Energy Balance Models for Planetary and Lunar Climates

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University of Minnesota Mathematics of Climate Seminar

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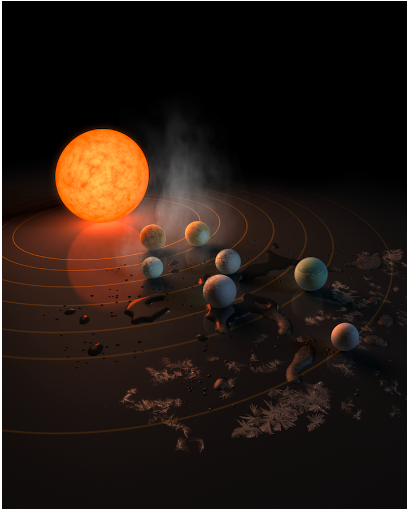
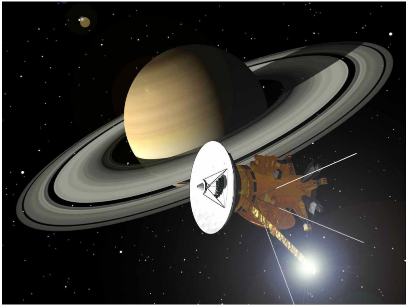
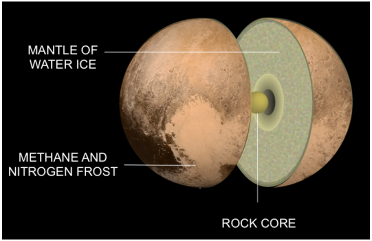
Earth is cool but other places are cooler...

