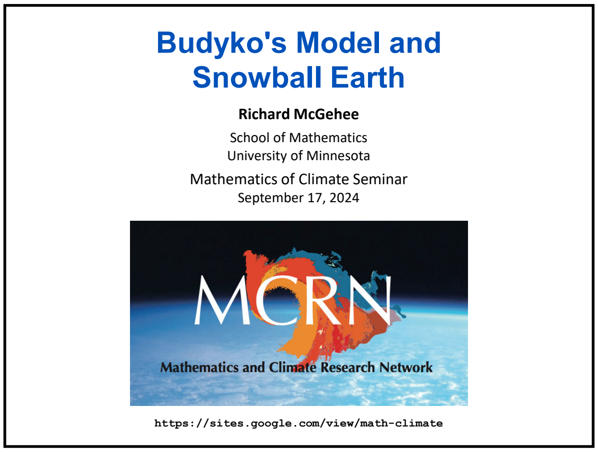


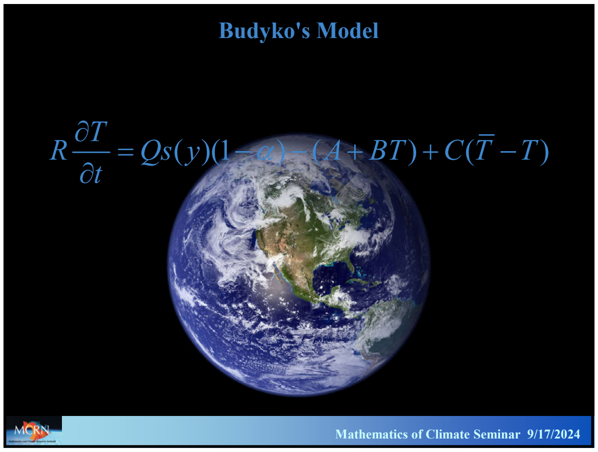
Budyko's Model and Snowball Earth

Richard McGehee
School of Mathematics
University of Minnesota

Mathematics of Climate Seminar
September 17, 2024

MCRN
Mathematics and Climate Research Network

https://sites.google.com/view/math-climate



Budyko's Model

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\bar{T}-T)$$

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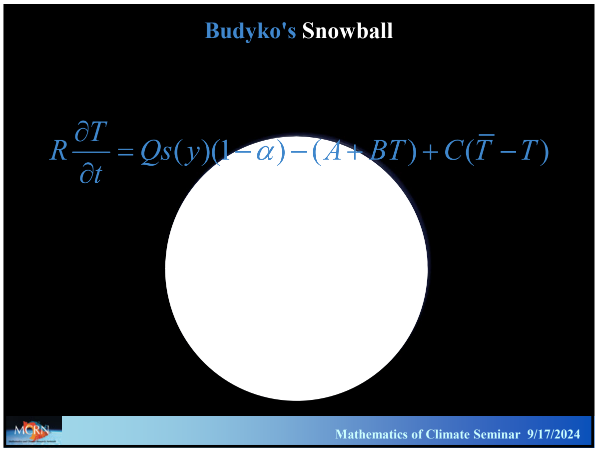

Budyko's Snowball

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\bar{T}-T)$$

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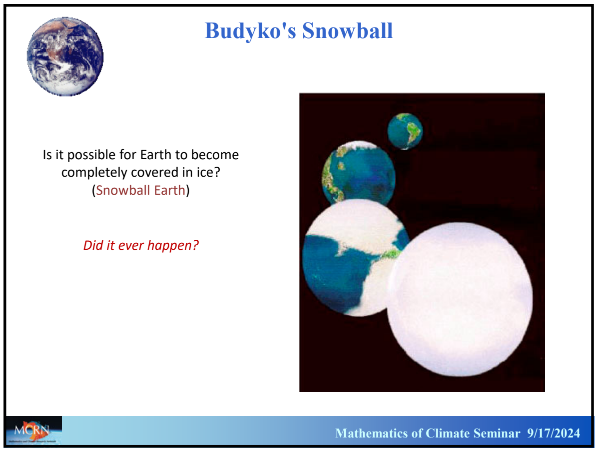

Budyko's Snowball

Is it possible for Earth to become completely covered in ice?
(Snowball Earth)

Did it ever happen?

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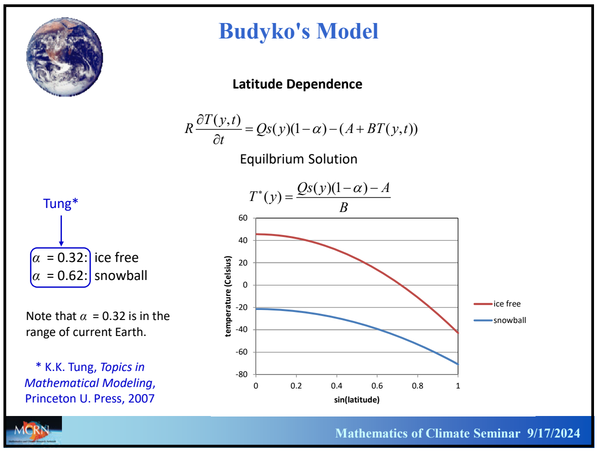

Budyko's Model

Latitude Dependence

$$R\frac{\partial T(y,t)}{\partial t} = Qs(y)(1-\alpha) - (A+BT(y,t))$$

Equilibrium Solution

$$T^*(y) = \frac{Qs(y)(1-\alpha)-A}{B}$$

Tung*

$\boxed{\alpha = 0.32:}$ ice free
$\boxed{\alpha = 0.62:}$ snowball

Note that $\alpha = 0.32$ is in the range of current Earth.

