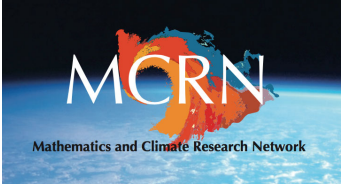



An Introduction to Earth's Heat Imbalance

Richard McGehee
School of Mathematics
University of Minnesota
Mathematics of Climate Seminar
September 3, 2024




<https://sites.google.com/view/math-climate>




Earth's Heat Imbalance

Is the climate changing?




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Earth's Heat Imbalance

Is the climate changing?

Floods
Droughts
Fires
Heat



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Earth's Heat Imbalance **floods**

Climate Emergency?

More than 300,000 in emergency shelters after Bangladesh floods
Aljazeera August 25, 2024
Relentless monsoon rains and flooding have submerged vast areas in Bangladesh, damaging homes and infrastructure.



[aljazeera.com/gallery/2024/8/25/more-than-300000-bangladeshis-in-emergency-shelters-after-floods](https://www.aljazeera.com/gallery/2024/8/25/more-than-300000-bangladeshis-in-emergency-shelters-after-floods)



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Earth's Heat Imbalance **floods**

Climate Emergency?
THE WALL STREET JOURNAL


The Flooding in Vermont Is 'Historic and Catastrophic,' Governor Says



<https://www.cnn.com/2023/07/11/us/northeast-storms-flooding-excessive-rainfall-tuesday/index.html>




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Earth's Heat Imbalance **droughts**


Climate Emergency?

Hoover Dam, USA




www.economist.com/energy-opinion/2021/05/20/like-mead-likely-1-er-2-shortage-2023-impact-wisdom/5183361001/

Loire River, France




Europe's Key Rivers Fall to Critical Levels, Aggravating Energy Crunch
Low water levels and heavy temperatures threaten power and energy, as Europe faces more energy crisis in Europe

www.wsj.com/articles/europe-key-rivers-fall-to-critical-levels-aggravating-energy-crisis-11660216954




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Earth's Heat Imbalance **fires**
Climate Emergency?




Canada



<https://www.nytimes.com/2023/09/04/global/canada-wildfires-climate-change.html>

New York City



<https://www.cnn.com/us/live-news/canada-wildfires-us-air-quality-04-09-23/index.html>

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Earth's Heat Imbalance **heat**
Climate Emergency?




The New York Times
Phoenix's Month in Hell: A 31-Day Streak of Record Heat Ends

A continuous stretch of days reaching or exceeding 110 degrees has filled emergency rooms. On Monday, the city hit 108 degrees, breaking the run, but setting a new, brutal record.