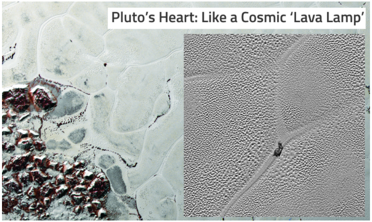


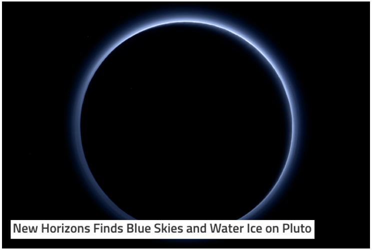


Photos from nasa.gov

Pluto and Charon



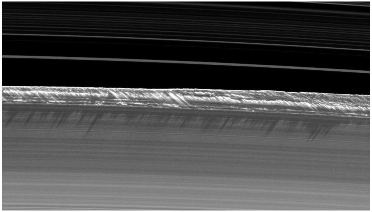


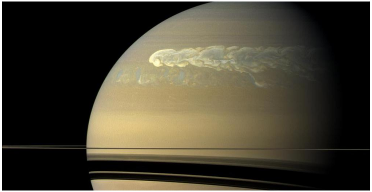


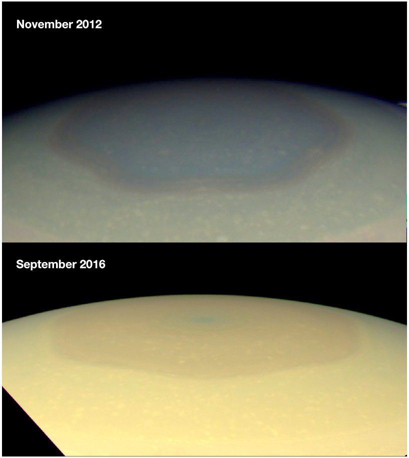
Pluto 'Paints' its Largest Moon Red

Photos from nasa.gov

NASA's Cassini Mission: Saturn

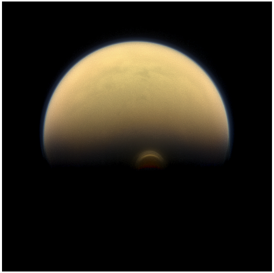


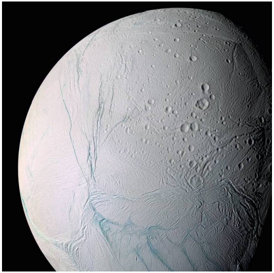


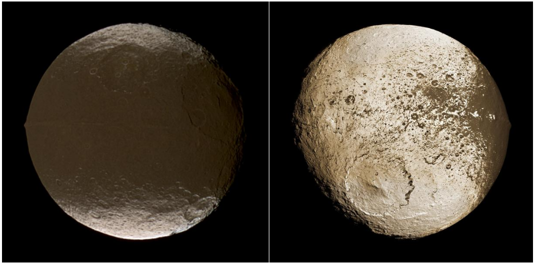


Photos from nasa.gov

NASA's Cassini Mission: Titan, Enceladus, and Iapetus







Photos from nasa.gov

Europa

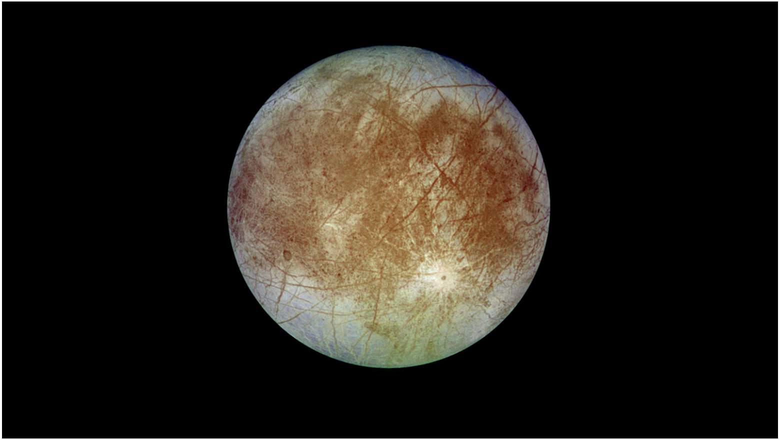


Photo from nasa.gov

The TRAPPIST-1 System

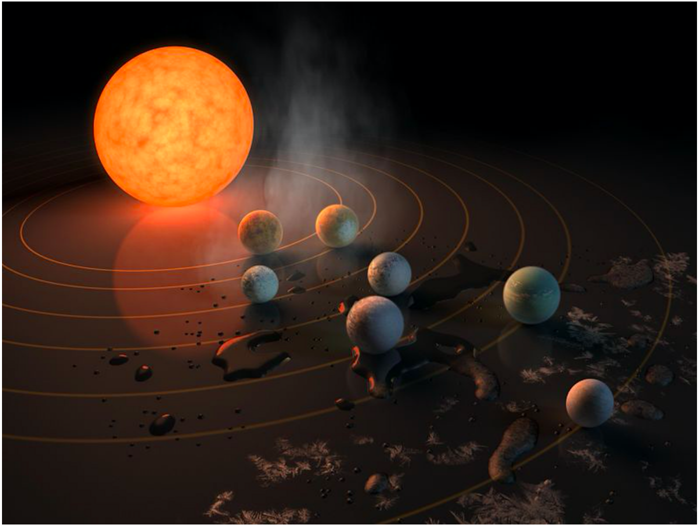


Photo from nasa.gov

Budyko's Model

Recall the Budyko-Widiasih equation for Earth's energy balance:

$$\frac{\partial}{\partial t}T = \frac{1}{R}\left(Qs(y)(1-\alpha(\eta,y)) - (A+BT(y,\eta)) - C\left(T(y,\eta)-\overline{T}\right)\right)$$

for $y, \eta \in [0,1]$ and with dynamic ice line

$$\dot{\eta} = \rho(T(\eta,\eta) - T_c)$$

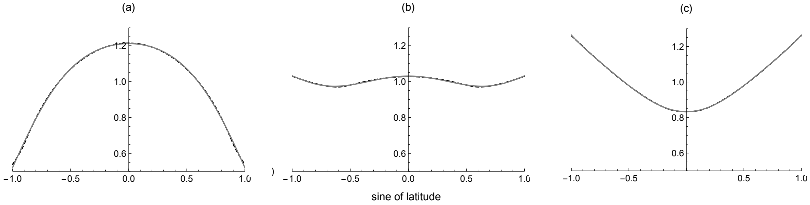
and piecewise constant albedo function

$$\alpha(y,\eta) = \begin{cases} \alpha_w & y < \eta \\ \alpha_0 & y = \eta \\ \alpha_i & y > \eta \end{cases}$$

Insolation Distribution Depends on Obliquity

The insolation distribution as a function of y depends on the planet's obliquity, β, and is given by

$$s(y,\beta) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\sin\gamma - y\cos\beta\right)^2}\,d\gamma$$



Nadeau and McGehee, "A simple formula for a planet's mean annual insolation by latitude."

