Homoclinic Bifurcation in Welander's Model

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Outline

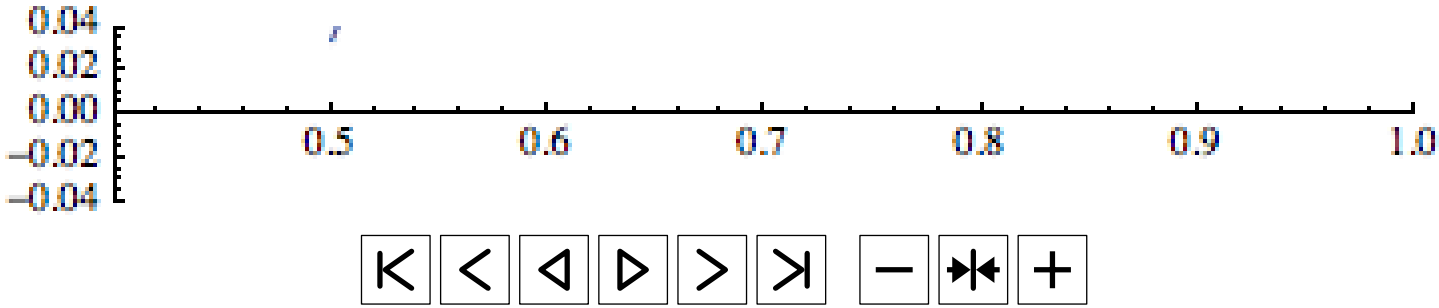
- A recap of the model
- A local picture of the bifurcation
- A global picture of the bifurcation
- The corresponding smooth phenomena

Welander's Model, nondimensionalized, coordinate changed

The Model:

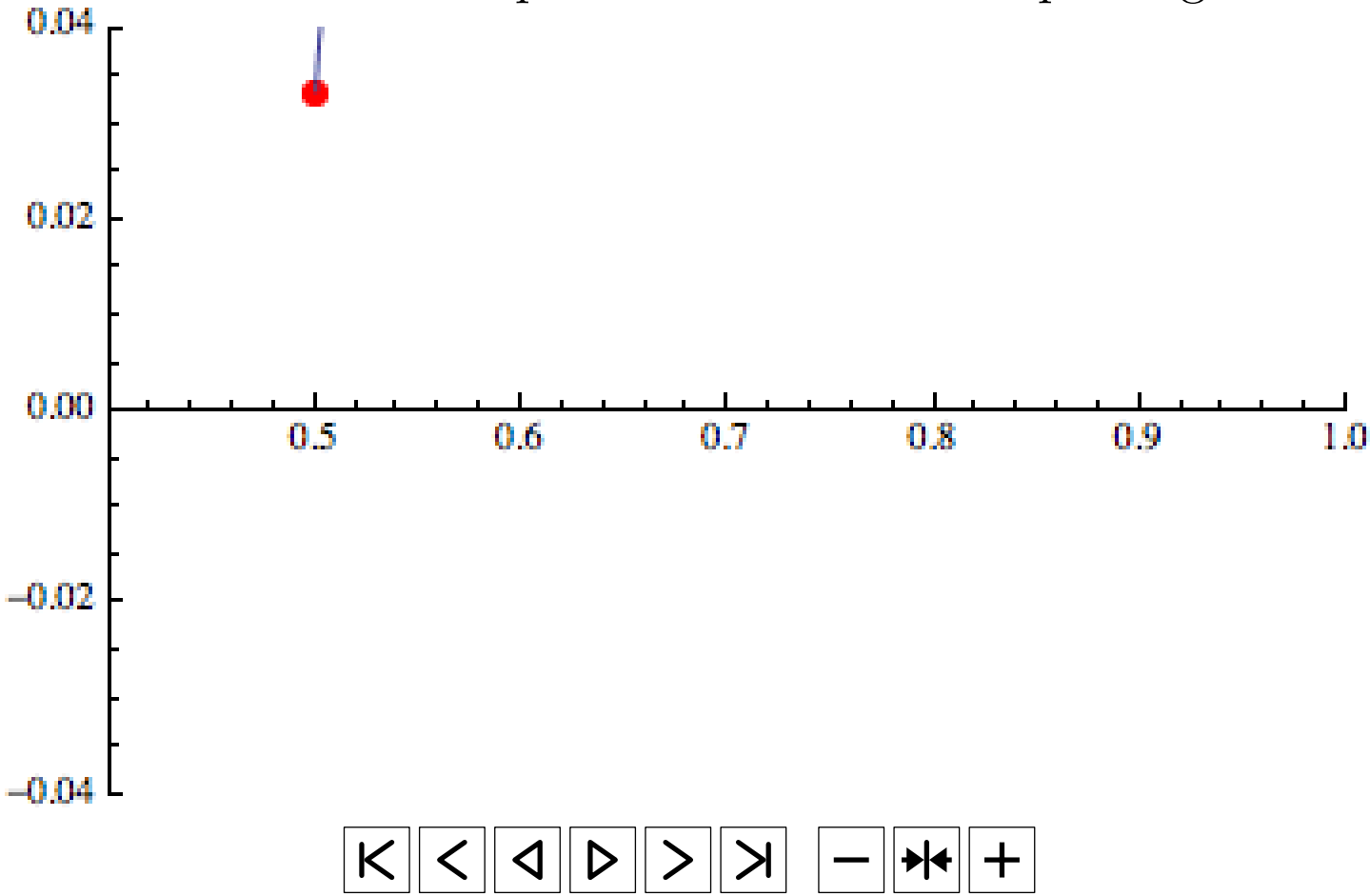
$$\begin{aligned}
\dot{x} &= 1 - x - k(y)x \\
\dot{y} &= \beta - \beta\varepsilon - \varepsilon k(y) - \alpha - (\beta + k(y))y - (\alpha\beta - \alpha)x \\
k &= \tfrac{1}{\pi}\tan^{-1}\left(\tfrac{y}{a}\right) + \tfrac{1}{2} \to \begin{cases} 1 & y > 0 \\ 0 & y < 0 \end{cases}
\end{aligned}$$