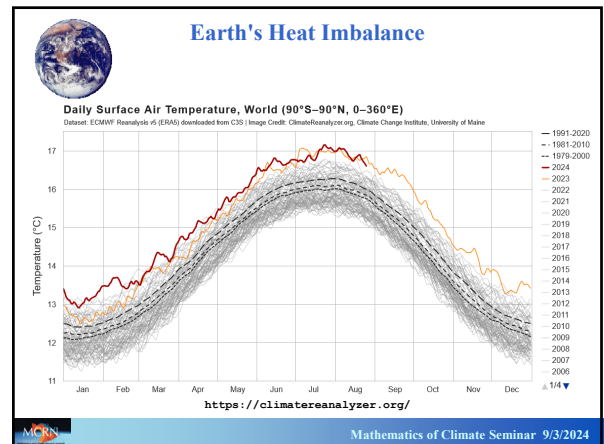
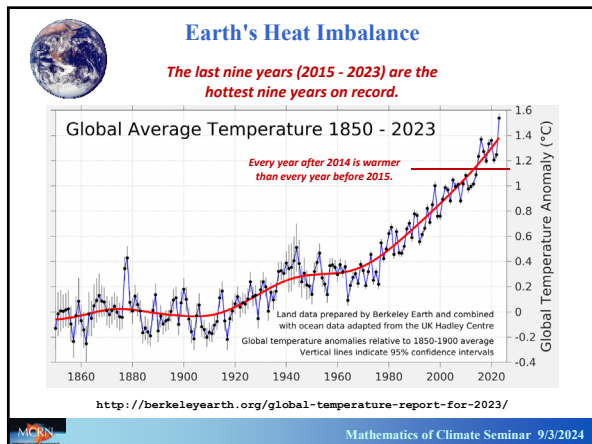
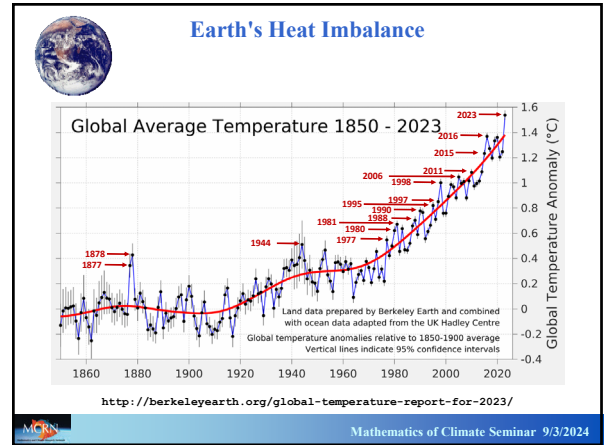
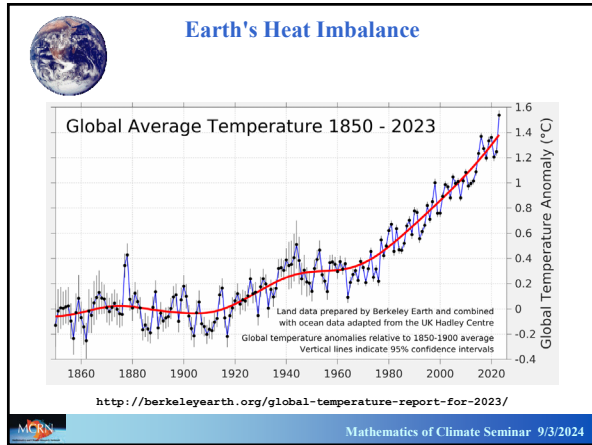


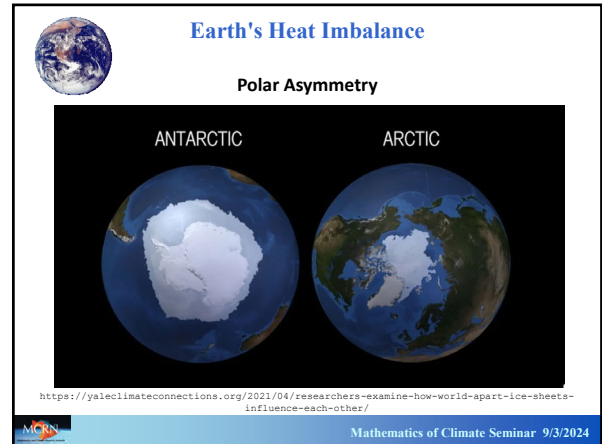
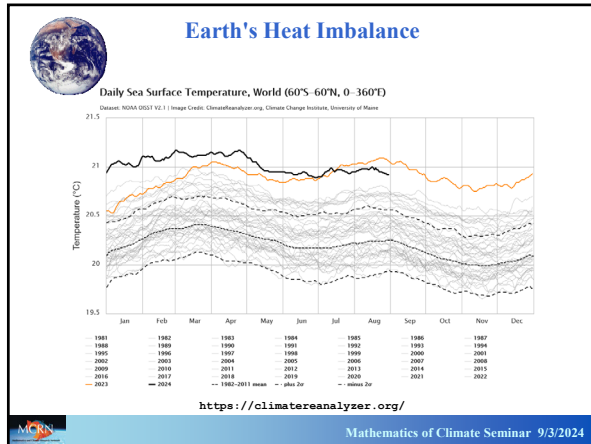
<https://www.accuweather.com/en/weather-news/phoenix-heat-wave-continues-the-most-intense-on-record-for-the-city/1562182>



<https://www.capitoltimes.com/news/2023/06/06/astudy-phoenix-faces-heat-10-or-more-days-of-heat-wave-10-days-at-a-time/>

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Earth's Heat Imbalance

Is the climate changing?

