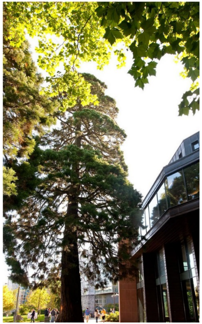




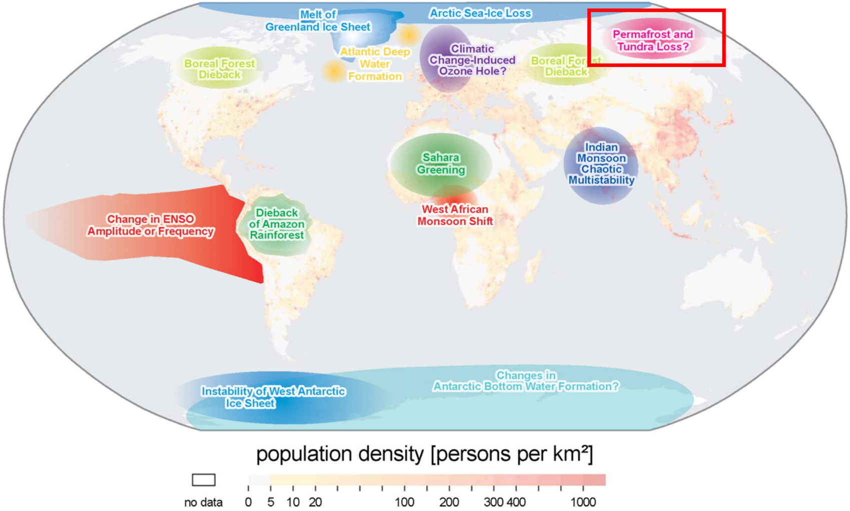
A TRADITION OF INDEPENDENT THINKING

UCC
University College Cork, Ireland
Coláiste na hOllscoile Corcaigh

Rate-Induced Tipping of the Compost Bomb: Sizzling Summers, Heteroclinic Canards and Metastable Zombie Fires

Eoin O’Sullivan, Kieran Mulchrone, Sebastian Wieczorek
University College Cork
8th November 2022

Tipping Points



T. M. Lenton, H. Held, E. Kriegler, J. W. Hall, W. Lucht, S. Rahmstorf, and H. J. Schellnhu-ber. Tipping elements in the earth's climate system. Proc. Nat. Acad. Sci., 105(6):1786–1793, 2008.

Tipping Points

- Bifurcation-induced (B-tipping)
- Noise-induced (N-tipping)
- Rate-induced (R-tipping)

Ashwin P, Wieczorek S, Vitolo R, Cox P. 2012 Tipping points in open systems: bifurcation, noise-induced and rate-dependent examples in the climate system. Phil. Trans. R. Soc. A 370, 1166–1184

Rate-Induced Tipping

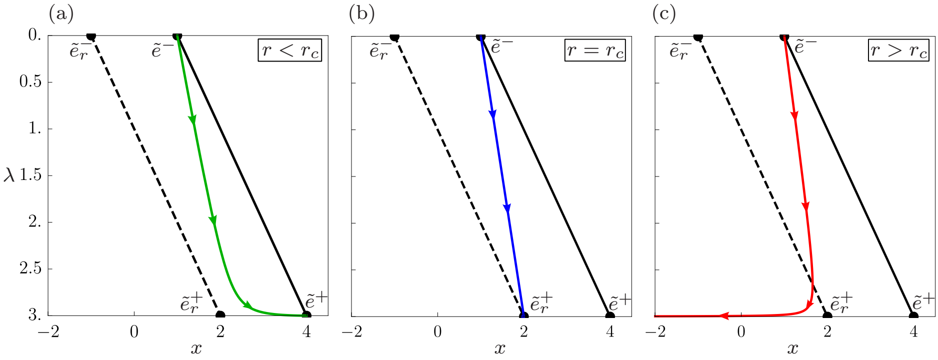
Simple example:

$$\frac{dx}{dt} = 1 - (x - \lambda)^2$$
$$\frac{d\lambda}{dt} = r\lambda(3 - \lambda),$$

For $x \in \mathbb{R}$ and $\lambda \in [0, 3]$.

