The mid-Pleistocene transition as a Hopf bifurcation

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Outline

- Korobeinikov & McNabb's “data”
- The mid-Pleistocene transition
- The model
- Analysis: stability & a supercritical Hopf bifurcation
- Comments

History of climate a la Korobeinikov & McNabb

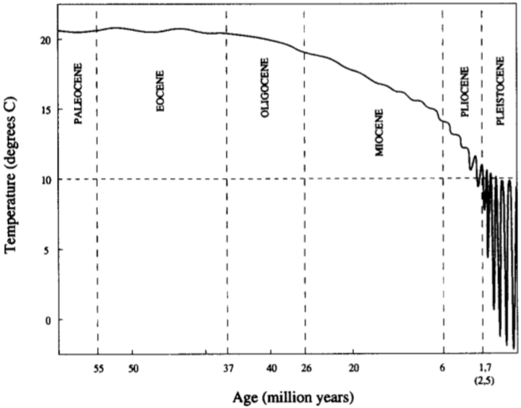
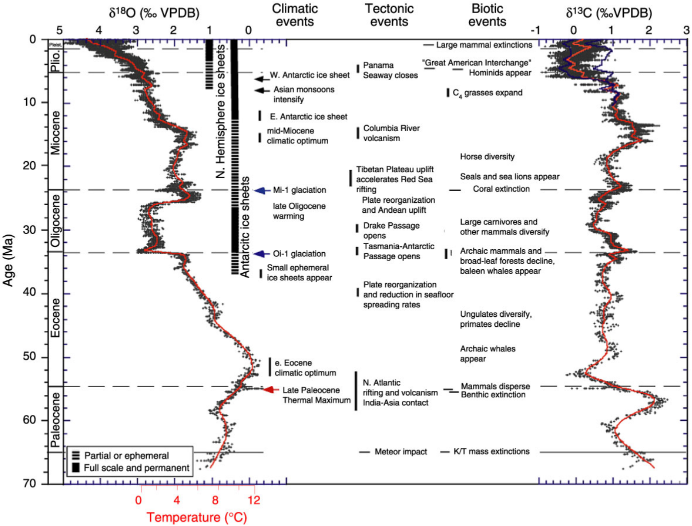


Figure 1. Mean annual temperature in central Europe for the last 50 million years (adapted from Andersen and Borns [1], fig. 1-19).

Korobeinikov, Andrei; McNabb, Alex. Journal of Applied Mathematics and Decision Sciences vol. 5 issue 4 December 1, 2001. p. 201-214

History of climate a la foraminifera



J. Zachos, et al. 2001. Trends, Rhythms, and Aberrations in Global Climate 65 Ma to Present. Science: Vol. 292 no. 5517 pp. 686-693 DOI: 10.1126/science.1059412

Korobeinikov & McNabb's sources

- K&M cite *The Ice Age World: An Introduction to Quaternary History and Research with Emphasis on North America and Northern Europe During the Last 2.5 Million Years* by B.G. Andersen & H.W. Borns Jr., Scandinavian University Press 1994.

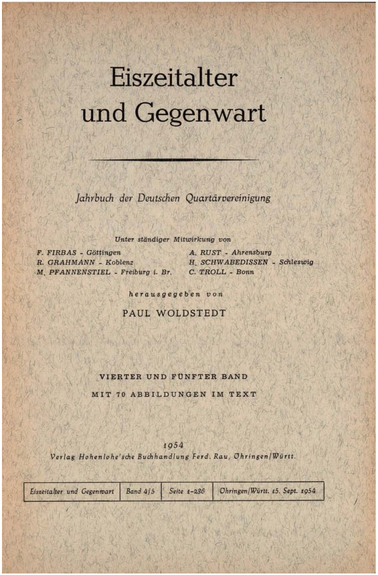
- *The Ice Age World* graciously thanks
    - The Research Council of Norway
    - Norsk Hydro A.S.
    - Saga Petroleum A.S.
    - Statoil

  for their generous contributions.

- That said, *The Ice Age World* seems like a decent enough book.

- *The Ice Age World* got its plot from

  > Woldstedt, P., 1954: "Die Klimakurve des Tertiärs und quartärs in Mitteleuropa". *Eiszeitalter und Gegenwart*, Bd.4/5, pp. 5-9.