Floods
 Droughts
 Fires
 Heat

Is it a crisis or an emergency?

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Earth's Heat Imbalance

Climate crisis or climate emergency?

The two elements that distinguish crisis from emergency are immediacy and the need for decisive change. An **emergency** is acute — the undesirable outcome is here, right now. **Immediate action is required.** A crisis may also call for action, but the purpose of the action is to prevent a possible outcome.

google.com

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Earth's Heat Imbalance

What determines the Earth's surface temperature?

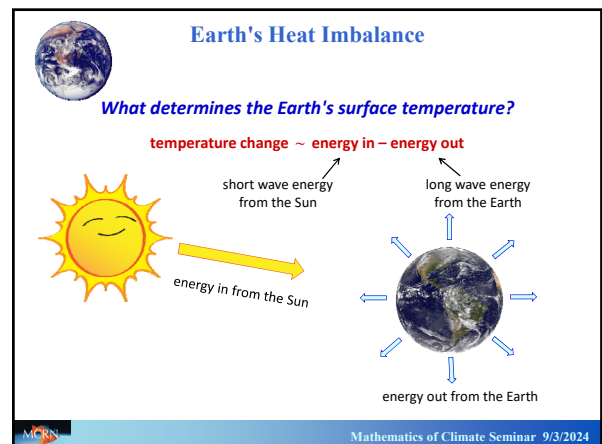
Conservation of Energy
 Heat is a form of energy.
 Temperature measures heat.

temperature change ~ energy in – energy out

short wave energy from the Sun long wave energy from the Earth

Everything else is detail.

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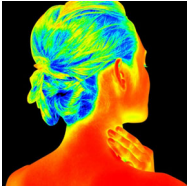


Earth's Heat Imbalance

Black-Body Radiation

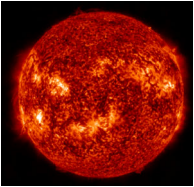
Every object emits electromagnetic radiation according to its temperature. The intensity of the radiation (power flux) is approximated by a theoretical object called a *perfect black body*.

human



<https://1ststalkscience.ca/educational-resources/backgrounders/thermal-imaging>

sun



<https://solarsystem.nasa.gov/solar-system/sun/overview/>

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Earth's Heat Imbalance

Black-Body Radiation

Stefan-Boltzmann Law

$$F = \sigma T^4$$

power flux (W/m²) ← F ← temperature (K)

Stefan-Boltzmann constant
 $\sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

0 K = -273°C = "absolute zero"

watt = joule per second = "power"

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Earth's Heat Imbalance

Black-Body Radiation

Stefan-Boltzmann Law

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power flux (W/m²) ← F ← temperature (K)

Stefan-Boltzmann constant
 $\sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

0 K = -273°C = "absolute zero"

Reasonable approximation:
Every body in the solar system radiates energy according to this law.

Let's try the Sun.

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Earth's Heat Imbalance

Stefan-Boltzmann Law


$$F = \sigma T^4$$

power flux (W/m²) ← F ← temperature (K)

Stefan-Boltzmann constant
 $\sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

Example

surface temperature of the Sun: 5780K
 power flux: $5.67 \times 10^{-8} \times (5780)^4 = 6.33 \times 10^7 \text{ W/m}^2$
 total solar power output: $6.33 \times 10^7 \times 4\pi(r_s)^2$,
 where r_s = radius of the sun = $6.96 \times 10^8 \text{ m}$
 total solar output: $3.85 \times 10^{26} \text{ W}$
 230 nanoseconds = time it takes for the Sun to produce the equivalent of the annual global electricity production.