Slyman K, Jones CK. 2022 Rate and Noise-Induced Tipping Working in Concert. arXiv:2210.00873

Rate-Induced Tipping



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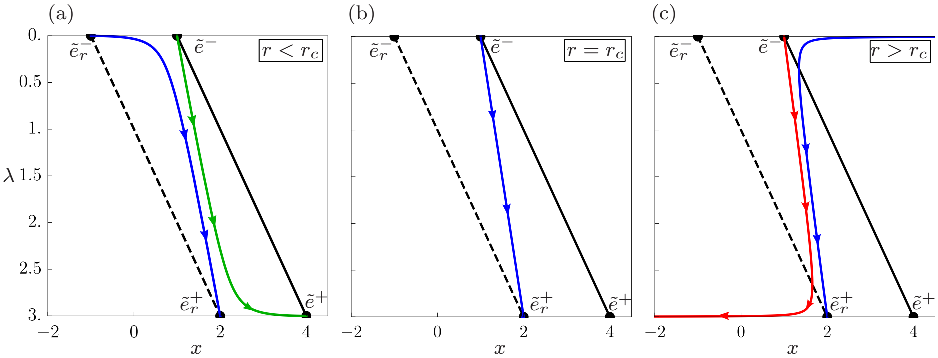
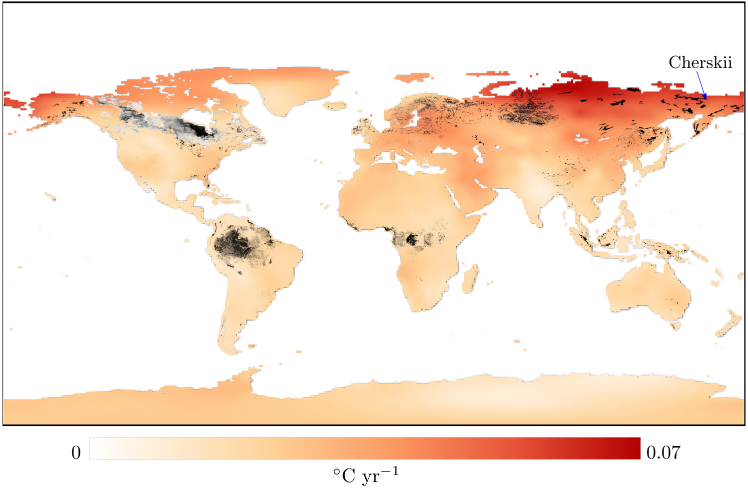
Rate-Induced Tipping



Slyman K, Jones CK. 2022 Rate and Noise-Induced Tipping Working in Concert. arXiv:2210.00873

Rate-Induced Tipping



Zombie Fires

Scholten, R.C., Jandt, R., Miller, E.A. et al. Overwintering fires in boreal forests. Nature 593, 399–404 (2021)

London Fires

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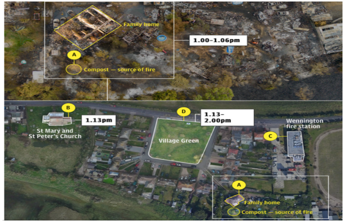
Wennington fire: Compost blaze that devastated village started just yards from fire station

Local firefighters were out on another call when flames ripped through their community on Britain's hottest ever day



News

London fires: How did Wennington fire start, was it a compost heap and did extreme heat start other fires?

London firefighters had their busiest day on Tuesday since the second world war, according to mayor of London, Sadiq Khan.

Mirror

The east London blaze is said to have been caused by spontaneous combustion (Image: Getty Images)

NEWS | POLITICS | FOOTBALL | CELEBS | TV | MONEY | TRAVEL

Man claims Wennington fire started when 'compost heap spontaneously combusted'

More than 100 firefighters battled a huge blaze in east London this afternoon which threatened to engulf several home as temperatures reached 40C in the country

By Dan Warburton, News Reporter & Antony Thrower, News Reporter
23:45, 19 Jul 2022 | UPDATED 18:44, 20 Jul 2022

4 COMMENTS

Soil-Carbon Model

The starting point is a 3 process, 2 variable model:

$$\epsilon\frac{dT}{dt} = -\frac{\lambda}{A}(T - T_a(rt)) + C\, r_s(T)$$
$$\frac{dC}{dt} = \Pi - C_s\, r_s(T).$$