Eiszeitalter und Gegenwart

Jahrbuch der Deutschen Quartärvereinigung

Unter ständiger Mitwirkung von

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PAUL WOLDSTEDT

VIERTER UND FÜNFTER BAND
MIT 70 ABBILDUNGEN IM TEXT

1954
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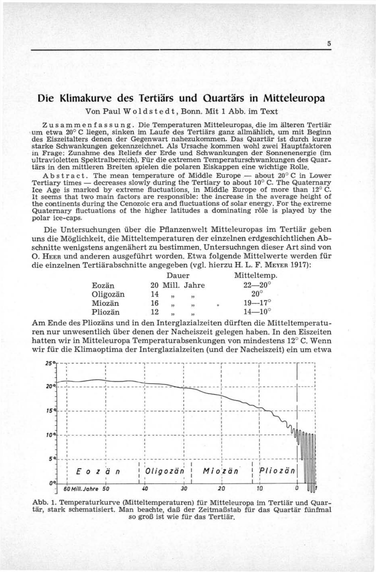


Die Klimakurve des Tertiärs und Quartärs in Mitteleuropa

Von Paul Woldstedt, Bonn. Mit 1 Abb. im Text

Zusammenfassung. Die Temperaturen Mitteleuropas, die im älteren Tertiär um etwa 20° C liegen, sinken im Laufe des Tertiärs ganz allmählich, um mit Beginn des Eiszeitalters denen der Gegenwart nahezukommen. Das Quartär ist durch kurze starke Schwankungen gekennzeichnet. Als Ursache kommen wohl zwei Hauptfaktoren in Frage: Zunahme des Reliefs der Erde und Schwankungen der Sonnenenergie (im ultravioletten Spektralbereich). Für die extremen Temperaturschwankungen des Quartärs in den mittleren Breiten spielen die polaren Eiskappen eine wichtige Rolle.

Abstract. The mean temperature of Middle Europe — about 20° C in Lower Tertiary times — decreases slowly during the Tertiary to about 10° C. The Quaternary Ice Age is marked by extreme fluctuations, in Middle Europe of more than 12° C. It seems that two main factors are responsible: the increase in the average height of the continents during the Cenozoic era and fluctuations of solar energy. For the extreme Quaternary fluctuations of the higher latitudes a dominating rôle is played by the polar ice-caps.

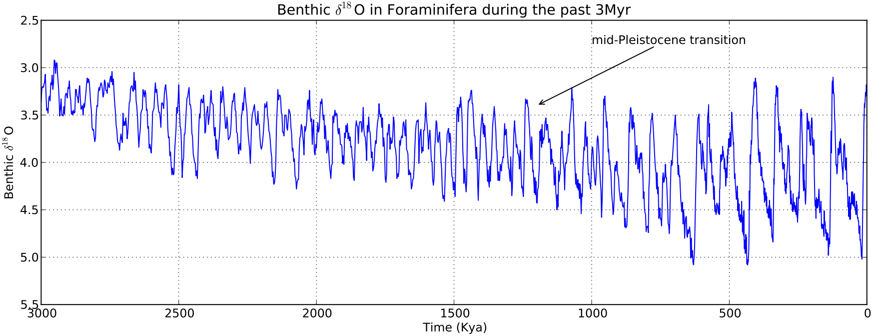
Die Untersuchungen über die Pflanzenwelt Mitteleuropas im Tertiär geben uns die Möglichkeit, die Mitteltemperaturen der einzelnen erdgeschichtlichen Abschnitte wenigstens angenähert zu bestimmen. Untersuchngen dieser Art sind von O. Heer und anderen ausgeführt worden. Etwa folgende Mittelwerte werden für die einzelnen Tertiärabschnitte angegeben (vgl. hierzu H. L. F. Meyer 1917):

	Dauer	Mitteltemp.
Eozän	20 Mill. Jahre	22—20°
Oligozän	14 „ „	20°
Miozän	16 „ „	19—17°
Pliozän	12 „ „	14—10°

Am Ende des Pliozäns und in den Interglazialzeiten dürften die Mitteltemperaturen nur unwesentlich über denen der Nacheiszeit gelegen haben. In den Eiszeiten hatten wir in Mitteleuropa Temperaturabsenkungen von mindestens 12° C. Wenn wir für die Klimaoptima der Interglazialzeiten (und der Nacheiszeit) ein um etwa

Abb. 1. Temperaturkurve (Mitteltemperaturen) für Mitteleuropa im Tertiär und Quartär, stark schematisiert. Man beachte, daß der Zeitmaßstab für das Quartär fünfmal so groß ist wie für das Tertiär.

