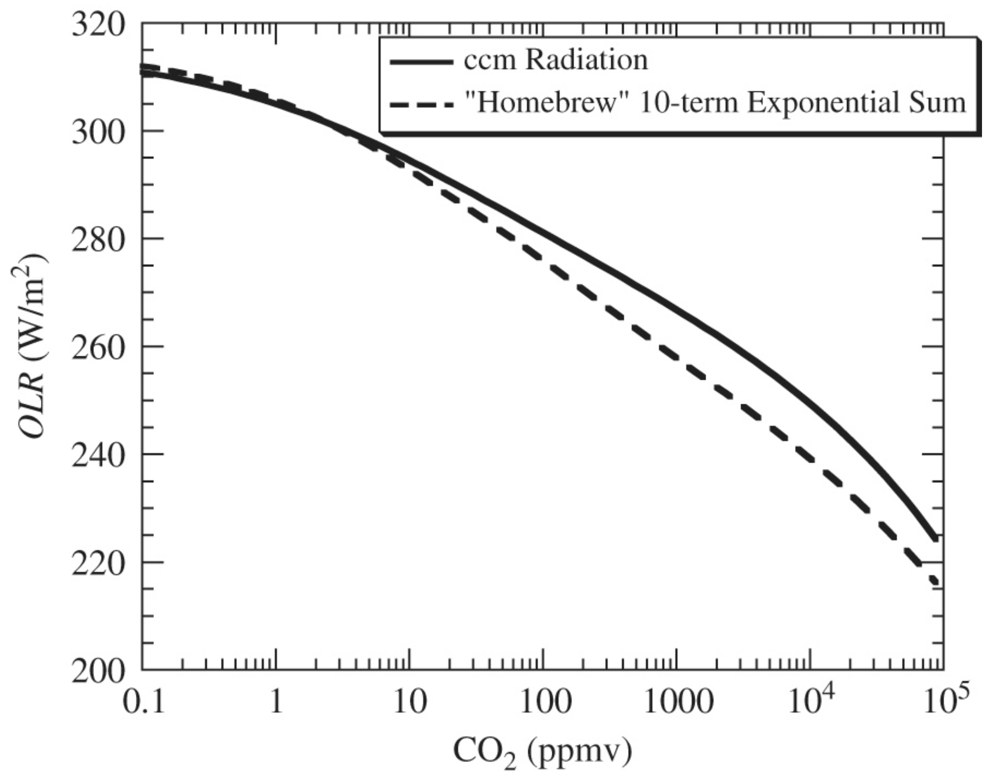




Absorption coefficient for CO_2 at 1 bar and 300K (bottom). Corresponding OLR for the three concentrations (top).

"The rate at which absorption decays with distance determines the rate at which the OLR decreases as the greenhouse gas concentration is made larger."

--R. Pierrehumbert, PPC, p. 219



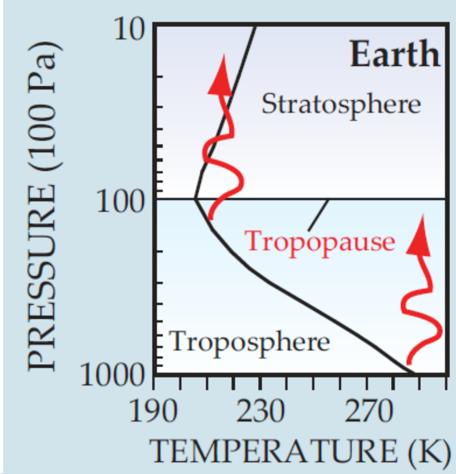
The *OLR* vs CO_2 concentration for CO_2 in a dry air atmosphere. $T_s = 273K$ held fixed. This curve gives the amount of absorbed solar radiation needed to maintain the surface temperature at freezing.