Oscillations In Welander's Model



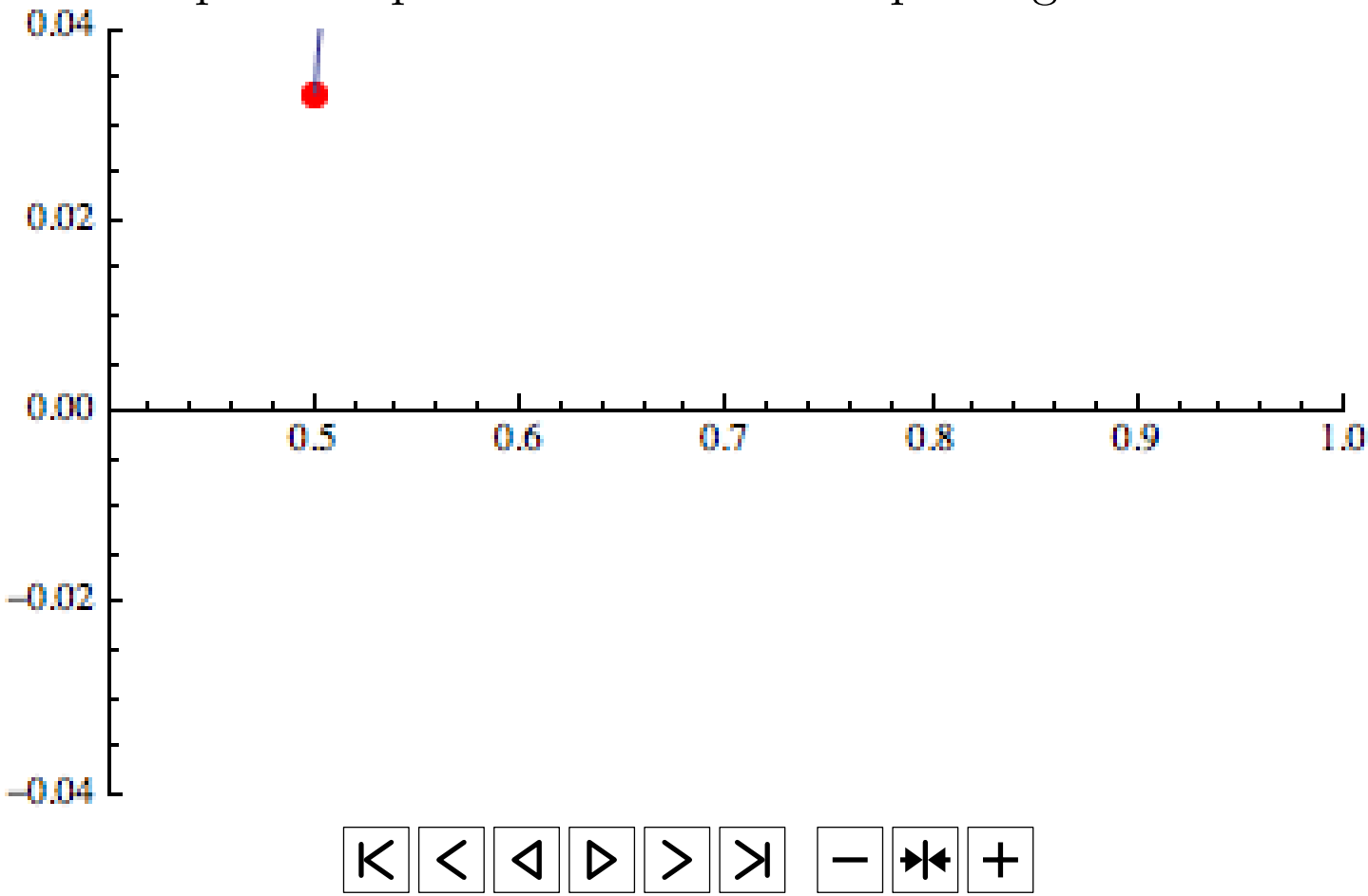
Oscillations in Welander's Model

Recall that oscillations in Welander's Model are caused by attraction to virtual equilibria. When the equilibria become "real" we expect them to be globally attracting. What happens at the point where the virtual equilibrium crosses the splitting manifold?