* K.K. Tung, *Topics in Mathematical Modeling*, Princeton U. Press, 2007

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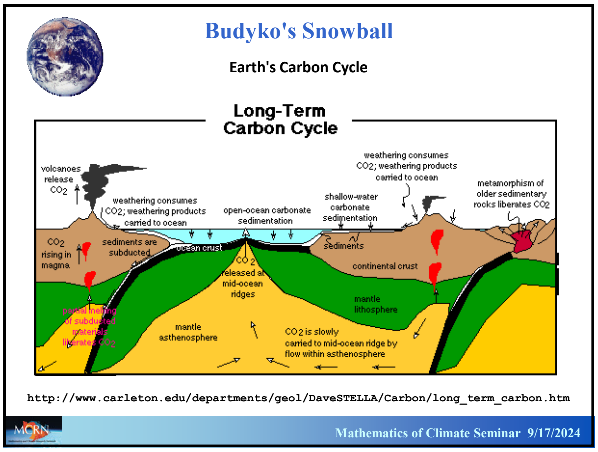

Budyko's Snowball

Earth's Carbon Cycle

http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm

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Budyko's Snowball

Earth's Carbon Cycle

Volcanos emit CO_2.

Silicate weathering carries carbon to the ocean.

Carbon sinks to the bottom of the ocean by the "biological pump" and by the precipitation of calcium carbonate.

The carbon is captured in the sediment.

The sediment is subducted beneath the continental crust by plate tectonics.

Volcanic activity released carbon from the carbonate rocks in the form of CO_2.

Repeat.

Budyko's Snowball

Earth's Carbon Cycle

Silicate Weathering

Rainwater containing dissolved CO_2 falling on silicate rocks replaces a silicon atom with a carbon atom, ultimately producing calcium carbonate (limestone) and silicon dioxide (quartz). For example, calcium silicate (Wollastonite)**:**

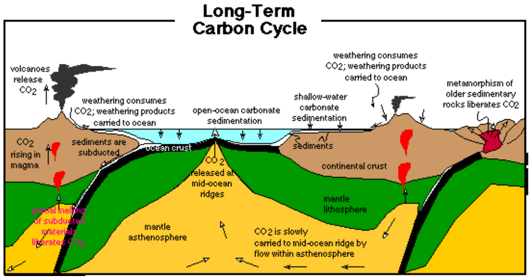
$$CaSiO_3 + CO_2 \to CaCO_3 + SiO_2$$

Under volcanic conditions, the carbon atom is replaced by a silicon atom, completing the long term carbon cycle.

$$CaCO_3 + SiO_2 \to CaSiO_3 + CO_2$$

Budyko's Snowball

Earth's Carbon Cycle



http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm

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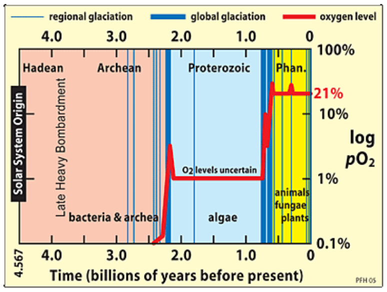
Budyko's Snowball

There is evidence that Snowball Earth has occurred, the last time about 600 million years ago.



http://www.snowballearth.org/when.html

Budyko's Snowball

The continents were clustered near the equator.

Hoffman & Schrag, *Snowball Earth*, SCIENTIFIC AMERICAN, January 2000, 68-75

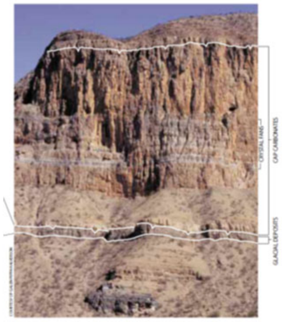
Budyko's Snowball

“Ice-rafted debris” occurred in ocean sediments near the equator, indicating large equatorial glaciers calving icebergs.

Hoffman & Schrag, *Snowball Earth*, SCIENTIFIC AMERICAN, January 2000, 68-75

Budyko's Snowball

Large limestone deposits
"cap carbonates"
are found immediately above the
glacial debris, indicating a rapid
warming period following the
snowball.



Hoffman & Schrag, *Snowball Earth*, SCIENTIFIC AMERICAN, January 2000, 68-75

Budyko's Snowball

If Earth ever was a snowball, how did we escape?

Idea:

When the Earth is ice-covered, silicate weathering almost disappears, but volcanic activity continues, allowing for a build-up of CO_2 in the atmosphere.