The mid-Pleistocene transition

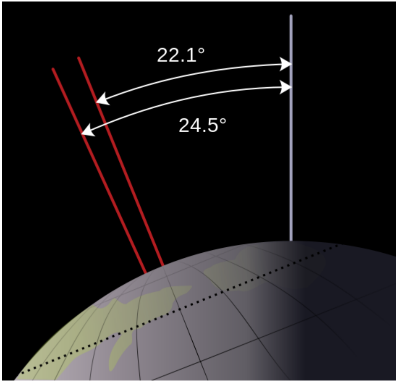


- Before 1.2 Mya, glacial cycles had a period of 41Kyr.
- Since 1.2 Mya, the period has been 100Kyr.
- **The 100,000 year problem:** what happened?

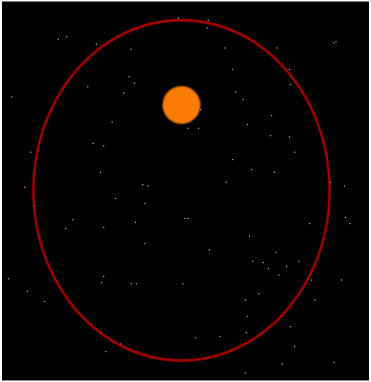
Lisiecki, L.E. and M.E. Raymo. 2005. A Pliocene-Pleistocene stack of 57 globally distributed benthic D18O records. Paleoceanography, Vol. 20, PA1003, doi:10.1029/2004PA001071.

Milankovitch cycles

- Glacial cycles are driven by Milankovitch cycles (+ feedback effects).
- The two that we care about today are:



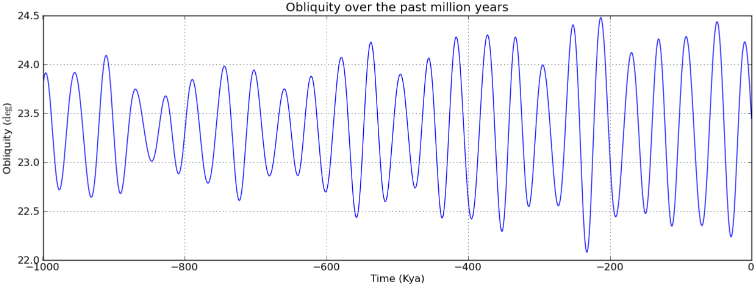
(c) Obliquity (axial tilt)



(d) Eccentricity

Milankovitch cycles

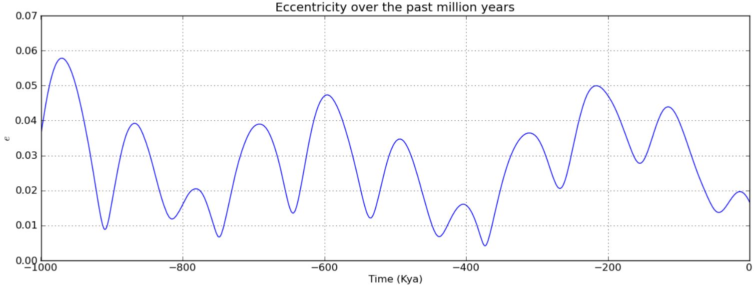
Obliquity: 41Kyr period.



Berger A. and Loutre M.F., 1991. Insolation values for the climate of the last 10 million years. Quaternary Sciences Review, Vol. 10 No. 4, pp. 297-317, 1991.

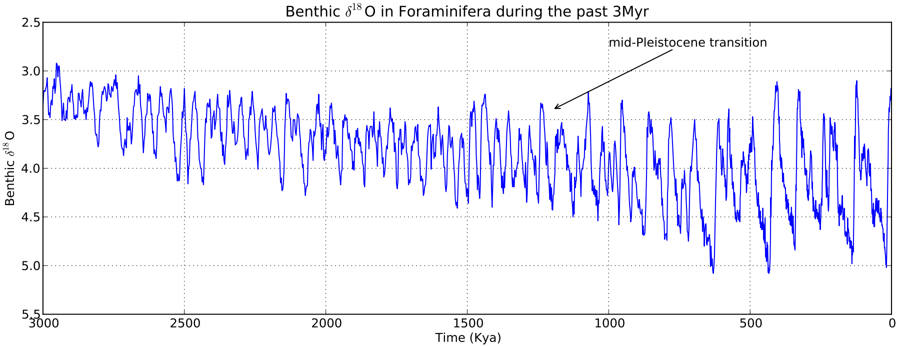
Milankovitch cycles

Eccentricity: 100Kyr period.