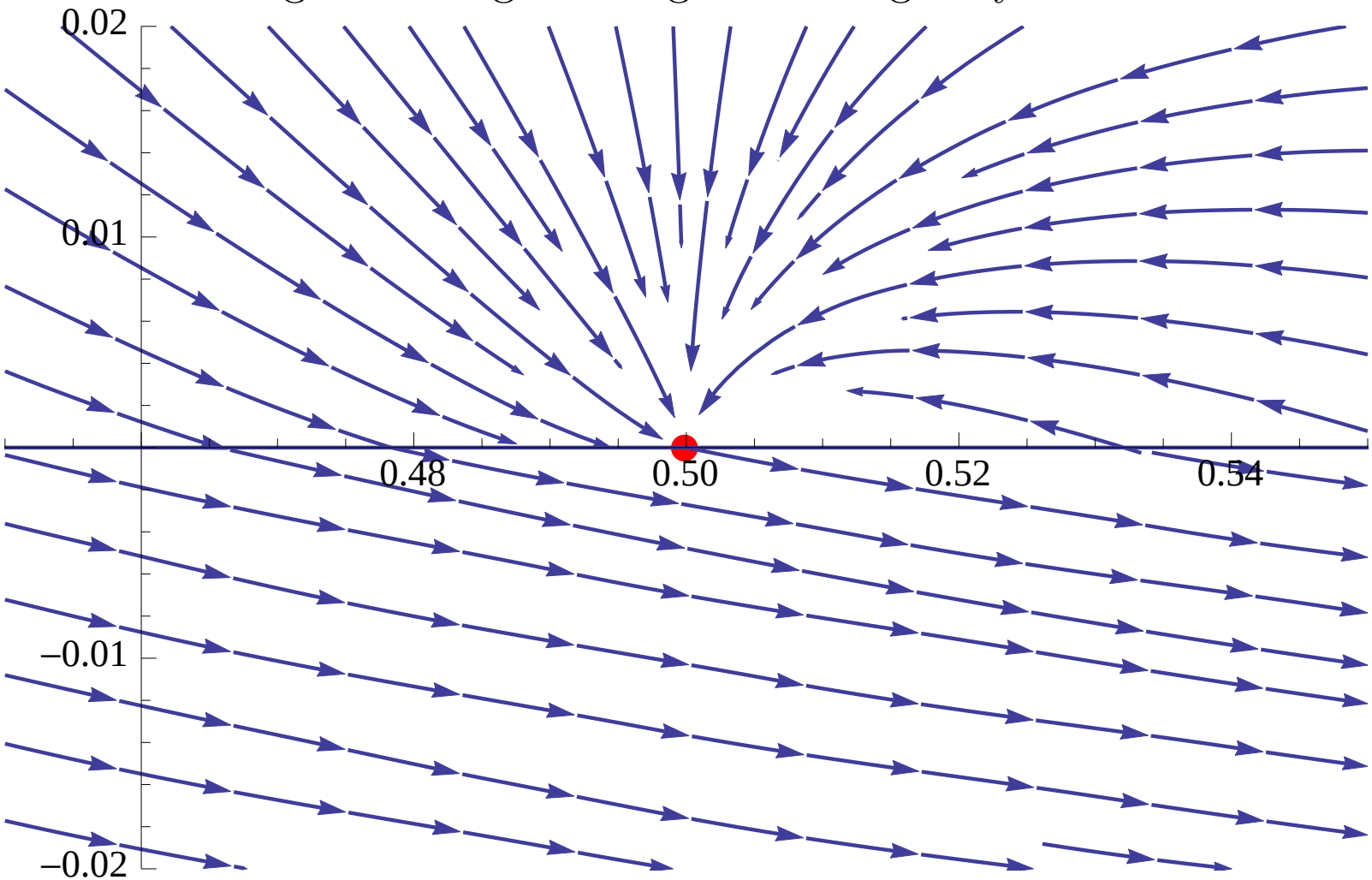
A local picture

The boundary collision can be thought of as a stability transition as a pseudoequilibrium leaves the splitting manifold.