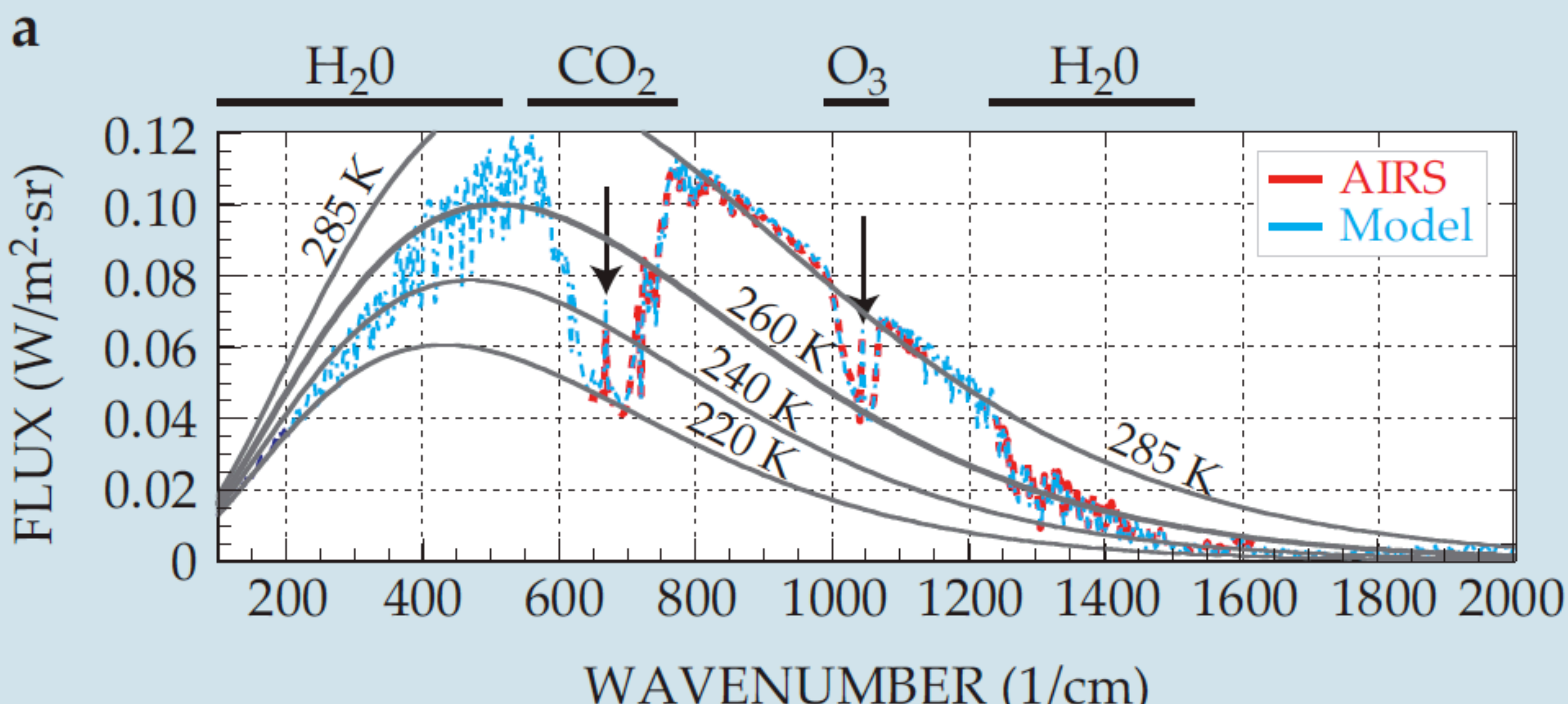
OLR goes down approximately in proportion to the logarithm of the CO_2 concentration.

(PPC, p. 265)

The left panel compares a computed global-mean, annual-mean emission spectrum for Earth (blue) with observations from the satellite-borne AIRS instrument (red); both are superimposed over a series of Planck distributions.