If the ice starts to melt, the ocean opens up and starts to reabsorb the CO_2, which warms the planet, causing more ice to melt, causing more CO_2 to be absorbed in the ocean, *etc.* Rapid positive feedback leads to an ice-free Earth.

Budyko's Model

Budyko's Equation

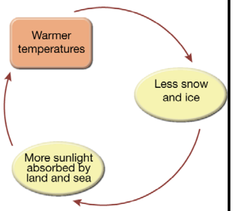
M. I. Budyko, "The effect of solar radiation variations on the climate of the Earth," Tellus XXI, 611-619 , 1969.

$$\overline{T} = \int_0^1 T(y)dy$$

$$\underset{\text{heat capacity}}{R}\overset{\text{surface temperature}}{\frac{\partial T}{\partial t}} = \underbrace{\underset{\text{insolation}}{Q}\overset{\text{sin(latitude)}}{s(y)}(1-\underset{\text{albedo}}{\alpha})} - \underbrace{(A+BT)}_{\text{OLR}} + \underbrace{C(\overline{T}-T)}_{\text{heat transport}}$$

Budyko's Snowball

Ice-albedo Feedback



temperature warms
ice melts
albedo decreases
more sunlight absorbed
temperature warms
REPEAT

Why would it stop?

Where is this feedback in Budko's equaton?

`http://www.i-fink.com/melting-polar-ice/`

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\overline{T}-T)$$

α shouldn't be a constant.

Budyko's Snowball

Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\overline{T}-T)$$

Where is the ice-albedo feedback?

Ice-albedo Feedback

albedo of ice: 0.62
albedo of land and water: 0.32

Assumption: there is a single boundary between ice and no ice occurring at $y = \eta$.

$$\alpha = \alpha(y,\eta) = \begin{cases} \alpha_1 = 0.32, & y < \eta \\ \alpha_2 = 0.62, & y > \eta \end{cases}$$

Budyko's Snowball

Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\overline{T}-T)$$

Where is the ice-albedo feedback?

Ice-albedo Feedback

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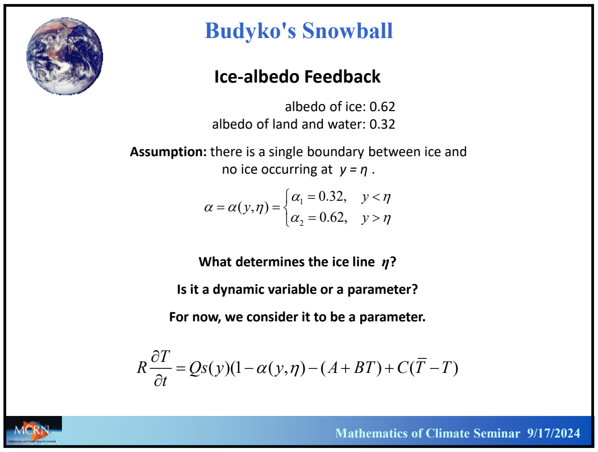
$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta) - (A+BT) + C(\overline{T}-T)$$



Budyko's Snowball

Ice-albedo Feedback

albedo of ice: 0.62
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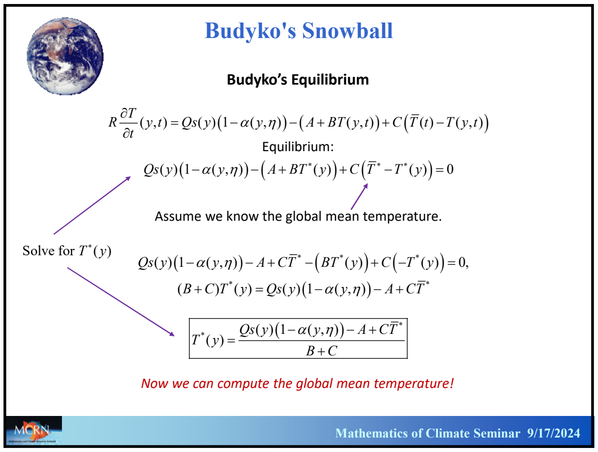
$$\alpha = \alpha(y,\eta) = \begin{cases} \alpha_1 = 0.32, & y < \eta \\ \alpha_2 = 0.62, & y > \eta \end{cases}$$

What determines the ice line η?

Is it a dynamic variable or a parameter?

For now, we consider it to be a parameter.

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\overline{T} - T)$$



Budyko's Snowball

Budyko's Equilibrium

$$R\frac{\partial T}{\partial t}(y,t) = Qs(y)\big(1-\alpha(y,\eta)\big) - \big(A+BT(y,t)\big) + C\big(\overline{T}(t) - T(y,t)\big)$$

Equilibrium:

$$Qs(y)\big(1-\alpha(y,\eta)\big) - \big(A+BT^*(y)\big) + C\big(\overline{T}^* - T^*(y)\big) = 0$$

Assume we know the global mean temperature.