Insolation Distribution Approximation

We can write the Legendre series expansion in y

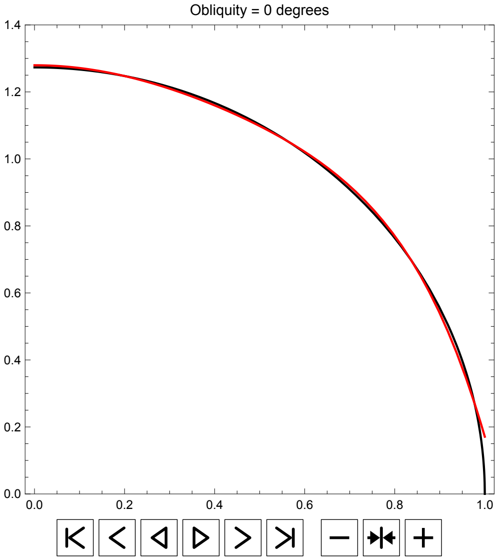
$$s(y,\beta) = \frac{2}{\pi^2}\int_0^{2\pi}\sqrt{1-\left(\sqrt{1-y^2}\sin\beta\sin\gamma - y\cos\beta\right)^2}\,d\gamma$$
$$= \sum_{n=0}^{\infty} B_{2n}(\beta)P_{2n}(y)$$

which turns out to have a really nice form

$$s(y,\beta) = \sum_{n=0}^{\infty} A_{2n}P_{2n}(\cos\beta)P_{2n}(y)$$

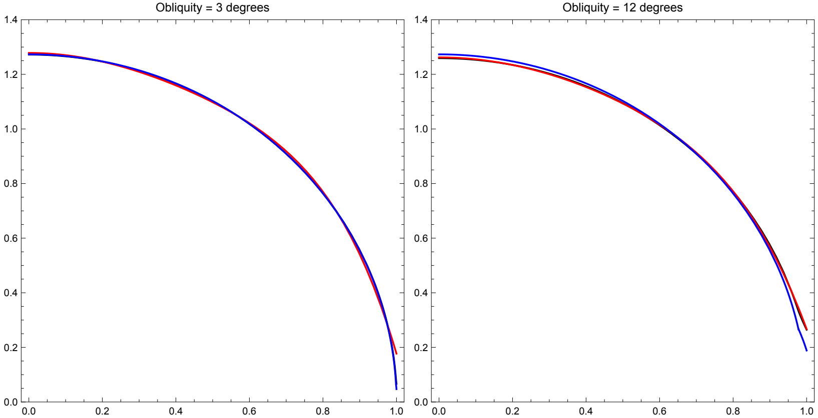
admitting a simple polynomial approximation to the sixth degree.

Sixth Degree Approximation Great for $\beta \geq 12^\circ$



Ojakangas-Stevenson Model for Small Obliquity

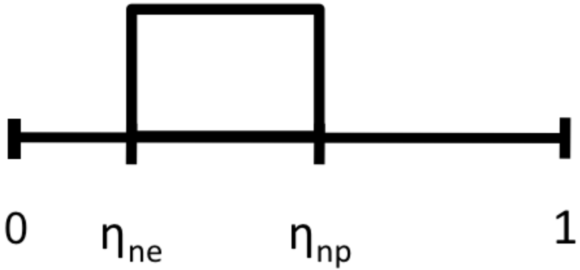
$$s(y,\beta) \approx \begin{cases} \frac{4\sqrt{1-y^2}}{\pi} & \arccos(y) > \beta \\ \frac{\sqrt{2(\beta^2-\arccos^2(y))}}{\pi} & \arccos(y) \leq \beta \end{cases}$$