R. Pierrehumbert, Infrared radiation and planetary temperature, *Physics Today* (January 2011).





$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A + BT) - C\left(T - \int_0^1 T dy\right)$$

T 5° x 5°

OLR 15° x 15°

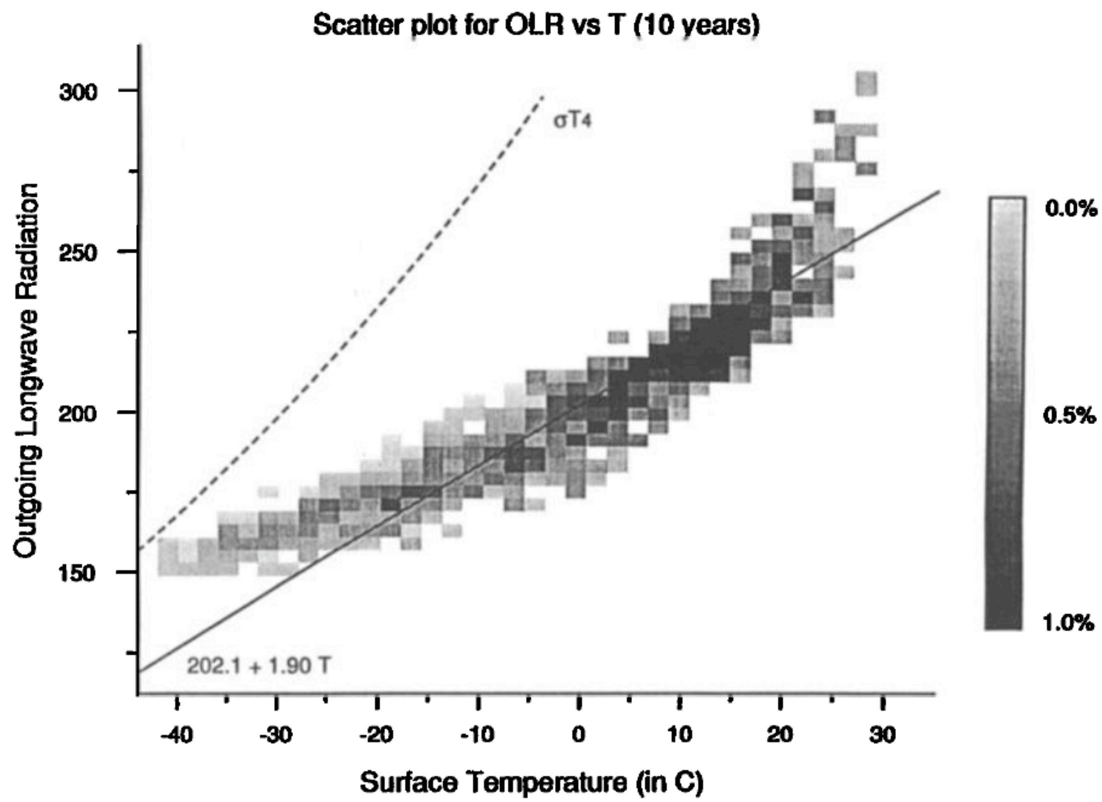


Fig. 2. Scatter plot of OLR versus surface temperature from 30°N to 90°N from the 10-year data set. The scale on the right indicates the percent of total. Note that there is a cosine of latitude weighting to account for the differing grid point areas.

C. Graves et al, New parameterizations and sensitivities for simple climate models, *J. Geophysical Research* **98** (1993), 5025-5036.