T - Soil Temperature

T_a - Air Temperature

C - Soil Carbon

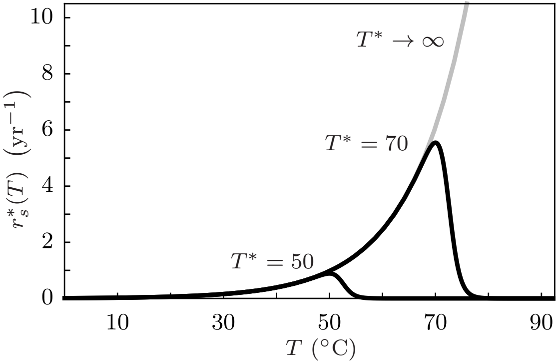
$r_s(T)$ - Microbial Respiration

Luke, C.M. and Cox, P.M. (2011), Soil carbon and climate change: from the Jenkinson effect to the compost-bomb instability. European Journal of Soil Science, 62: 5-12.

Wieczorek S., Ashwin P., Luke C. M. and Cox P. M. 2011, Excitability in ramped systems: the compost-bomb instability. Proc. R. Soc. A.4671243–1269

Clarke, J., Huntingford, C., Ritchie, P. et al. The compost bomb instability in the continuum limit. Eur. Phys. J. Spec. Top. (2021).

Modified Microbial Respiration

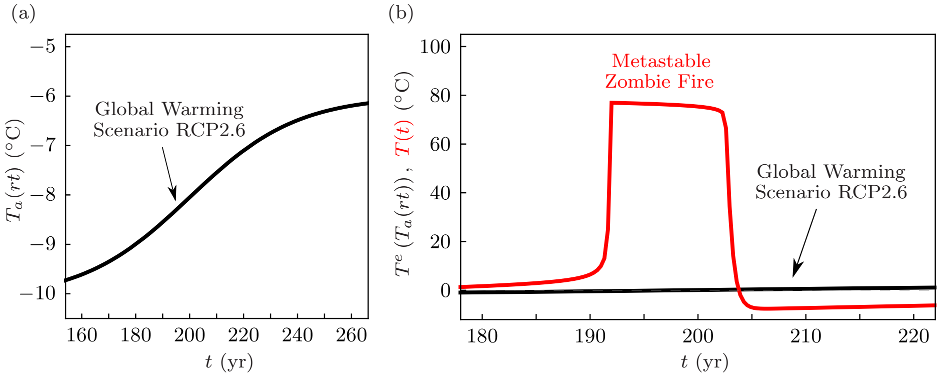


$$r_s(T) = r_s(0)\exp\left[\alpha T_s\right]$$

$$r_s^*(T) = \frac{r_s(0)\left(e^{-c\alpha b} + e^{\alpha b}\right)}{e^{c\alpha(T_s - b)} + e^{-\alpha(T-b)}}$$

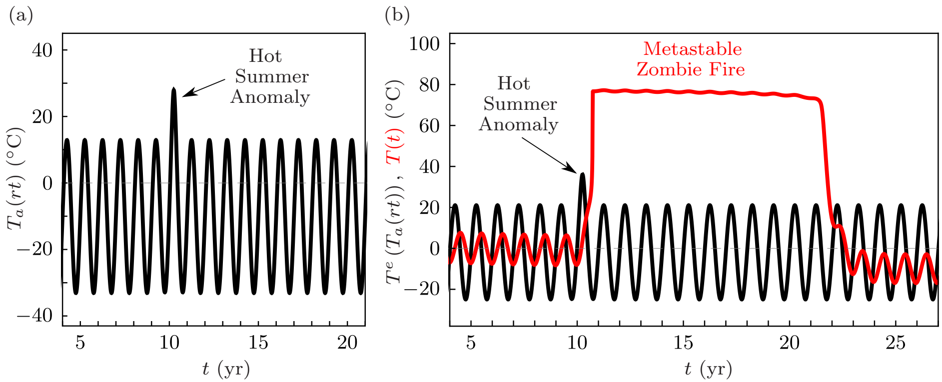
$$b = T^* + \frac{\ln c}{\alpha + c\alpha}.$$

Cherskii Climate Change



Chersky Climate: Average Temperature, Weather by Month, Chersky water temperature - climate-data.org

Sizzling Siberian Summer



Chersky Climate: Average Temperature, Weather by Month, Chersky water temperature - climate-data.org