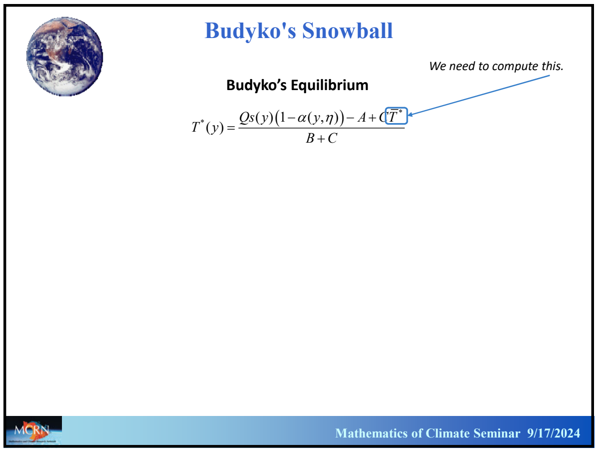
Solve for $T^*(y)$

$$Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^* - \big(BT^*(y)\big) + C\big(-T^*(y)\big) = 0,$$

$$(B+C)T^*(y) = Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*$$

$$\boxed{T^*(y) = \frac{Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*}{B+C}}$$

Now we can compute the global mean temperature!

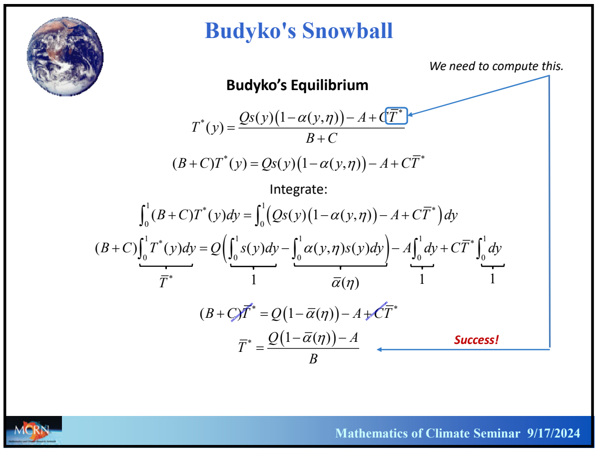


Budyko's Snowball

We need to compute this.

Budyko's Equilibrium

$$T^*(y) = \frac{Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\boxed{\overline{T}^*}}{B+C}$$



Budyko's Snowball

We need to compute this.

Budyko's Equilibrium

$$T^*(y) = \frac{Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\boxed{\overline{T}^*}}{B+C}$$

$$(B+C)T^*(y) = Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*$$

Integrate:

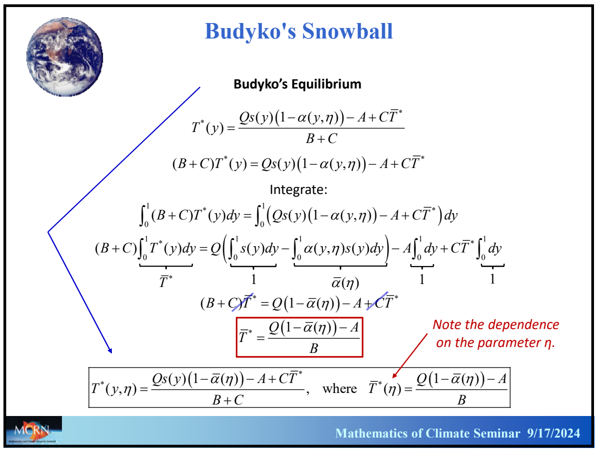
$$\int_0^1 (B+C)T^*(y)dy = \int_0^1 \Big(Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*\Big)dy$$

$$(B+C)\underbrace{\int_0^1 T^*(y)dy}_{\overline{T}^*} = Q\Big(\underbrace{\int_0^1 s(y)dy}_{1} - \underbrace{\int_0^1 \alpha(y,\eta)s(y)dy}_{\overline{\alpha}(\eta)}\Big) - A\underbrace{\int_0^1 dy}_{1} + C\overline{T}^*\underbrace{\int_0^1 dy}_{1}$$

$$(B+\cancel{C)}\overline{T}^* = Q\big(1-\overline{\alpha}(\eta)\big) - A\cancel{+C}\overline{T}^*$$

$$\overline{T}^* = \frac{Q\big(1-\overline{\alpha}(\eta)\big) - A}{B}$$

Success!



Budyko's Snowball

Budyko's Equilibrium

$$T^*(y) = \frac{Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*}{B+C}$$

$$(B+C)T^*(y) = Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*$$

Integrate:

$$\int_0^1 (B+C)T^*(y)dy = \int_0^1 \Big(Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*\Big)dy$$