Laskar, J., Fienga, A., Gastineau, Manche, H.: 2011a, La2010: a new orbital solution for the long-term motion of the Earth, Astronomy & Astrophysics, Volume 532, id.A89, 15 pp.

The mid-Pleistocene transition



Lisiecki, L.E. and M.E. Raymo. 2005. A Pliocene-Pleistocene stack of 57 globally distributed benthic D18O records. Paleoceanography, Vol. 20, PA1003, doi:10.1029/2004PA001071.

Open questions around the mid-Pleistocene transition

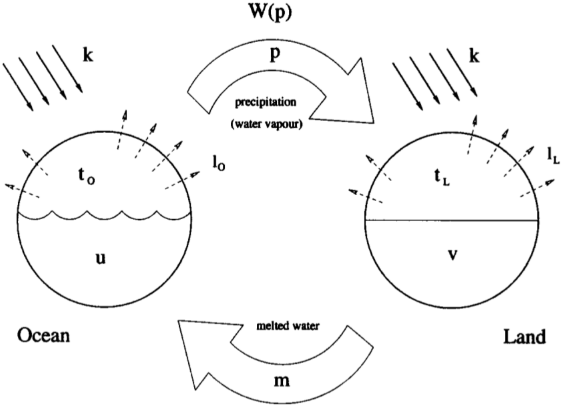
- What role did eccentricity play in the MPT?
  - Nonlinear, if any.
  - Eccentricity's effect on insolation is weak, compared to the other orbital variations.

- Is the 100Kyr period an artifact of (2×41) and (3×41)Kyr cycles?
  - e.g. Huybers

- Is the 100Kyr period inherent to the planet's climate system?
  - e.g. Maasch & Saltzman
  - e.g. **Korobeinikov & McNabb!**

Korobeinikov & McNabb's model

Korobeinikov & McNabb's system models the "ocean-land-atmosphere" system:



Korobeinikov, Andrei; McNabb, Alex. Journal of Applied Mathematics and Decision Sciences vol. 5 issue 4 December 1, 2001. p. 201-214

Some assumptions

- Global water stocks are divided betwen two large reservoirs O and L
  - O and L continuously exchange water and heat (mostly latent heat).
  - O contains all ocean water, with total mass u.
  - L contains all land water (snow & ice), with total mass v.
  - $u + v = V =$ constant.
- O and L exchange water via precipitation (rain, hail, snow):
  - Snowfall increases v
  - Rainfall and glacial melts decreases v.
  - $\dot{v} =$ (rate of snowfall) $-$ (rate at which ice & snow melts) $= s - m$
- Atmosphere water vapor (and corresponding heat) is ignored.
  - So the model pays attention to $\sim 99\%$ of water on Earth's surface.

The model

Goal:

1. Find an expression for $\dot{q}_O$, where $q_O(t)$ is the heat content of the ocean.
2. Then write the mean temperature t_O of the ocean proportionally to q_O. That is, write
$$C_O t_O = q_O,$$
where C_O is the total mean heat capacity of the ocean.
3. Then do the same for land: $C_L t_L = q_L$.
4. Giving us an ODE
$$\begin{aligned} \dot{v} &= s - m \\ \dot{t}_O &= g(v, t_O, t_L, \lambda) \\ \dot{t}_L &= h(v, t_O, t_L, \lambda) \end{aligned}$$

The model

Ocean's heat content:

$$\dot{q}_O = \overbrace{E_O}^{\text{insolation}} - \underbrace{\gamma_v p}_{\text{heat from precipitation}} - \overbrace{W(p)}^{\text{transport heat}} - \underbrace{I_O}_{\text{re-radiation}}$$

where

$p(t)$: total precipitation rate (snowfall + rainfall)

γ_v : latent heat of vaporization

$W(p)$: heat spent on work to transport water of mass p from the ocean onto the land

The model

Land's heat content:

$$\dot{q}_L = \overbrace{E_L}^{\text{insolation}} + \gamma_v p + \underbrace{\gamma_m s}_{\text{heat from snow}} - \gamma_m m - \overbrace{I_L}^{\text{re-radiation}}$$

where

$p(t)$: total precipitation rate (snowfall + rainfall)
γ_v : latent heat of vaporization
γ_m : latent heat of melting