Two Ice Lines

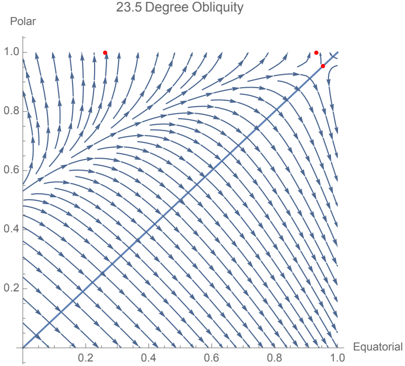
Want to be able to account for general obliquity, so add another ice line equation

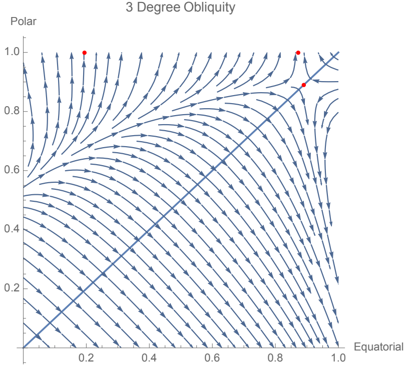
$$\dot{\eta}_e = \rho(T(\eta,\eta) - T_c)$$
$$\dot{\eta}_p = \rho(T_c - T(\eta,\eta))$$



Obliquity Changes Number and Location of Equilibria

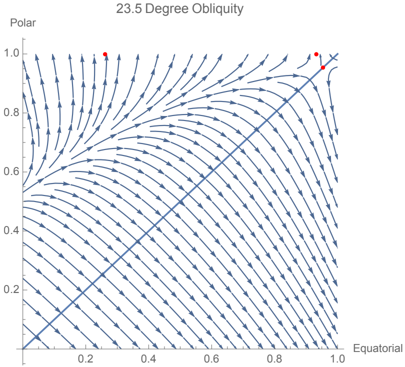
"Jupiter Earth"

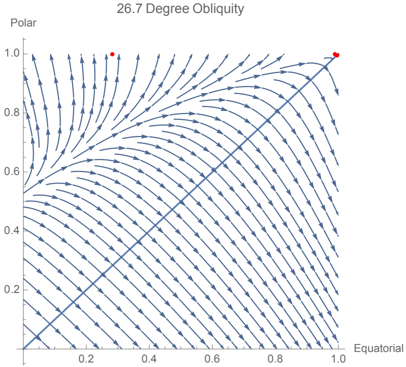




Obliquity Changes Number and Location of Equilibria

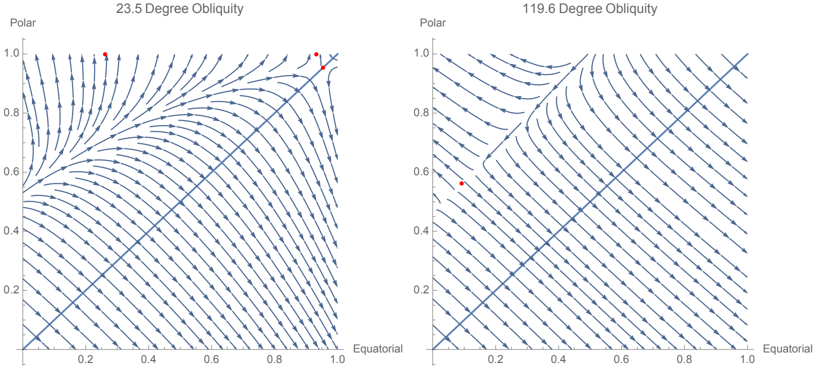
"Saturn Earth"





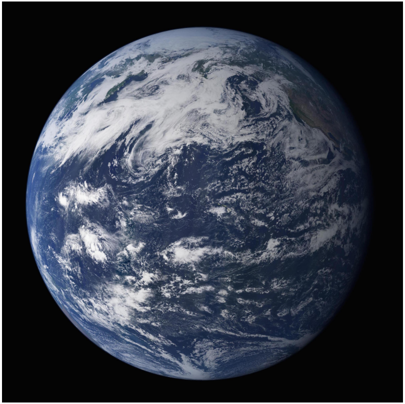
Obliquity Changes Number and Location of Equilibria