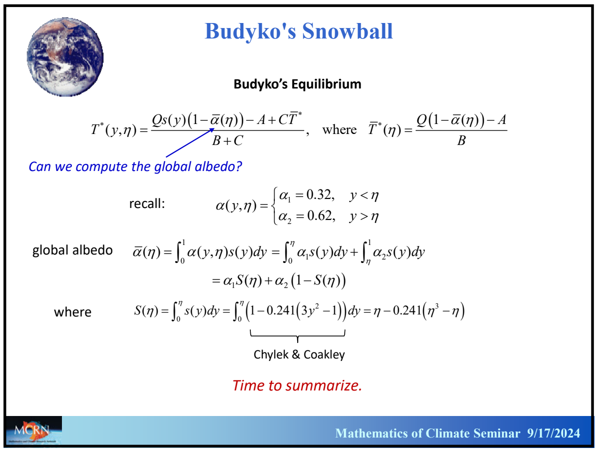
$$(B+C)\underbrace{\int_0^1 T^*(y)dy}_{\overline{T}^*} = Q\Big(\underbrace{\int_0^1 s(y)dy}_{1} - \underbrace{\int_0^1 \alpha(y,\eta)s(y)dy}_{\overline{\alpha}(\eta)}\Big) - A\underbrace{\int_0^1 dy}_{1} + C\overline{T}^*\underbrace{\int_0^1 dy}_{1}$$

$$(B+\cancel{C)}\overline{T}^* = Q\big(1-\overline{\alpha}(\eta)\big) - A\cancel{+C}\overline{T}^*$$

$$\boxed{\overline{T}^* = \frac{Q\big(1-\overline{\alpha}(\eta)\big) - A}{B}}$$

Note the dependence on the parameter η.

$$\boxed{T^*(y,\eta) = \frac{Qs(y)\big(1-\overline{\alpha}(\eta)\big) - A + C\overline{T}^*}{B+C}, \quad \text{where} \quad \overline{T}^*(\eta) = \frac{Q\big(1-\overline{\alpha}(\eta)\big) - A}{B}}$$



Budyko's Snowball

Budyko's Equilibrium

$$T^*(y,\eta) = \frac{Qs(y)\big(1-\overline{\alpha}(\eta)\big) - A + C\overline{T}^*}{B+C}, \quad \text{where} \quad \overline{T}^*(\eta) = \frac{Q\big(1-\overline{\alpha}(\eta)\big) - A}{B}$$

Can we compute the global albedo?

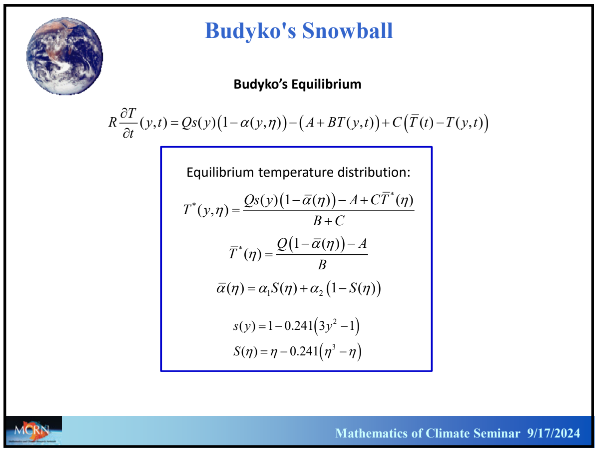
recall: $\alpha(y,\eta) = \begin{cases} \alpha_1 = 0.32, & y < \eta \\ \alpha_2 = 0.62, & y > \eta \end{cases}$

global albedo $\overline{\alpha}(\eta) = \int_0^1 \alpha(y,\eta)s(y)dy = \int_0^\eta \alpha_1 s(y)dy + \int_\eta^1 \alpha_2 s(y)dy$

$$= \alpha_1 S(\eta) + \alpha_2\big(1 - S(\eta)\big)$$

where $S(\eta) = \int_0^\eta s(y)dy = \int_0^\eta \underbrace{\Big(1 - 0.241\big(3y^2 - 1\big)\Big)}_{\text{Chylek \& Coakley}}dy = \eta - 0.241\big(\eta^3 - \eta\big)$

Time to summarize.



Budyko's Snowball

Budyko's Equilibrium

$$R\frac{\partial T}{\partial t}(y,t)=Qs(y)\left(1-\alpha(y,\eta)\right)-\left(A+BT(y,t)\right)+C\left(\overline{T}(t)-T(y,t)\right)$$

Equilibrium temperature distribution:

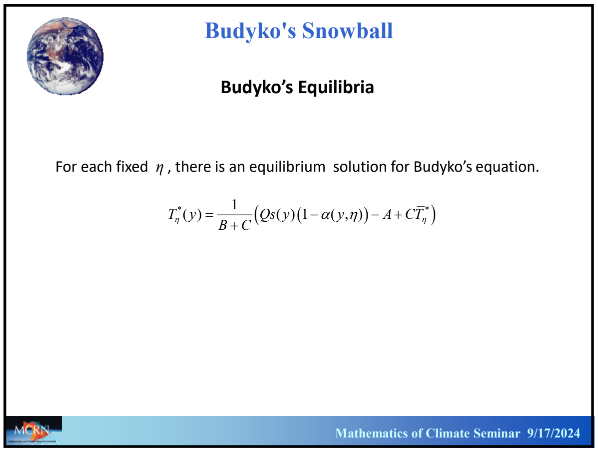
$$T^*(y,\eta)=\frac{Qs(y)\left(1-\overline{\alpha}(\eta)\right)-A+C\overline{T}^*(\eta)}{B+C}$$