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Earth's Heat Imbalance

Insolation

(Incoming solar Radiation)

How much energy from the Sun is hitting the Earth?

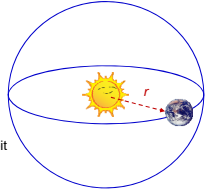
Solar flux at a distance r from the sun:

$$F = \frac{6.33 \times 10^7 4\pi r_s^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_s}{r}\right)^2 \text{ W/m}^2$$

$r_s = 6.96 \times 10^8 \text{ m}$
 $r = 1.5 \times 10^{11} \text{ m}$

$$F = 1368 \text{ W/m}^2$$

solar flux at Earth's orbit



Average surface flux: $\frac{F \times \pi r_e^2}{4\pi r_e^2} = \frac{F}{4} = 342 \text{ W/m}^2$

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The Climate Emergency

Insolation

(Incoming solar Radiation)

Solar flux at a distance r from the sun:

$$F = \frac{6.33 \times 10^7 4\pi r_s^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_s}{r}\right)^2 \text{ W/m}^2$$

$r_s = 6.96 \times 10^8 \text{ m}$
 $r = 1.5 \times 10^{11} \text{ m}$

$$F = 1368 \text{ W/m}^2$$

← solar flux at Earth's orbit

Power intercepted by the Earth: $F \times \pi r_e^2 \text{ W}$
 Earth's surface area: $4\pi r_e^2 \text{ m}^2$

Average surface flux: $\frac{F \times \pi r_e^2}{4\pi r_e^2} = \frac{F}{4} = 342 \text{ W/m}^2$

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Earth's Heat Imbalance

Insolation


Solar flux at a distance r from the sun:

$$F = \frac{6.33 \times 10^7 \cdot 4\pi r_s^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_s}{r}\right)^2 \text{ W/m}^2$$

$r_s = 6.96 \times 10^8 \text{ m}$
 $r = 1.5 \times 10^{11} \text{ m}$
 $F = 1368 \text{ W/m}^2$

Power intercepted by the Earth:
 $F \times \pi r_e^2 \text{ W}$, $r_e = \text{radius of Earth} = 6.37 \times 10^6 \text{ m}$
 Power intercepted: $1.74 \times 10^{17} \text{ W}$

Biologically Stored Energy
 total coal reserves: 10^{15} kg
 energy content: $3 \times 10^7 \text{ J/kg}$
 total energy in coal reserves: $3 \times 10^{22} \text{ J}$
= 2 days of insolation



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Earth's Heat Imbalance

Insolation

Global Average Insolation
 intercepted flux: $F = 1368 \text{ W/m}^2$
 Earth cross-section: πr_e^2
 surface area: $4\pi r_e^2$
 average flux: $1368/4 = 342 \text{ W/m}^2 = Q$

Simple Model
 Assume that Earth is a perfectly thermally conducting black body.
 $Q = \sigma T^4$
 $T = (Q/\sigma)^{1/4} = (342/5.67 \times 10^{-8})^{1/4}$
 $= 279 \text{ K} = 6^\circ \text{ C} = 43^\circ \text{ F}$

Dynamics
 $R \frac{dT}{dt} = Q - \sigma T^4$
 heat capacity \rightarrow $R \frac{dT}{dt}$ \leftarrow stable equilibrium

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Earth's Heat Imbalance

Insolation

Global Average Insolation
 intercepted flux: $F = 1368 \text{ W/m}^2$
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heat imbalance

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Earth's Heat Imbalance

What determines the Earth's surface temperature?

Conservation of Energy
 Heat is a form of energy.
 Temperature measures heat.

heat imbalance

temperature change \sim energy in - energy out

short wave energy from the Sun \rightarrow energy in \leftarrow long wave energy from the Earth \rightarrow energy out

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Earth's Heat Imbalance

What determines the Earth's surface temperature?