Comparison with Medium Complexity PDE Model

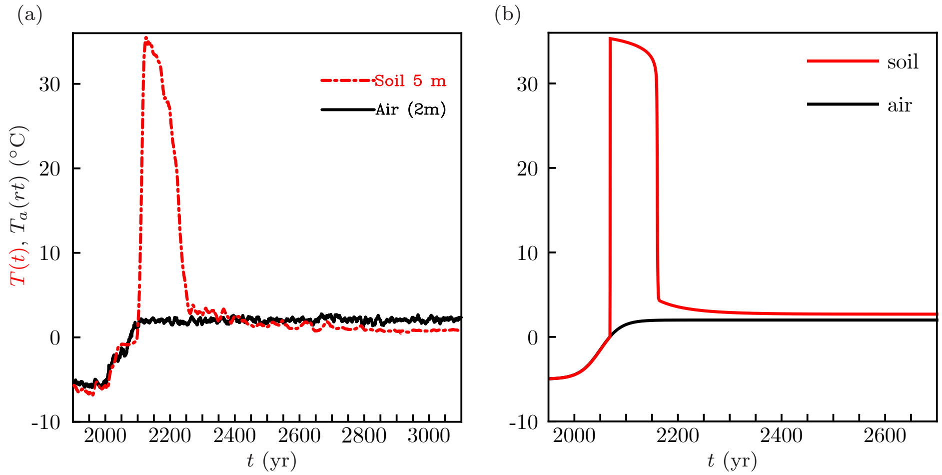
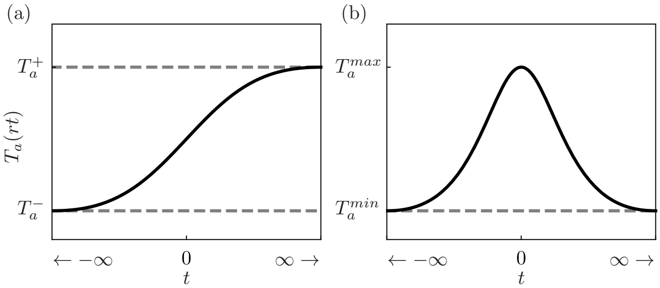


Figure: Qualitative agreement with model of [Khvorostyanov et al. Tellus (2008), 60B] (left).

External Inputs



(a) Global Warming

$$T_a(rt) = \frac{T_a^+}{2}(\tanh{(rt)} + 1).$$

(b) Hot Summer Anomaly

$$T_a(rt) = (T_a^{max})\,\mathrm{sech}(rt)$$

Compactification

Introduce a third dependent variable:

$$s = \tanh\left(\frac{\nu}{2} rt\right), \quad s \in (-1, 1),$$

And extend the vector field to $\{s = \pm 1\}$ as follows:

$$T_a^{(\nu)}(s) = \begin{cases} T_a\left(2\tanh^{-1}(s)/\nu\right) & \text{for } s \in (-1,1), \\ T_a^+ & \text{for } s = 1, \\ T_a^- & \text{for } s = -1. \end{cases}$$

S. Wieczorek, C. Xie, and C. K. R. T. Jones. Compactification for asymptotically autonomous dynamical systems: theory, applications and invariant manifolds. Nonlinearity, 34(5):2970–3000, May 2021

Autonomous Compactified System

This gives the autonomous *compactified system*

$$\begin{aligned}
\epsilon\,\frac{dT}{dt} &= f_1(T, C, T_a^{(\nu)}(s)),\\
\frac{dC}{dt} &= f_2(T, C),\\
\frac{1}{r}\,\frac{ds}{dt} &= \frac{\nu}{2}(1 - s^2),
\end{aligned}$$

with equilibria $\tilde{e}^-$ (saddle) and $\tilde{e}^+$ (sink) in the planes $\{s = \pm 1\}$.