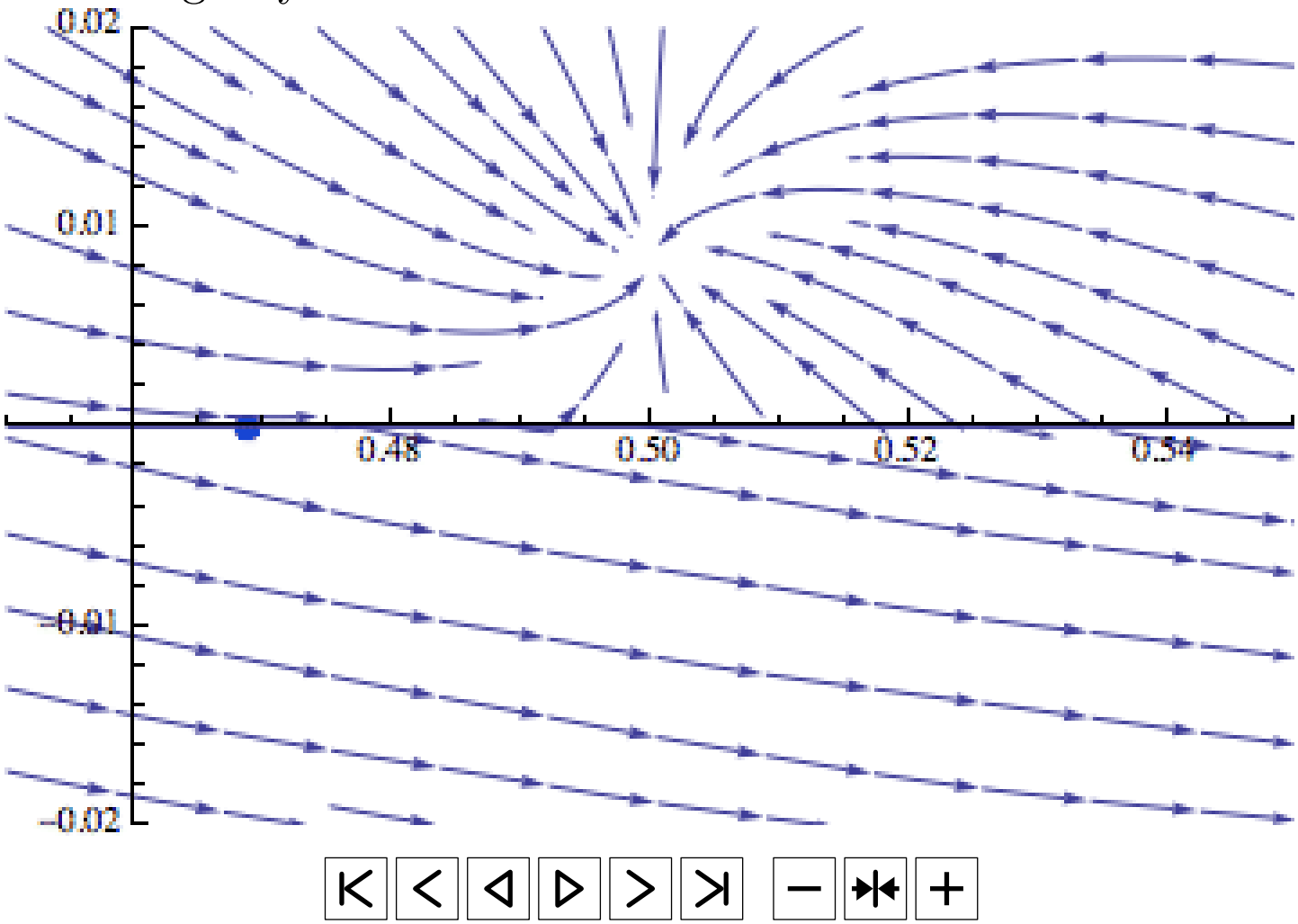
A local picture

Local stability at the bifurcation point is nonstandard. No eigenvalues go through the imaginary axis.