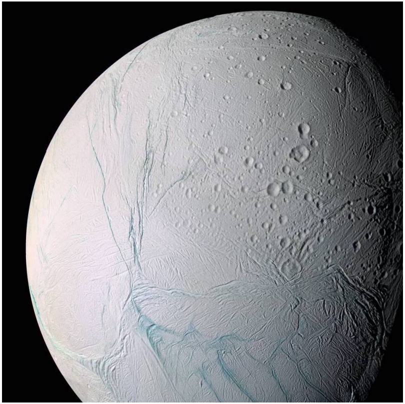
"Pluto Earth"



Finding the Budyko Constants *A*, *B*, and *C*

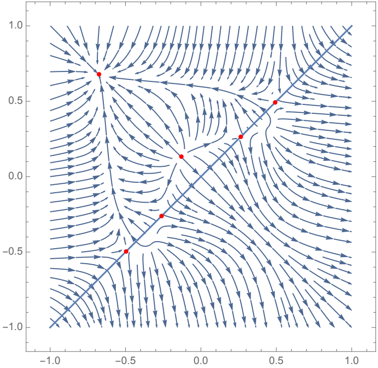
- For Earth, *A* and *B* were determined empirically from satellite data [Tung]
- Does the heat transport term even make sense for modeling 'icy' planets and moons?





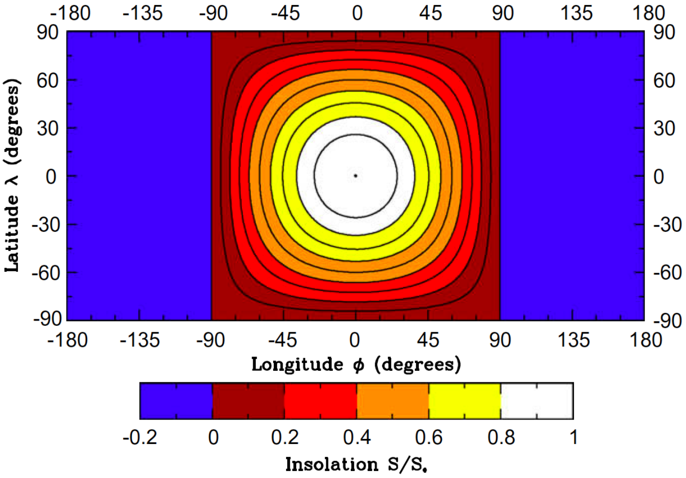
Emma Jaschke: Adapting Budyko's Model to Pluto

Preliminary Results: In a two ice-line model, the only stable ice-line configurations are a large ice belt and ice-free Pluto.



South Ice Line vs. North Ice Line Vector Plot

Mean Annual Insolation from Synchronous Rotation



A. Dobrovolskis, "Insolation patterns on synchronous exoplanets with obliquity."

A Longitudinal Budyko-Widiasih Model for 1:1 Resonance

We'll use the same form as before

$$\frac{\partial}{\partial t}T = \frac{1}{R}\left(Qs(\phi)(1-\alpha(\phi,\mu)) - (A + BT(\phi,\mu)) - C\left(T(\phi,\mu) - \overline{T}\right)\right)$$

for $\phi, \mu \in [0, \pi]$, but considering the longitudinal changes in the ice line

$$\dot{\mu} = \rho(T(\mu,\mu) - T_c)$$

and piecewise constant albedo function, dependent on longitude

$$\alpha(\phi,\mu) = \begin{cases} \alpha_w & \phi < \mu \\ \alpha_0 & \phi = \mu \\ \alpha_i & \phi > \mu \end{cases}$$

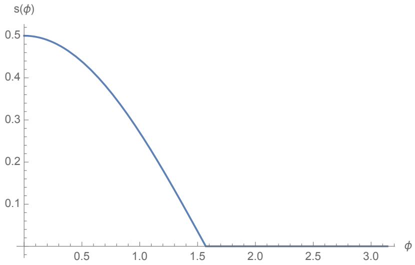
A Longitudinal Insolation Distribution

In the 1:1 case for a circular orbit, mean annual insolation is given by

$$s(\phi, \lambda) = \frac{\cos(\phi)\cos(\lambda) + |\cos(\phi)\cos(\lambda)|}{8}$$