Global Warming: 1 Fast 2 Slow

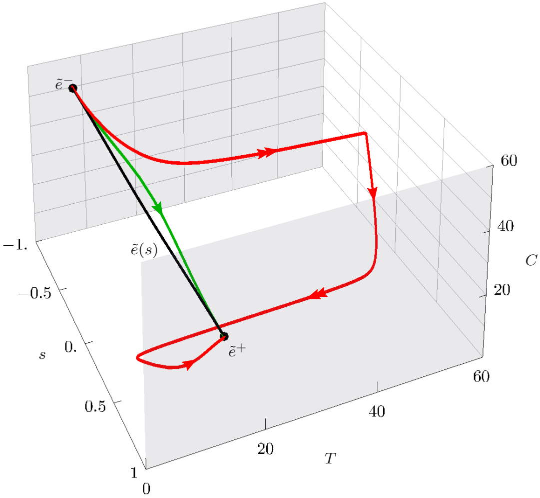
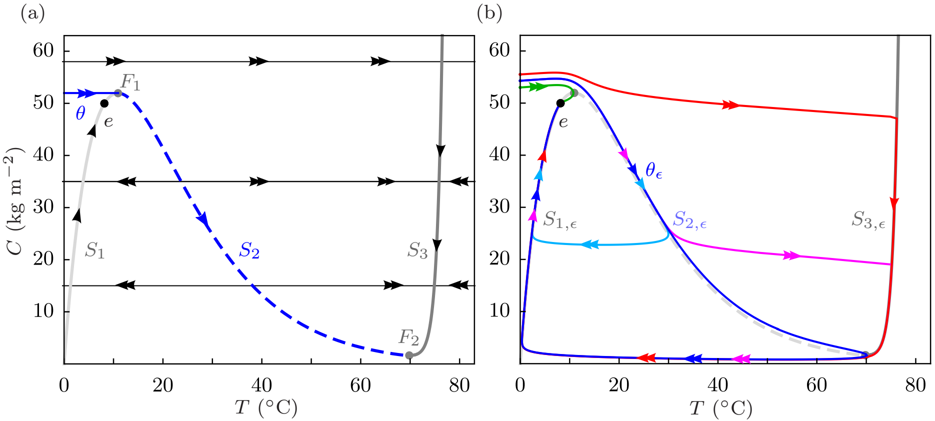


Figure: Solutions for $\epsilon \approx 0.064$.

The Frozen System: 1 Fast 1 Slow



The Frozen System condition is $r = 0$.

The singular limit $\epsilon \to 0$ gives the critical manifold S.

Global Warming: 1 Fast 2 Slow

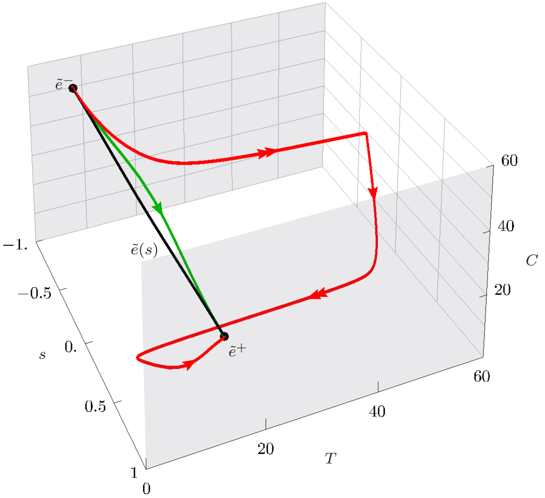


Figure: Solutions for $\epsilon \approx 0.064$.

