Stommel's Ocean Circulation Box Model

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- Density is affected by temperature and salinity
- Temperature and salinity work in opposing ways. Which effect dominates circulation?

The First Model

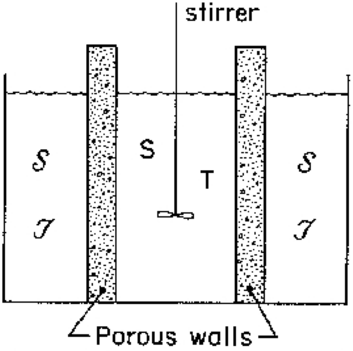

Fig. 1. The idealized experiment, consisting of a well stirred vessel of water with temperature T and salinity S (in general variable in time) separated by porous walls from and outside vessel whose temperature $\mathcal{T}$ and salinity $\mathcal{S}$ are maintained at constant values.

Tellus XIII (1961), 2

$$
\begin{aligned}
\frac{dT}{dt} &= c(\mathcal{T} - T) \\
\frac{dS}{dt} &= d(\mathcal{S} - S)
\end{aligned}
$$

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δ is considered to be small. This means that salinity changes are slower than temperature changes.

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Notice: $x, y \to 1$, but $y \to 1$ more quickly. Recall: $x = 1 = y \Rightarrow x = \mathcal{S}$, $y = \mathcal{T}$.

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$R = \frac{\beta \mathcal{S}}{\alpha \mathcal{T}}$ R measures the effect of salinity vs. temperature at the final equilibrium, $x = y = 1$.

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$$\frac{d\rho}{d\tau} = \rho_0(\alpha\mathcal{T})(-\frac{dy}{d\tau} + R\frac{dx}{d\tau}) = \rho_0(\alpha\mathcal{T})(y - 1 + R\delta(1 - x))$$

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Consider the case: $R\delta < 1$, $R > 1$, $x_0 = y_0 = 0$ When $\tau = 0$, $\frac{d\rho}{d\tau} = \rho_0(\alpha\mathcal{T})(-1 + R\delta) < 0$ Hidden assumption: $\mathcal{T} > 0$ When $\tau \to \infty$, $\rho = \rho_0(1 + \alpha\mathcal{T}(-1 + R)) > \rho_0$ The point: Temperature is the dominant effect at first, but after sufficient time, salinity effects take over, and the final density is higher than the initial density.

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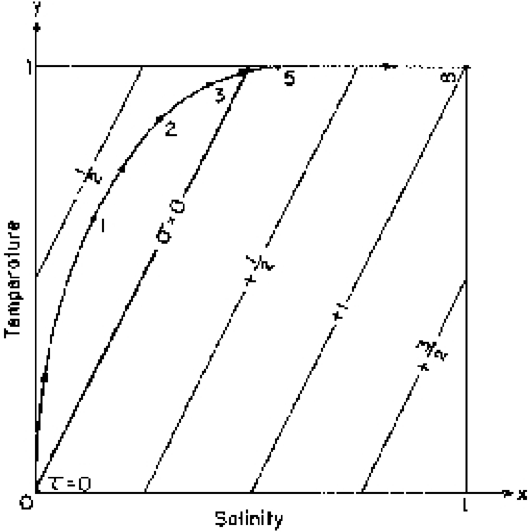

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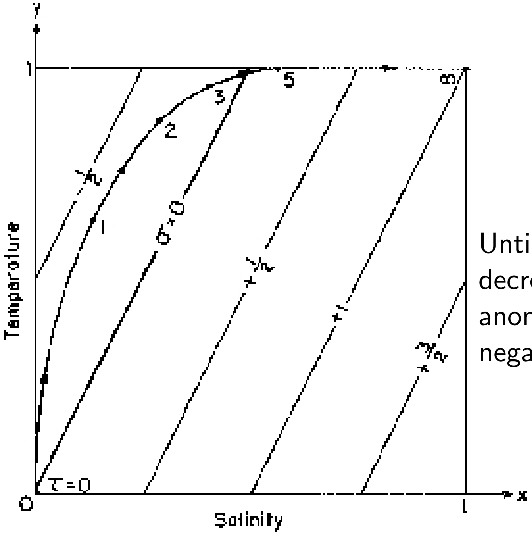

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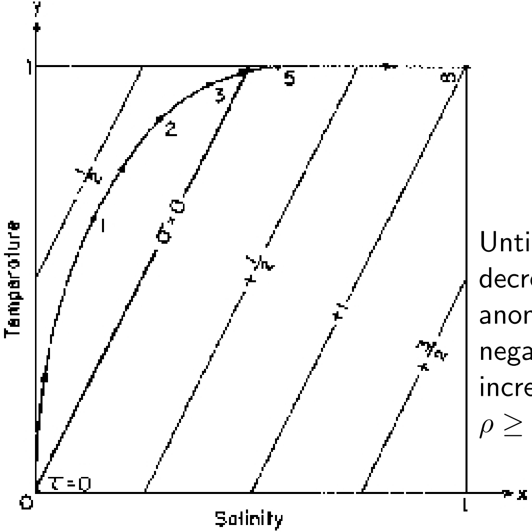

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