$$\overline{T}^*(\eta)=\frac{Q\left(1-\overline{\alpha}(\eta)\right)-A}{B}$$

$$\overline{\alpha}(\eta)=\alpha_1 S(\eta)+\alpha_2\left(1-S(\eta)\right)$$

$$s(y)=1-0.241\left(3y^2-1\right)$$

$$S(\eta)=\eta-0.241\left(\eta^3-\eta\right)$$

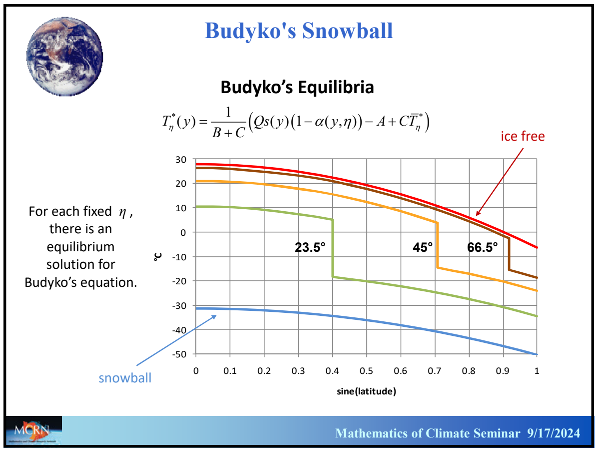


Budyko's Snowball

Budyko's Equilibria

For each fixed η, there is an equilibrium solution for Budyko's equation.

$$T_\eta^*(y)=\frac{1}{B+C}\left(Qs(y)\left(1-\alpha(y,\eta)\right)-A+C\overline{T}_\eta^*\right)$$



Budyko's Snowball

Budyko's Equilibria

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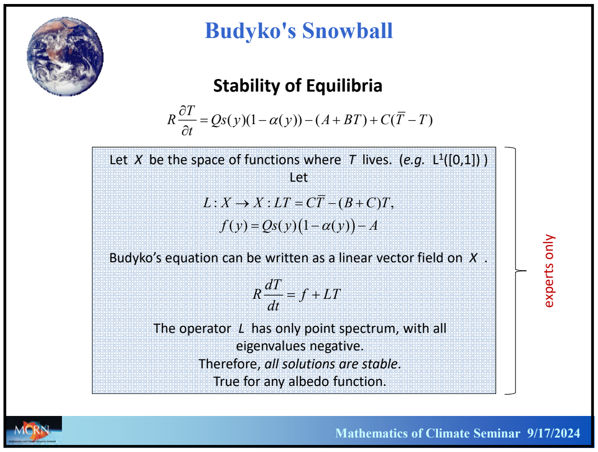
For each fixed η, there is an equilibrium solution for Budyko's equation.



Budyko's Snowball

Stability of Equilibria

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y))-(A+BT)+C(\overline{T}-T)$$

Let X be the space of functions where T lives. (*e.g.* $L^1([0,1])$)
Let

$$L:X\to X:LT=C\overline{T}-(B+C)T,$$

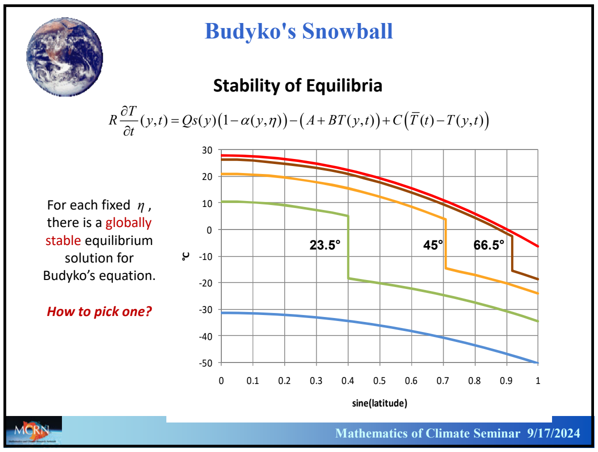
$$f(y)=Qs(y)\left(1-\alpha(y)\right)-A$$

Budyko's equation can be written as a linear vector field on X .

$$R\frac{dT}{dt}=f+LT$$

The operator L has only point spectrum, with all eigenvalues negative.
Therefore, *all solutions are stable*.
True for any albedo function.

experts only



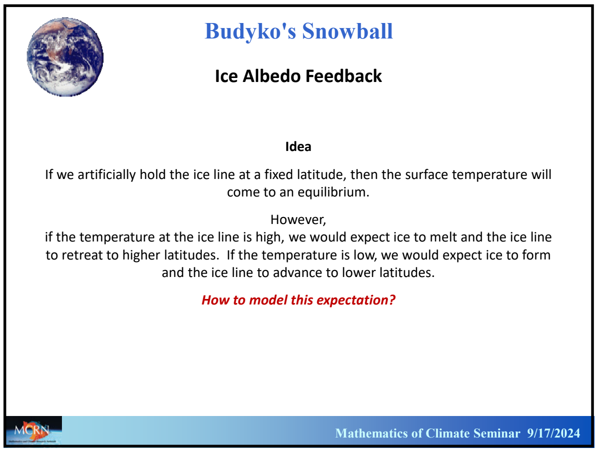
Budyko's Snowball

Stability of Equilibria

$$R\frac{\partial T}{\partial t}(y,t)=Qs(y)\left(1-\alpha(y,\eta)\right)-\left(A+BT(y,t)\right)+C\left(\overline{T}(t)-T(y,t)\right)$$

For each fixed η, there is a globally stable equilibrium solution for Budyko's equation.