Global Warming: 1 Fast 2 Slow

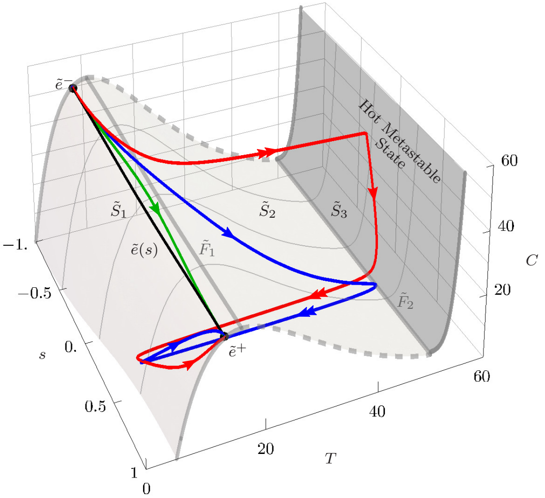
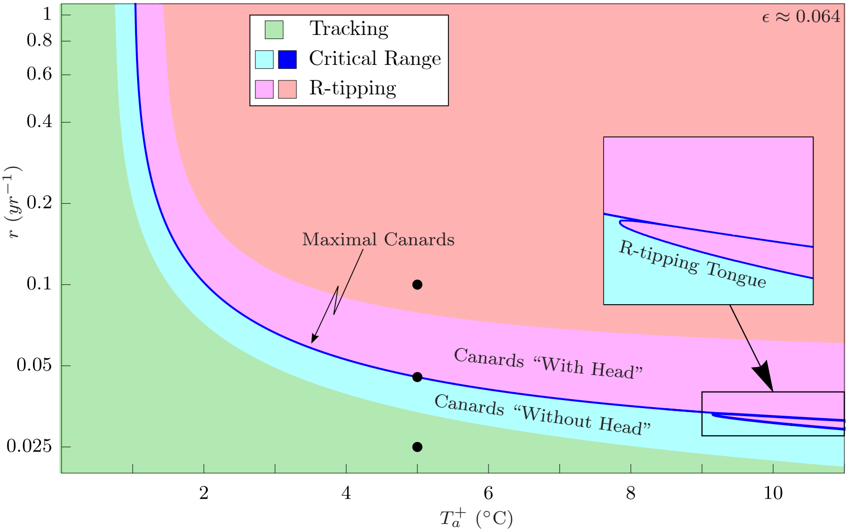
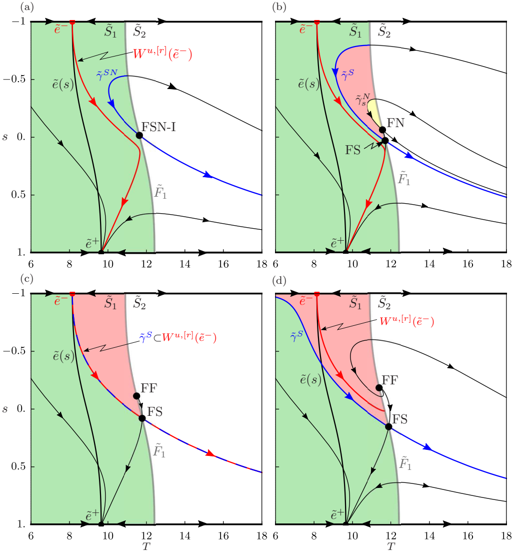


Figure: Solutions for $\epsilon \approx 0.064$ with $\tilde{S}$ overlaid.