$$\text{Insolation} = OLR$$

$$Q = \sigma T^4$$

$$Q = 240 \Rightarrow T \approx 255K$$

(***T*** global average temperature)

$$Q = \sigma T^4 - \overline{G} = \sigma T^4 - 150 \Rightarrow T \approx 288K$$

Let time vary:

$$R\frac{dT}{dt} = Q - OLR = Q - (\sigma T(t)^4 - G(t))$$

$$G(t) = \overline{G} + \beta \ln\left(\frac{C}{C_0}\right)$$

$$C = C(t) = \text{atmospheric concentration of } CO_2 \text{ (ppm)}$$

After A. Hogg, Glacial cycles and carbon dioxide: A conceptual model, *Geophysical Res. Letters* **35** (2008)

$$R\frac{dT}{dt} = Q - OLR = Q - (\sigma T(t)^4 - G(t)) \qquad G(t) = \overline{G} + \beta \ln\left(\frac{C}{C_0}\right)$$

$$C = C(t) = \text{atmospheric concentration of } \mathrm{CO_2} \text{ (ppm)}$$

Linear approximation: $T_0 = 273 \quad , \; T \;{}^\circ\mathrm{C}, \; T/T_0 \ll 1$

$$T^4 = (T_0 + T)^4 = T_0^4(1 + T/T_0)^4 \approx T_0^4(1 + 4T/T_0)$$

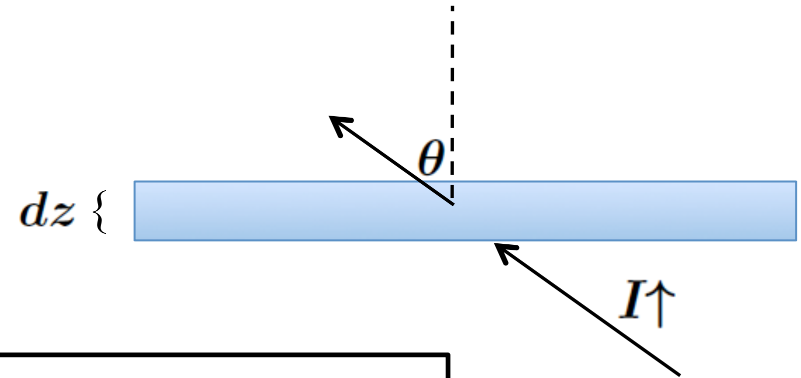
Kelvin!

$$OLR = \sigma T^4 - G \approx \sigma T_0^4 + 4\sigma T_0^3 T - \left(\overline{G} + \beta \ln\left(\frac{C}{C_0}\right)\right) = BT + A,$$

$$A = A(t) = \sigma T_0^4 - \left(\overline{G} + \beta \ln\left(\frac{C(t)}{C_0}\right)\right)$$

After A. Hogg, Glacial cycles and carbon dioxide: A conceptual model, *Geophysical Res. Letters* **35** (2008)

Basic equations for radiative flux*



$$dI\uparrow = -\kappa\rho_r I\uparrow \sec\theta dz + \kappa\rho_r I_b \sec\theta dz$$

Schwarzschild equation for radiative transfer without scattering

z height above surface

κ absorption coefficient

ρ_r density of radiating gas

I_b blackbody radiation flux from Planck

Energy absorbed from the incoming beam by the absorbing gas.

Energy emitted by the layer of gas.

- assume: given z, T and p are known
- consider $\kappa = \kappa(\nu, z)$
- consider $I_b = I_b(\nu, z)$

*G. Plass, Infrared radiation in the atmosphere, *Amer. J. Physics* **24** (1956), 303-321.

Basic equations for radiative flux

$$dI\uparrow = -\kappa\rho_r I\uparrow \sec\theta dz + \kappa\rho_r I_b \sec\theta dz$$

new independent variable: mass of radiating gas per unit area

$$u = \int_z^\infty \rho_r dz = \int_z^\infty c\rho dz, \quad du = -\rho_r dz$$

ρ total density of atmosphere at height z

$c = \rho_r/\rho$ fractional concentration of radiating gas

$u = 0$ at t.o.a

$$dI\uparrow = \kappa I\uparrow \sec\theta du - \kappa I_b \sec\theta du$$

- $\kappa = \kappa(u)$
- $I_b = I_b(u)$

Basic equations for radiative flux

$$dI\uparrow = \kappa I\uparrow \sec\theta du - \kappa I_b \sec\theta du$$

$$u = \int_z^\infty \rho_r dz = \int_z^\infty c\rho dz$$

Transmission function between "heights" $u_0 < u_1$:

$$\tau(u_0, u_1) = \exp\left(-\sec\theta \int_{u_0}^{u_1} \kappa du\right)$$

- $\kappa \approx 0 \Rightarrow \tau \approx 1$
- $\kappa \gg 1 \Rightarrow \tau \approx 0$