How to pick one?



Budyko's Snowball

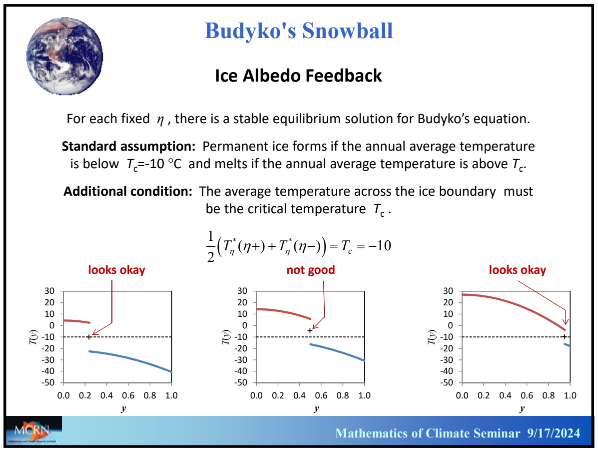
Ice Albedo Feedback

Idea

If we artificially hold the ice line at a fixed latitude, then the surface temperature will come to an equilibrium.

However,
if the temperature at the ice line is high, we would expect ice to melt and the ice line to retreat to higher latitudes. If the temperature is low, we would expect ice to form and the ice line to advance to lower latitudes.

How to model this expectation?



Budyko's Snowball

Ice Albedo Feedback

For each fixed η, there is a stable equilibrium solution for Budyko's equation.

Standard assumption: Permanent ice forms if the annual average temperature is below T_c=-10 °C and melts if the annual average temperature is above T_c.

Additional condition: The average temperature across the ice boundary must be the critical temperature T_c .

$$\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=T_c=-10$$

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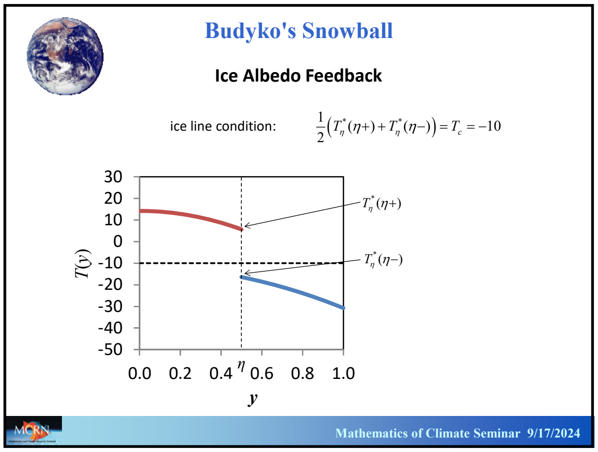

Budyko's Snowball

Ice Albedo Feedback

ice line condition: $\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=T_c=-10$

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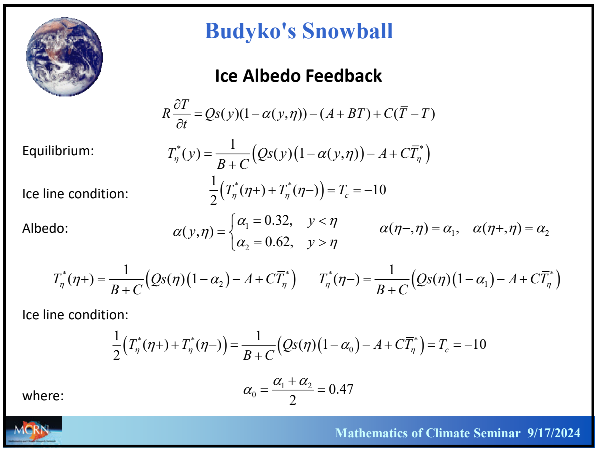

Budyko's Snowball

Ice Albedo Feedback

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

Equilibrium: $T_\eta^*(y)=\frac{1}{B+C}\left(Qs(y)\left(1-\alpha(y,\eta)\right)-A+C\overline{T}_\eta^*\right)$

Ice line condition: $\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=T_c=-10$

Albedo: $\alpha(y,\eta)=\begin{cases}\alpha_1=0.32, & y<\eta\\ \alpha_2=0.62, & y>\eta\end{cases}$ $\qquad \alpha(\eta-,\eta)=\alpha_1, \quad \alpha(\eta+,\eta)=\alpha_2$

$$T_\eta^*(\eta+)=\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_2\right)-A+C\overline{T}_\eta^*\right) \qquad T_\eta^*(\eta-)=\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_1\right)-A+C\overline{T}_\eta^*\right)$$