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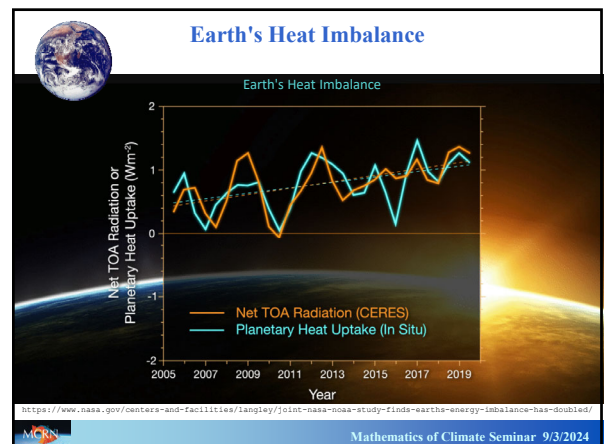
heat imbalance

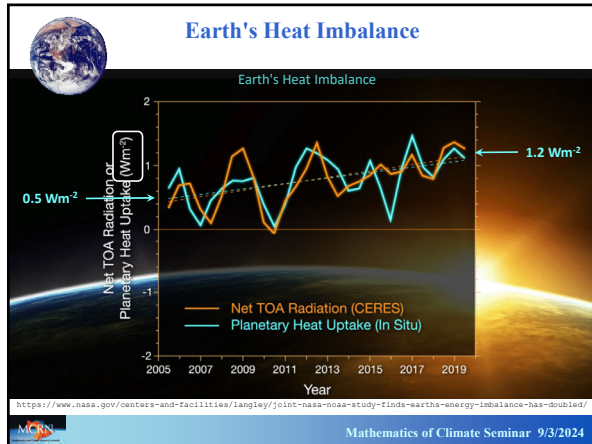
temperature change \sim energy in - energy out

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Can we measure heat imbalance?

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Earth's Heat Imbalance

2020 Heat Imbalance

$1 Wm^{-2}$

Surface area of Earth:
 $500,000,000 km^2 = 5 \times 10^{14} m^2$

Insolation over Earth's surface:
 $5 \times 10^{14} W = 500,000 GW$

How much is that?

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Earth's Heat Imbalance

2020 Heat Imbalance

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Insolation over Earth's surface:
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How much is that?

Earth's heat imbalance:
500,000 nuclear power plants

Prairie Island Nuclear Power Plant capacity: **1GW**

https://en.wikipedia.org/wiki/Prairie_Island_Nuclear_Power_Plant

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Earth's Heat Imbalance

2020 Yearly Heat Imbalance

Insolation over Earth's surface:
 $5 \times 10^{14} W = 5 \times 10^{14} \text{ Joules/second}$

seconds in a year:
 30 million = 3×10^7

Yearly Heat Imbalance
 $15 \times 10^{21} \text{ Joules} = 15 \text{ zj (zettajoules)}$

How much is that?

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Earth's Heat Imbalance

2020 Yearly Heat Imbalance

Insolation over Earth's surface:
 $5 \times 10^{14} W = 5 \times 10^{14} \text{ Joules/second}$

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Yearly Heat Imbalance
 $15 \times 10^{21} \text{ Joules} = 15 \text{ zj (zettajoules)}$

Recall:
 Biologically Stored Energy
 total coal reserves: 10^{15} kg
 energy content: $3 \times 10^7 \text{ J/kg}$
 total energy in coal reserves: $3 \times 10^{22} \text{ J}$
= 30 zj

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Earth's Heat Imbalance

Where is the extra heat going?

Mostly into the ocean.


How do we know?

