Winter Is Coming: A Dynamical Systems Approach to Better El Niño Predictions

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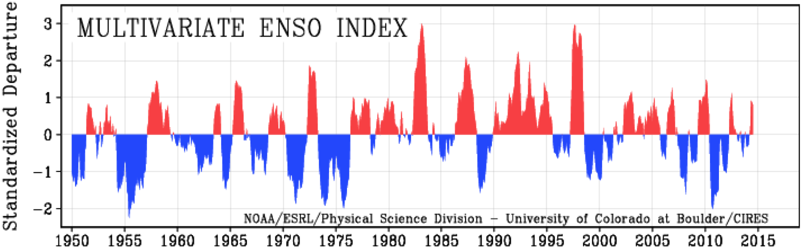
Outline

- What is ENSO?
- Why do we care?
- Conceptual ENSO Models
- Dynamical Systems

The Data



MULTIVARIATE ENSO INDEX
Standardized Departure
3
2
1
0
−1
−2
NOAA/ESRL/Physical Science Division – University of Colorado at Boulder/CIRES
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015

2014 Predictions

August:

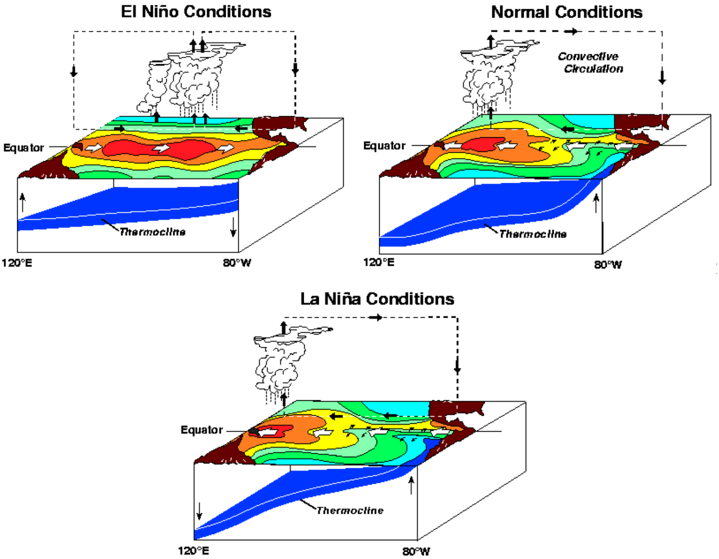
Talk of an El Niño year cools, but don’t despair yet about winter

While a 'super' El Niño looks to be off the table, what does develop this year might not deliver what many Canadians are hoping for

Don’t dismiss a 2014 ‘super’ El Niño just yet

September: No ENSO this year
October 9: El Niñ0 watch back on. Chances back over 65 % (NOAA).

Physical Cartoon



From: NOAA

Jin's Model - 1997

Recharge Oscillator Model:

$$\frac{dT_E}{dt} = RT_E + \gamma h_W - e_n(h_W + bT_E)^3$$
$$\frac{dh_W}{dt} = -rh_W - \alpha bT_E.$$

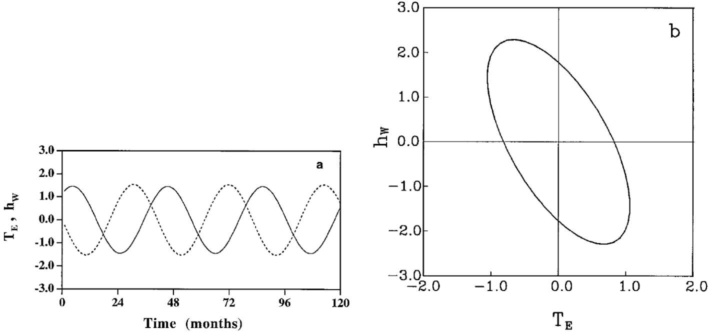
Variables:

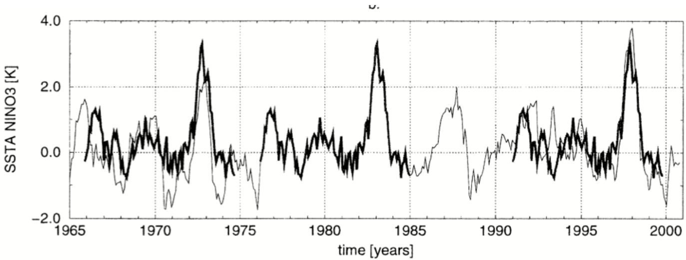
- T_E temperature anomaly in E Pacific
- h_W thermocline depth anomaly in W Pacific