Solution: $$I\uparrow(u) = I\uparrow(u_1)\tau(u, u_1) + \sec\theta \int_u^{u_1} \kappa(v) I_b(v) \tau(u, v) dv$$

upward flux at the lower boundary layer u_1 times the transmission between u and u_1

blackbody flux emitted by each layer of gas between u and u_1 times the transmission between u and the height of the emitting layer

$$u = \int_z^\infty \rho_r dz = \int_z^\infty c\rho dz$$

$$\tau(u_0, u_1) = \exp\left(-\sec\theta \int_{u_0}^{u_1} \kappa du\right)$$

Basic equations for radiative flux

Solution: $$I\uparrow(u) = I\uparrow(u_1)\tau(u,u_1) + \sec\theta \int_u^{u_1} \kappa(v) I_b(v)\tau(u,v)dv$$

Remove explicit dependence on absorption coefficient: Integrate by parts!

$$I\uparrow(u) = I\uparrow(u_1)\tau(u,u_1) + I_b(u) - I_b(u_1)\tau(u,u_1) + \int_u^{u_1} \tau(u,v)\frac{dI_b(v)}{dv}dv$$

Downward flux: $$dI\downarrow = -\kappa I\downarrow \sec\theta du + \kappa I_b \sec\theta du$$

$$I\downarrow(u) = I\downarrow(u_0)\tau(u_0,u) + I_b(u) - I_b(u_0)\tau(u_0,u) - \int_{u_0}^{u} \tau(v,u)\frac{dI_b(v)}{dv}dv$$

Basic equations for radiative flux

$$I\!\uparrow\!(u) = I\!\uparrow\!(u_1)\tau(u,u_1) + I_b(u) - I_b(u_1)\tau(u,u_1) + \int_u^{u_1} \tau(u,v)\frac{dI_b(v)}{dv}dv$$

$$I\!\downarrow\!(u) = I\!\downarrow\!(u_0)\tau(u_0,u) + I_b(u) - I_b(u_0)\tau(u_0,u) - \int_{u_0}^{u} \tau(v,u)\frac{dI_b(v)}{dv}dv$$

u_1 value of u at lower boundary (usually the surface)

u_0 value of u at upper boundary (usually t.o.a, so $u_0 = 0$)

Approximation: $I\!\downarrow\!(0) = 0$ $\quad I\!\uparrow\!(u_1) = I_b(u_1)$

solar infrared flux that reaches Earth very small relative to OLR

Earth's surface radiates as (almost) perfect blackbody in the infrared

Basic equations for radiative flux

$$I\uparrow(u) = I_b(u) + \int_u^{u_1} \tau(u,v)\frac{dI_b(v)}{dv}dv$$

$$I\downarrow(u) = I_b(u) - I_b(0)\tau(0,u) - \int_0^{u} \tau(v,u)\frac{dI_b(v)}{dv}dv$$

Infrared radiation does not come from a single level—a bit is contributed from each level (each having its own *T*).

A bit of this is absorbed at each intervening level of the atmosphere.

Radiation is emitted in all directions.

Rate of emission and absorption strongly depends on frequency.

Several greenhouse gasses.

Currently, Earth receives about 1 W/m^2 more from solar absorption than it emits to space as infrared (due to rapid rise of greenhouse gases).

Earth's temperature has not yet risen enough to restore the energy balance: takes time to first warm up the oceans and melt the ice. (PPC, p. 152)