Argo

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Earth's Heat Imbalance

Ocean Heat Content



part of the integrated global observation strategy

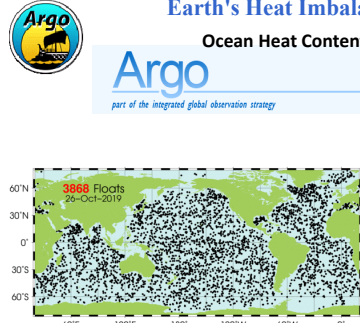
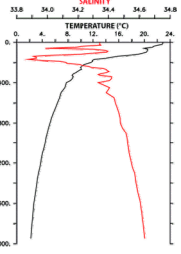


<http://www.argo.ucsd.edu/>

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Earth's Heat Imbalance

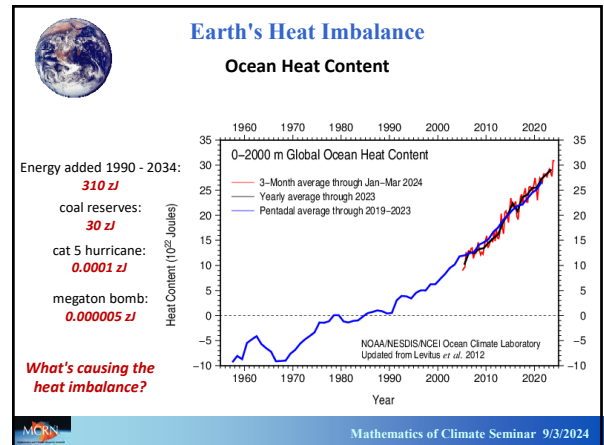
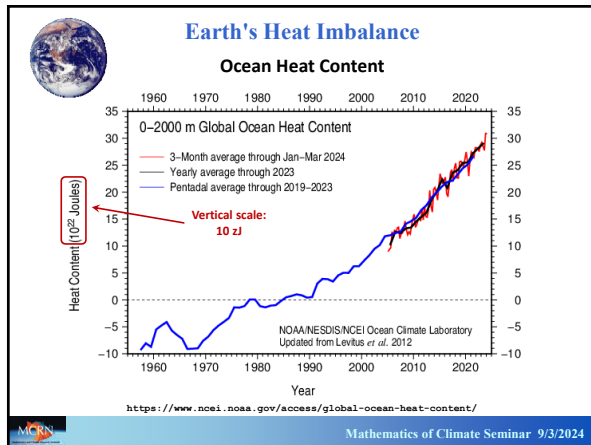
Ocean Heat Content

part of the integrated global observation strategy

<http://www.argo.ucsd.edu/>

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Earth's Heat Imbalance

What's causing the heat imbalance?

The Greenhouse Effect

Greenhouse gases (CO_2 , H_2O , CH_4) are transparent to visible light, but opaque to infrared light. The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.

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Earth's Heat Imbalance

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Who discovered the greenhouse effect?

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Earth's Heat Imbalance
What's causing the heat imbalance?




The Greenhouse Effect

Greenhouse gases (CO_2 , H_2O , CH_4) are transparent to visible light, but opaque to infrared light. The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.

Who discovered the greenhouse effect?

A mathematician!

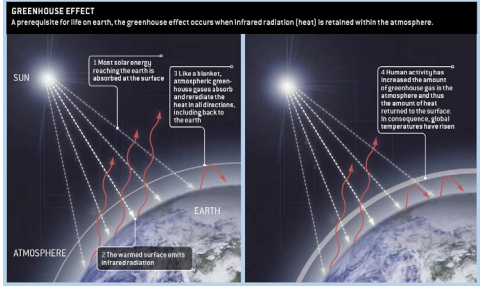
Joseph Fourier (1827), *Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires*, *Mémoires de l'Académie Royale des Sciences*, t. vii., p. 569.



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Earth's Heat Imbalance
The Greenhouse Effect

GREENHOUSE EFFECT
 A prerequisite for life on earth, the greenhouse effect occurs when infrared radiation (heat) is retained within the atmosphere.



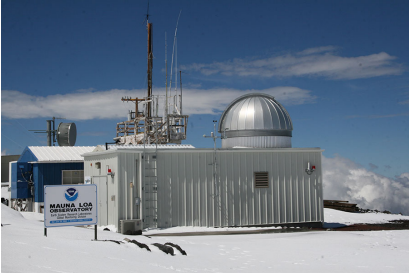
Gary Stix, *Scientific American* September 2006, pp.46-49



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Earth's Heat Imbalance
Can we measure greenhouse gasses?