Processes:

- Relaxation to mean state
- Upwelling
- Thermocline adjustment to wind-stress

Results


a
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
T_E , h_W
0
24
48
72
96
120
Time (months)
b
3.0
2.0
1.0
0.0
−1.0
−2.0
−3.0
h_W
−2.0
−1.0
0.0
1.0
2.0
T_E


4.0
2.0
0.0
−2.0
SSTA NINO3 [K]
1965
1970
1975
1980
1985
1990
1995
2000
time [years]

Supercharged Recharge Oscillator

$$
\begin{aligned}
\frac{dT_1}{dt} &= -\alpha(T_1 - T_r) - \delta\mu(T_2 - T_1)^2 \\
\frac{dT_2}{dt} &= -\alpha(T_2 - T_r) + \zeta\mu(T_2 - T_1)[T_2 - T_{sub}(T_1, T_2, h_1)] \\
\frac{dh_1}{dt} &= -r\left[h_1 - \frac{bL\mu}{2\beta}(T_2 - T_1)\right]
\end{aligned}
$$

Modifications from Recharge Oscillator model:

- Variables are no longer anomalies
- Subscript 1 indicates West, 2 indicates East
- Includes advection

Simulation vs. Data

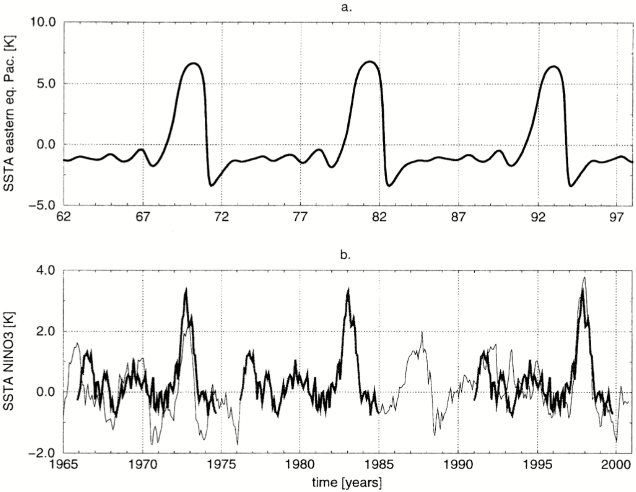


FIG. 11. (a) Simulated time series of the eastern equatorial temperature anomalies. The bifurcation parameters were set to $\zeta = 1.3$ and $\epsilon = 0.11$. (b) Thin: observed Niño-3 SSTA time series; thick: Niño-3 SSTA template from 1976 to 1985.

From: Timmerman, Jin, and Abshagen – 2003

MMOs

- Timmerman, Jin, Abshagen (2003): Shilnikov-type mechansim (saddle-focus)
- Other methods require multiple time-scales!

Change of variables:

- $S = T_2 - T_1$
- $T = T_1 - T_r$
- $h = h_1 - K$

New System

$$\frac{dS}{dt} = -\alpha S + \delta\mu S^2 + \zeta\mu S\left[S + T + Cf(S, T, h)\right]$$

$$\frac{dT}{dt} = -\alpha T - \delta\mu S^2$$

$$\frac{dh}{dt} = -r\left(h - K + \frac{bL\mu}{2\beta}S\right)$$

We want to analyze as a fast/slow system $\rightarrow$ need to non-dimensionalize!

Dimensionless System

$$x' = \varepsilon(x^2 - ax) + x\left[x + y - nz + d - c\left(x - \frac{x^3}{3}\right)\right]$$
$$y' = -\varepsilon(ay + x^2)$$
$$z' = m\left(k - z - \frac{x}{2}\right)$$

- $x \sim S$
- $y \sim T$
- $z \sim h$
- $\varepsilon = \delta/\zeta$
- $c = \frac{bL\mu(T_r - T_{r0})}{2h^*\beta}$
- $a = \frac{\alpha bL}{\delta\beta h^*}$
- $m = \frac{rbL}{\zeta\beta h^*}$
- n, k, d come from f

Dimensionless System

$$
\begin{aligned}
x' &= \varepsilon(x^2 - ax) + x\left[x + y - nz + d - c\left(x - \frac{x^3}{3}\right)\right] \\
y' &= -\varepsilon(ay + x^2) \\
z' &= m\left(k - z - \frac{x}{2}\right)
\end{aligned}
$$