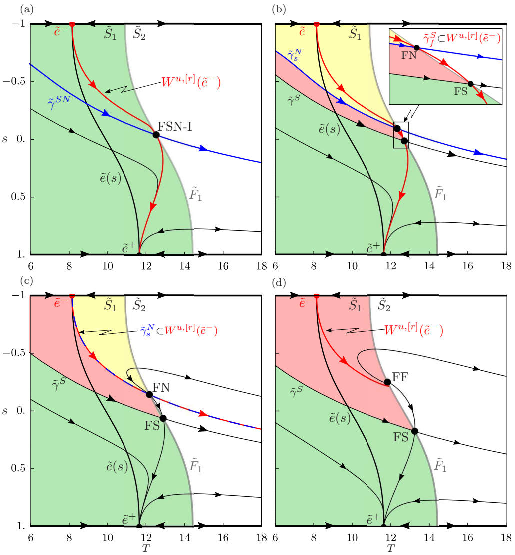
R-Tipping Diagram $\epsilon > 0$



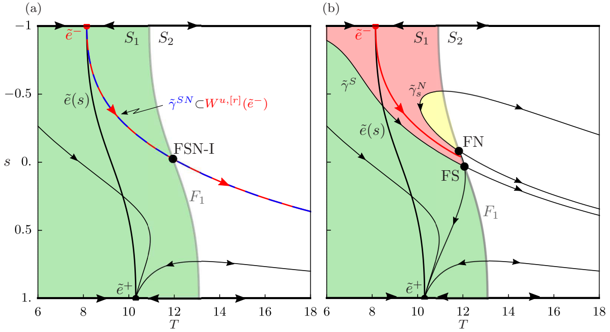
The Simple Case



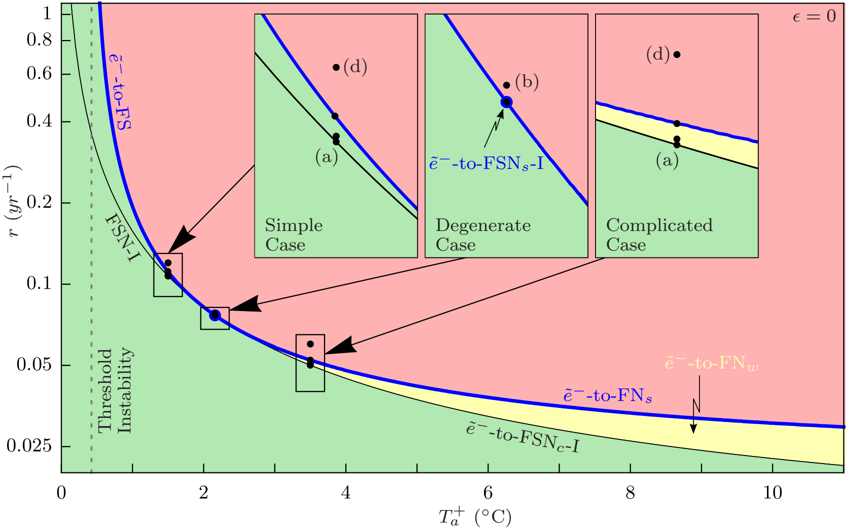
The Complicated Case



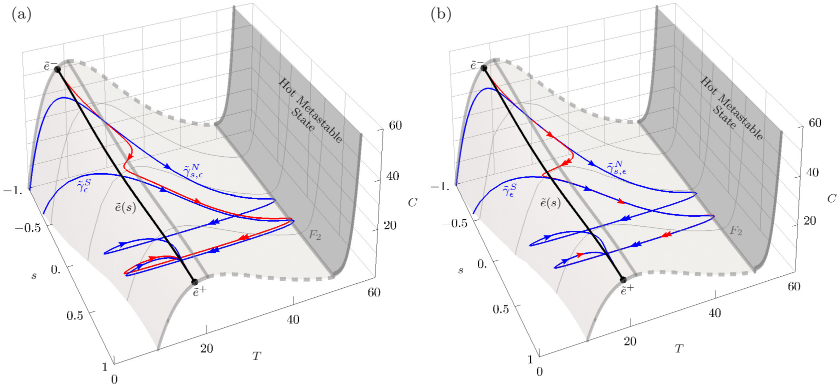
The Degenerate Case



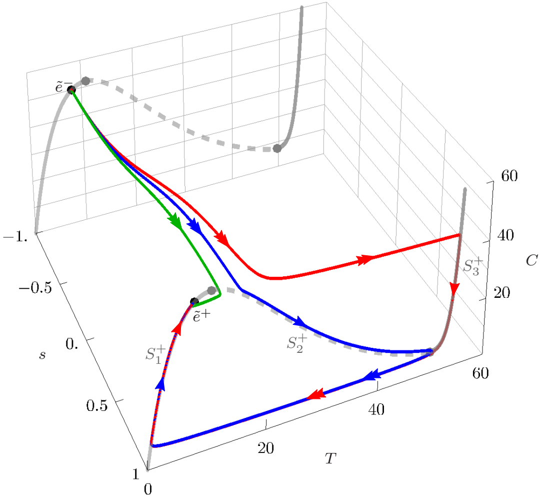
R-Tipping Diagram $\epsilon = 0$



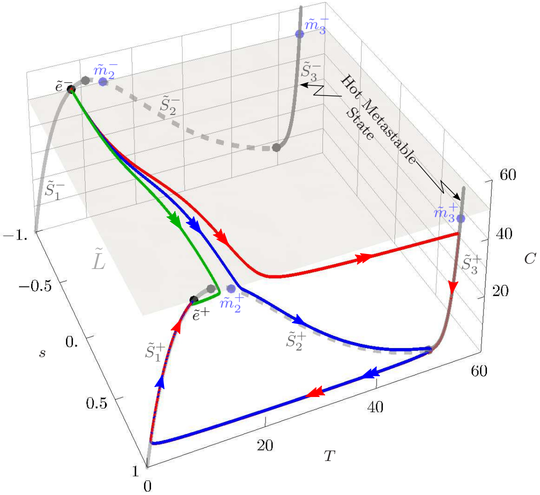
Secondary and Composite Canards



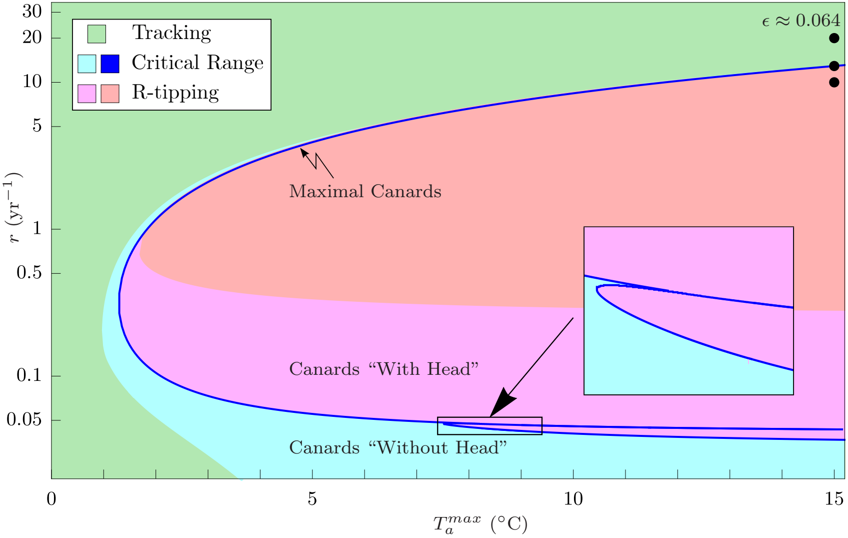