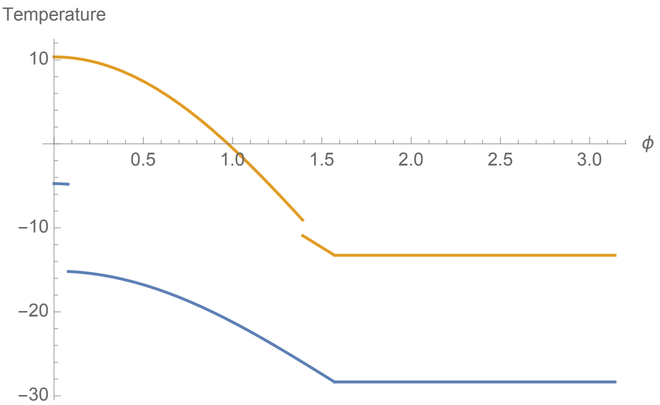
averaging over latitude yields

$$s(\phi, \lambda) = \frac{\cos(\phi) + |\cos(\phi)|}{4}$$



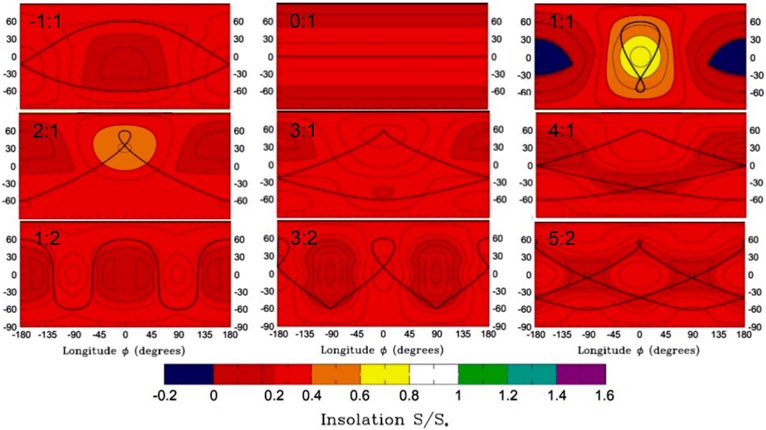
Longitudinal Temperature Profiles

There are two equilibrium ice line configurations:



Small Integer Spin-Orbit Resonances

Mean annual insolation distributions for $\beta = 60^\circ$ and $e = .2$:



A. Dobrovolskis, "Insolation on exoplanets with eccentricity and obliquity."

Some Questions about Insolation

- Can the Budkyo-Widiasih model be adapted to slowly rotating planets?
- Can we quantify the rates at which we can use the “rapidly spinning planet” method and approximation?
- How is insolation distributed on a despinning planet? Can this affect climate?

(1) M. Budyko. "The effect of solar radiation variations on the climate of the Earth." *Tellus*, **21**(5): 1969. 611-619.

(2) A. Dobrovolskis."Insolation patterns on synchronous exoplanets with obliquity." *Icarus* **204**: 2009. 1-10.

(3) A. Dobrovolskis."Insolation on exoplanets with eccentricity and obliquity." *Icarus* **226**: 2013. 760-776.

(4) H. Kaper and H. Engler. *Mathematics and Climate*, **Society for Industrial and Applied Mathematics**: 2013.

(5) A. Nadeau and R. McGehee. "A Simple Formula for a Planet's Mean Annual Insolation by Latitude." *Icarus*, **291**: 2017.

(6) R. McGehee and C. Lehman. "A Paleoclimate model of Ice Albedo Feedback Forced by Variations in Earth's Orbit." *SIAM J. Applied Dym. Sys.*, **11** (2): 2012.

(7) R. McGehee and E. Widiasih. "A Quadratic Approximation to Budyko's Ice-Albedo Feedback Model with Ice Line Dynamics." *SIAM J. Applied Dym. Sys.*, **13** (1): 2014.

(8) W. Ward. "Climatic Variations on Mars: Astronomical Theory of Insolation." *Journal of Geophysical Reseach*, **79**(24): 1974. 3375-3386.

(9) E. Widiasih. "Dynamics of the Budyko Energy Balance Model." *SIAM J. Applied Dym. Sys.*, **12** (4): 2013

Thank you!