- If ε is small (how small?), we have a fast/slow system.
- If m is also small, we have 1-fast, 2-slow OR 3 time-scale system.
- If $m = \mathcal{O}(1)$, we have a 2-fast, 1-slow system.

$$
\begin{aligned}
\varepsilon\dot{x} &= \varepsilon(x^2 - ax) + x\left[x + y - nz + d - c\left(x - \frac{x^3}{3}\right)\right] \\
\dot{y} &= -(ay + x^2) \\
\varepsilon\dot{z} &= m\left(k - z - \frac{x}{2}\right)
\end{aligned}
$$

Fast and Slow Dynamics

If we take the limit as $\varepsilon \to 0$, the two versions become: (1) The layer problem

$$\begin{aligned} x' &= x\left[x + y - nz + d - c\left(x - \frac{x^3}{3}\right)\right] \\ z' &= m\left(k - z - \frac{x}{2}\right) \\ y' &= 0 \end{aligned}$$

Or (2) The reduced problem

$$\begin{aligned} 0 &= x\left[x + y - nz + d - c\left(x - \frac{x^3}{3}\right)\right] \\ 0 &= m\left(k - z - \frac{x}{2}\right) \\ \dot{y} &= -(ay + x^2) \end{aligned}$$

The Critical Manifold ($\varepsilon = 0$)

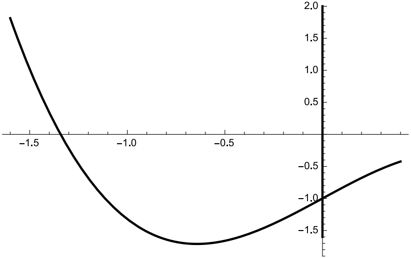
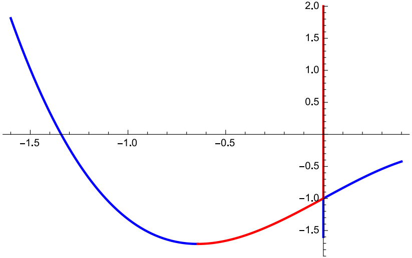


Figure: $a = 0.4, c = 4, d = 3.69, k = 1, m = 0.26, n = 2.69$

The Critical Manifold ($\varepsilon = 0$)



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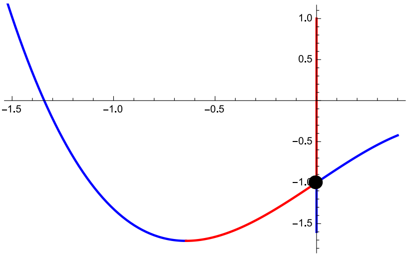

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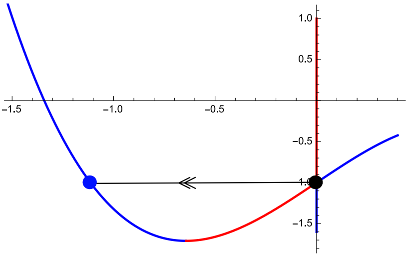

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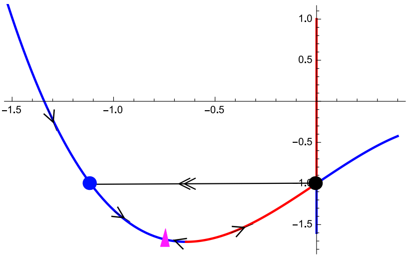

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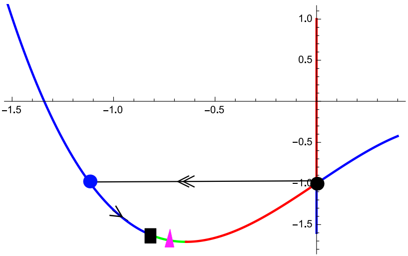

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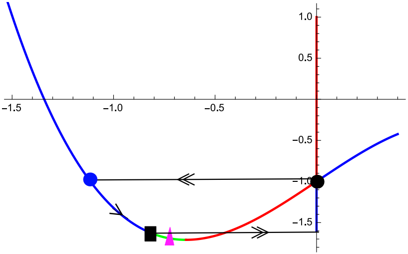