Hot Summer Anomaly: 2 Fast 1 Slow



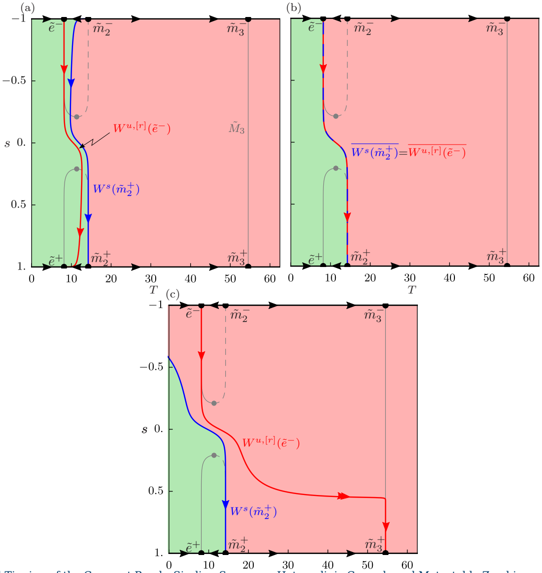
Hot Summer Anomaly: 2 Fast 1 Slow



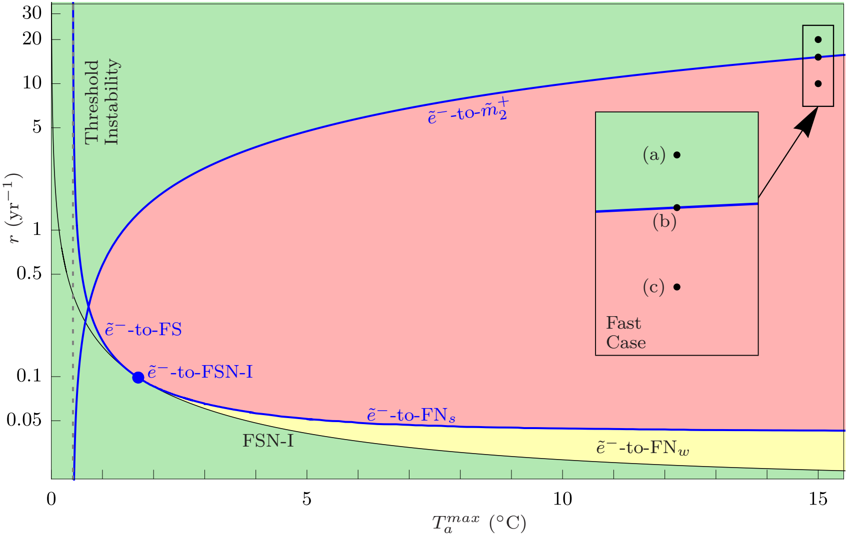
R-Tipping Diagram $\epsilon > 0$



The Fast Case



R-Tipping Diagram $\epsilon = 0$



Future Directions

- Additional Processes - ignition, hydrodynamics.
- Pattern formation in a spatially extended model.
- Theory of R-tipping due to Quasithresholds.
- Noisy R-tipping across Quasithresholds.