Mauna Loa Observatory



https://research.noaa.gov/Portals/0/EasyONNews/1502/2006060p587EDNmain10061mlo_sign_miller.jpg

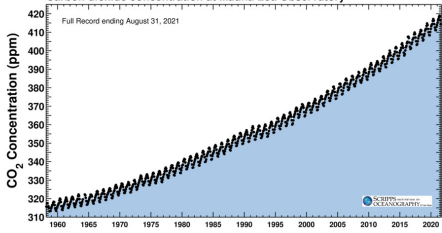



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Earth's Heat Imbalance
Can we measure greenhouse gasses?

Keeling Curve

August 28, 2021
 Carbon dioxide concentration at Mauna Loa Observatory



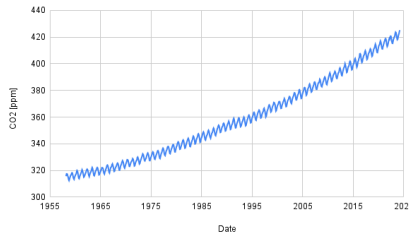
<https://keelingcurve.ucsd.edu/>



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Earth's Heat Imbalance

Mauna Loa Data



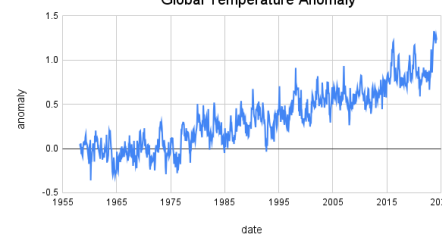
Data downloaded from https://scrippsco2.ucsd.edu/data/atmospheric_co2/primary_mlo_co2_record.html



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Earth's Heat Imbalance

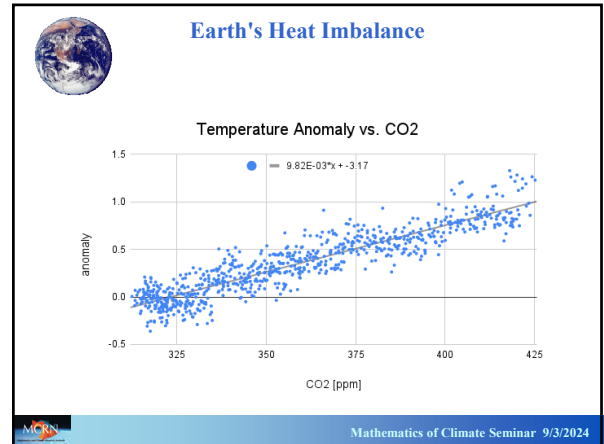
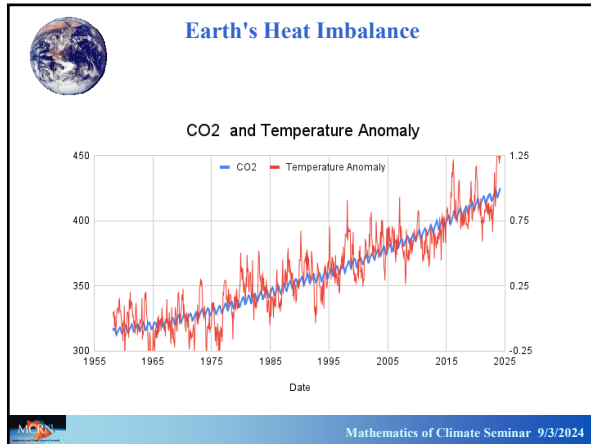
Global Temperature Anomaly



Data downloaded from <https://berkeleyearth.org/data/>

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Earth's Heat Imbalance

Coming Attraction

*Next Week:
Budyko's equation*

What determines the Earth's surface temperature?

temperature change ~ energy in – energy out

$$R \frac{\partial T}{\partial t} = Q_s(y)(1 - \alpha) - (A + BT) + C(\bar{T} - T)$$

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Earth's Heat Imbalance

Thanks for listening!

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Earth's Heat Imbalance

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