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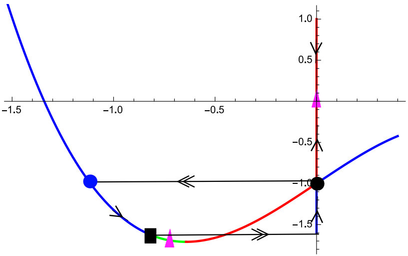

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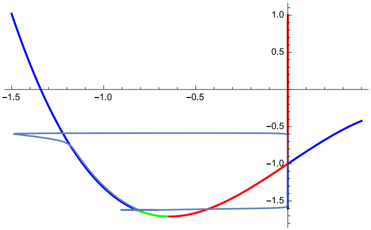
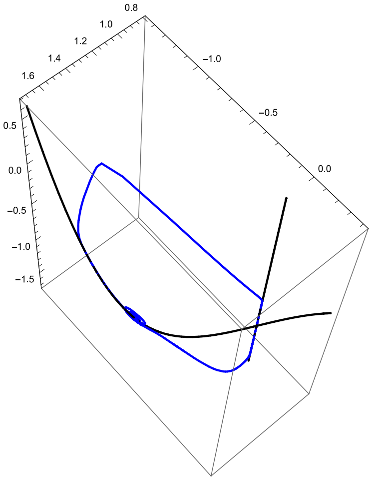
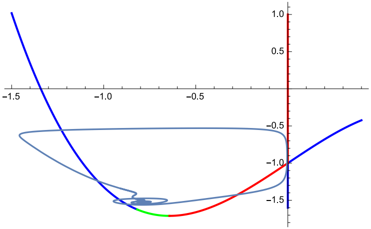
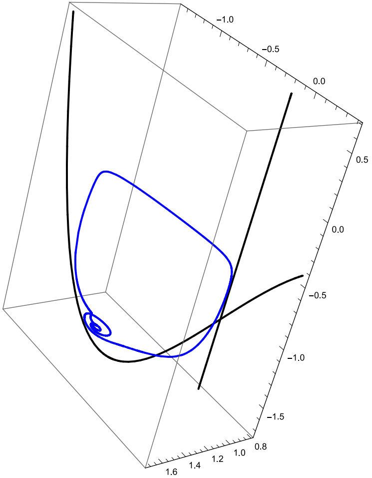
MMO Orbit: $\varepsilon = 0.001$



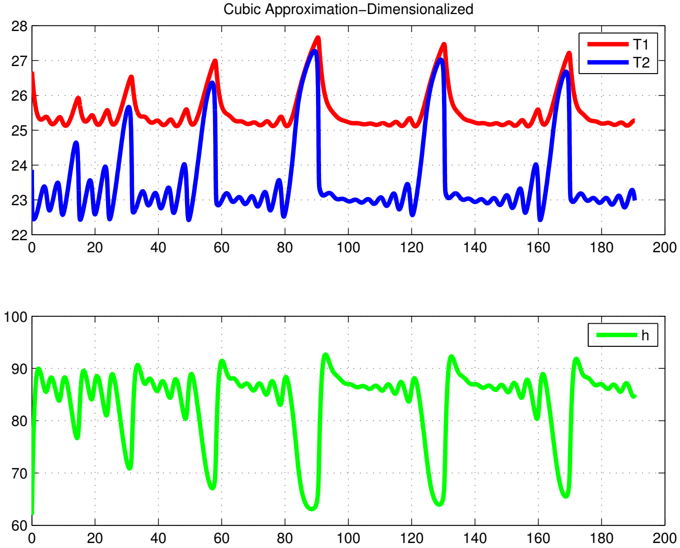


MMO Orbit: $\varepsilon = 0.1$

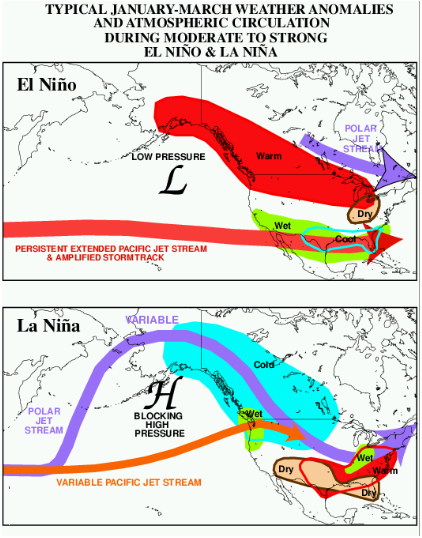




Re-dimensionalized Model Output



Consequences in the US



From: NOAA