The model

In sum:

$$\begin{aligned}
\dot{v} &= s - m \\
\dot{q}_O &= E_O - \gamma_v p - W(p) - I_O \\
\dot{q}_L &= E_L + \gamma_v p + \gamma_m s - \gamma_m m - I_L
\end{aligned}$$

So the system is in equilibrium when

$$s = m, \qquad E_O = W(p) + I_O + \gamma_v p, \qquad E_L = I_L - \gamma_v p.$$

More assumptions

- Total precipitation rate p is proportional to the difference between the mean ocean temperature t_O and the mean land temperature t_L:

$$p = s + r = a(t_O - t_L).$$

- Snowfall is proportional to total precipitation:

$$s = \delta p \qquad \text{where } \delta \cong 0.5.$$

- Neglect precipitation of transport: $W(p) = 0$.
  - Only about 0.7% of insolation is converted into the energy of motion of ocean currents, winds, and waves.
- The rate of snow melting m depends on t_L and surface area of the snow:

$$m = bt_L v^{\kappa}, \qquad \text{where } 0 < \kappa < 1.$$

Yet more assumptions

- The rate of re-radiation is proportional to temperature in the reservoirs:

$$I_O = ct_O, \qquad I_L = ct_L.$$

- Insolation absorbed by the ocean is proportional to insolation:

$$E_O = k\alpha$$

- Insolation absorbed by the land is proportional to insolation and inversely dependent on ice and snow surface area:

$$E_L = k\beta\left(1 - \frac{v^\kappa}{M}\right), \qquad \text{where } M > 0.$$

The model

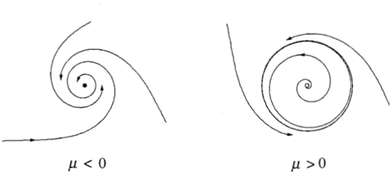
After the plethora of assumptions, we get:

$$\begin{aligned}
\dot{v} &= a\delta t_O - a\delta t_L - b t_L v^{\kappa}, \\
\dot{t}_O &= k\frac{\alpha}{C_O} + \frac{\gamma_v a}{C_O} t_L - \frac{(\gamma_v a + c)}{C_O} t_O, \\
\dot{t}_L &= k\frac{\beta}{C_L} - k\frac{\beta}{M C_L} v^{\kappa} + \frac{\gamma a}{C_L} t_O - \frac{\gamma a + c}{C_L} t_L + \frac{b\gamma_m}{C_L} t_L v^{\kappa}.
\end{aligned}$$

- By demanding some constraints on the parameters, the system has two equilibria Q_1 and Q_2, located in the positive area of the phase space.
- The authors say that these "conditions seem to be realistic."

Stability & a Hopf bifurcation

- Q_2 is unstable for all $k > 0$.
- For a condition on the parameters $\eta \geq 0$, Q_1 is stable for all $k > 0$.
- For $\eta < 0$, Q_1 admits a supercritical Hopf bifurcation with k as the bifurcation parameter.



Supercritical Hopf bifurcations occur when a pair of isolated, nonzero simple complex conjugate eigenvalues of the linearized system cross the imaginary axis from left to right.

Diagram credit: Steven Strogatz, Nonlinear Dynamics and Chaos: With Applications To Physics, Biology, Chemistry, And Engineering, Westview Press 2001, pg 250.

Some remarks

- Why is insolation the bifurcation parameter?
    - There's a lot of ice during an ice age.
    - To evaporate and transport all this ice, you need a lot of energy.
    - The Sun is the primary energy source for the earth's surface.
- Insolation is a bit of a misnomer: it's more about the amount of energy that reaches Earth's surface.
    - Only about 55% of insolation reaches Earth's surface.
    - Of course, many things cause variation in this (e.g., atmosphere composition).

The moral of the story

- Prior to 1.2Mya, the climate was relatively “stable”.
- After 1.2Mya, the climate became “unstable” and “chaotic”.
- This fits the general pattern of a supercritical Hopf bifurcation.

A few of my own comments

- The authors didn't get very specific about the parameters/constants.
- Their justification for certain assumptions was lacking.
- They claim that Milankovitch's theory requires glacial fluctuations in the northern hemisphere to be out-of-phase with those in the southern hemisphere. . . which is wrong.
- The purpose of the model is to show that the MPT could have been caused by "internal factors".

Thanks!