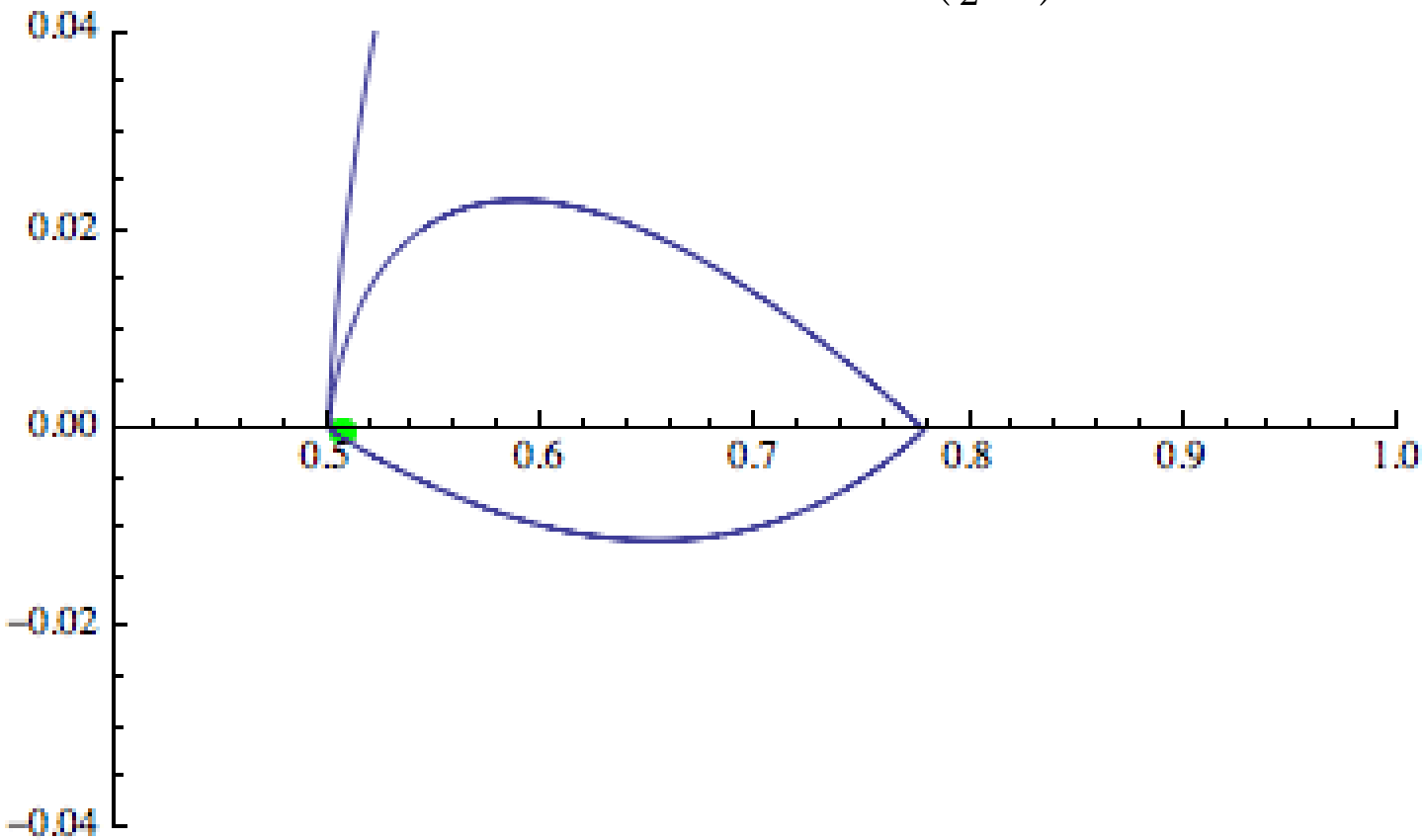
A local picture

The tangency also transitions between invisible and visible.



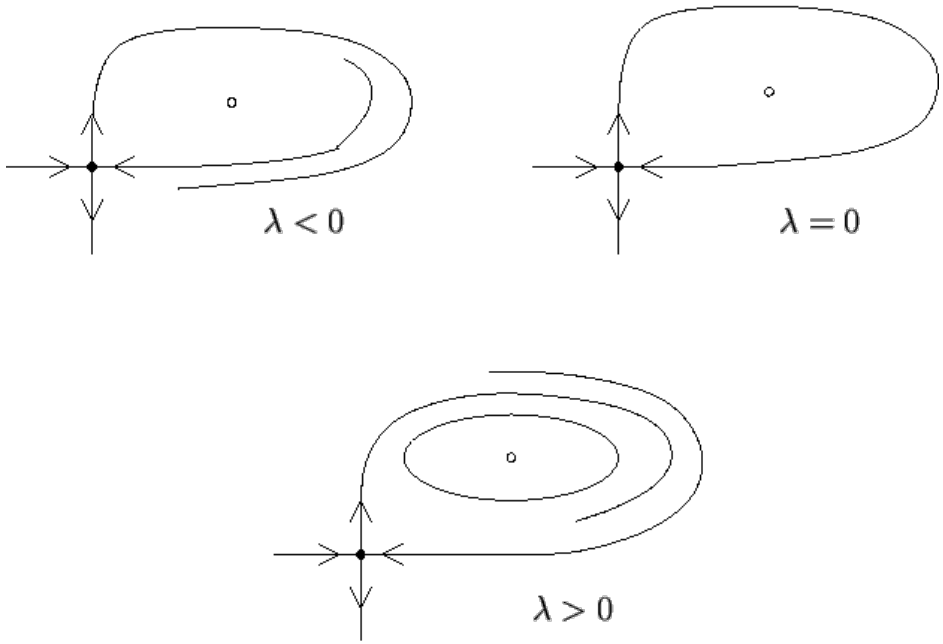
A global picture

The periodic orbit is destroyed through a homoclinic bifurcation. It is easy to show that the periodic orbit limits to a homoclinic orbit through the point $\left(\frac{1}{2}, 0\right)$.



However this bifurcation is not the result of a saddle equilibrium colliding with a periodic orbit!

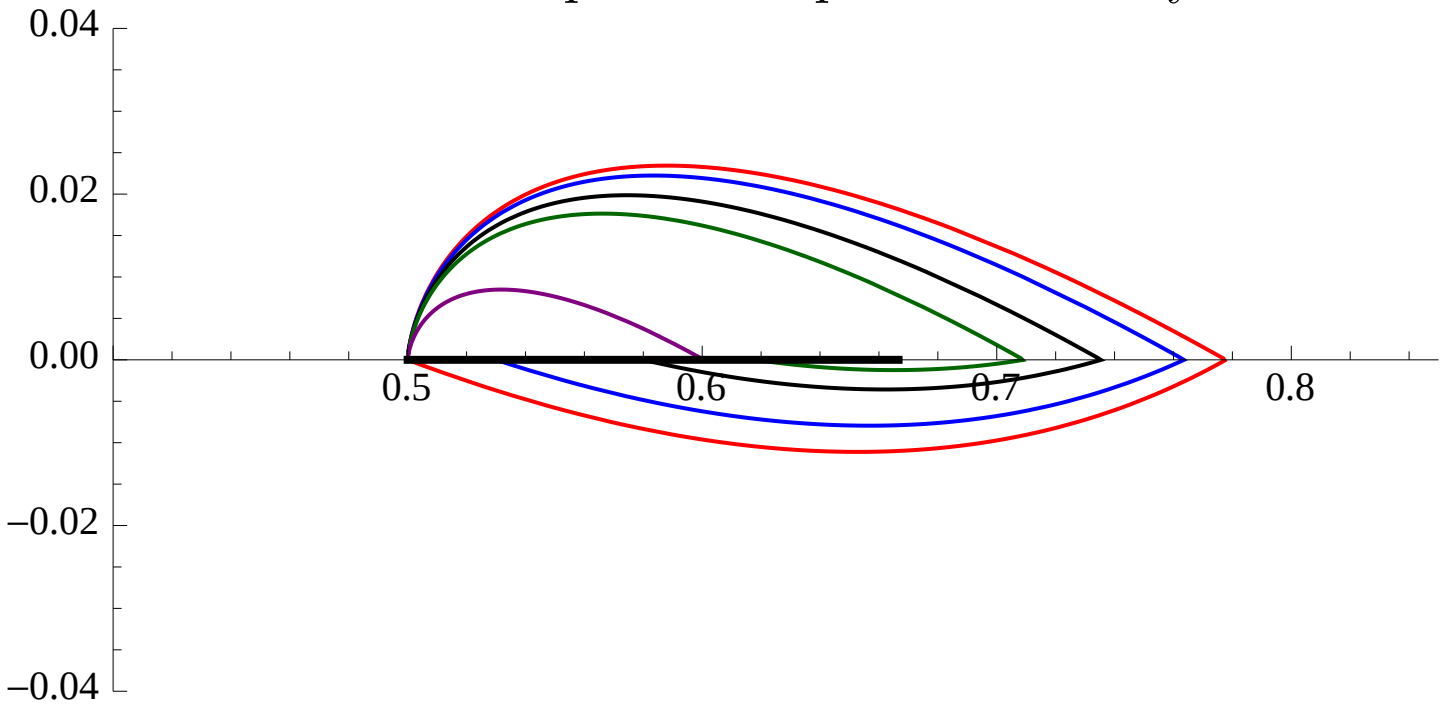
A global picture



https://me.ucsb.edu/ moehlis/APC591/tutorials/tutorial3/node3.html

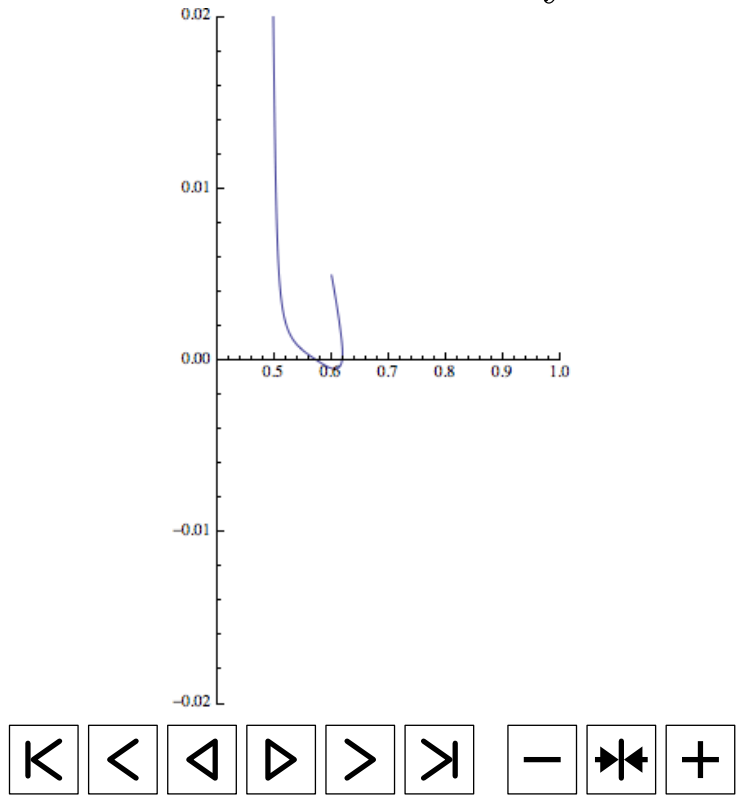
A global picture

Infinitely many homoclinic orbits go through the bifurcating equilibrium. There is no equilibrium point inside any of the orbits.



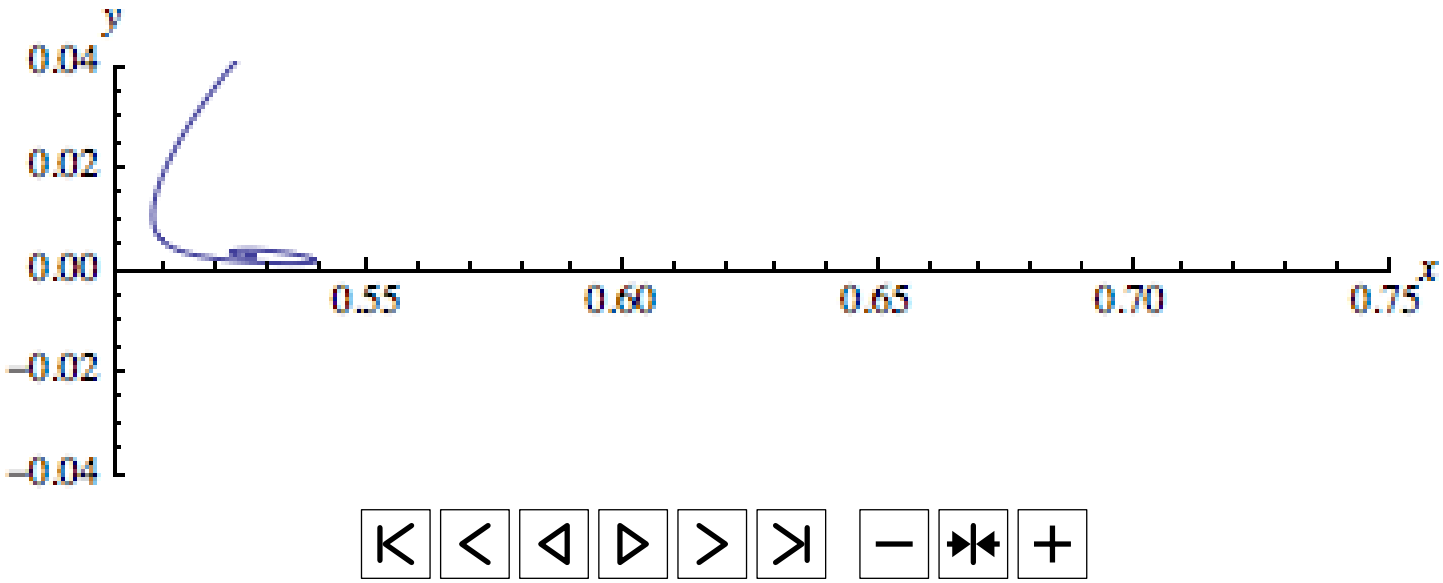
The smooth model

In the smooth model a subcritical Hopf bifurcation occurs, which gives an unstable periodic orbit. This can be seen by reversing time in the smooth system.



The Smooth Model

The unstable periodic orbit and the stable periodic orbit annihilate each other in a periodic orbit saddle node bifurcation.



Remarks and Questions

- In Welander's Model, the periodic orbit is destroyed through a nonsmooth homoclinic bifurcation.
- This bifurcation seems to be a degenerate limit of two phenomena in the smooth system, a subcritical Hopf bifurcation, and a periodic orbit saddle node.