Ice line condition:

$$\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_0\right)-A+C\overline{T}_\eta^*\right)=T_c=-10$$

where: $\alpha_0=\frac{\alpha_1+\alpha_2}{2}=0.47$

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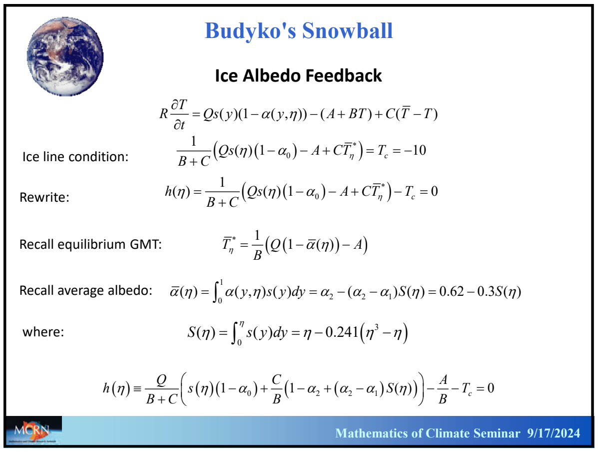

Budyko's Snowball

Ice Albedo Feedback

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

Ice line condition: $\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_0\right)-A+C\overline{T}_\eta^*\right)=T_c=-10$

Rewrite: $h(\eta)=\frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_0\right)-A+C\overline{T}_\eta^*\right)-T_c=0$

Recall equilibrium GMT: $\overline{T}_\eta^*=\frac{1}{B}\left(Q\left(1-\overline{\alpha}(\eta)\right)-A\right)$

Recall average albedo: $\overline{\alpha}(\eta)=\int_0^1\alpha(y,\eta)s(y)dy=\alpha_2-(\alpha_2-\alpha_1)S(\eta)=0.62-0.3S(\eta)$

where: $S(\eta)=\int_0^\eta s(y)dy=\eta-0.241\left(\eta^3-\eta\right)$

$$h(\eta)\equiv\frac{Q}{B+C}\left(s(\eta)\left(1-\alpha_0\right)+\frac{C}{B}\left(1-\alpha_2+\left(\alpha_2-\alpha_1\right)S(\eta)\right)\right)-\frac{A}{B}-T_c=0$$

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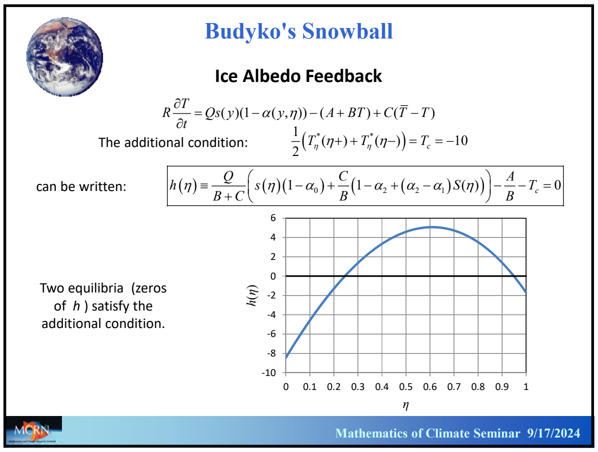

Budyko's Snowball

Ice Albedo Feedback

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

The additional condition: $\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=T_c=-10$

can be written: $\boxed{h(\eta)\equiv\frac{Q}{B+C}\left(s(\eta)\left(1-\alpha_0\right)+\frac{C}{B}\left(1-\alpha_2+\left(\alpha_2-\alpha_1\right)S(\eta)\right)\right)-\frac{A}{B}-T_c=0}$

Two equilibria (zeros of h) satisfy the additional condition.

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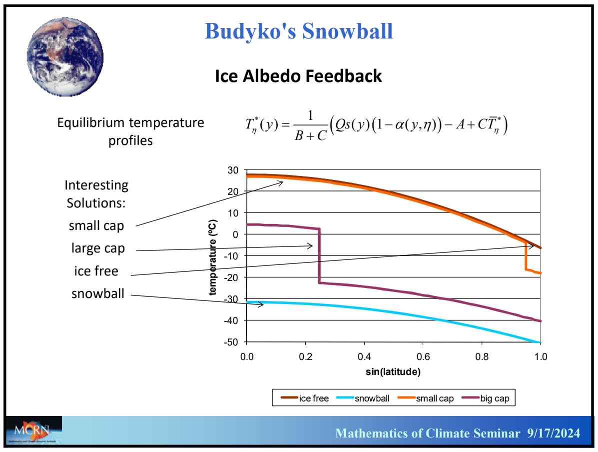

Budyko's Snowball

Ice Albedo Feedback

Equilibrium temperature profiles $T_\eta^*(y)=\frac{1}{B+C}\left(Qs(y)\left(1-\alpha(y,\eta)\right)-A+C\overline{T}_\eta^*\right)$

Interesting Solutions:

small cap

large cap

ice free

snowball

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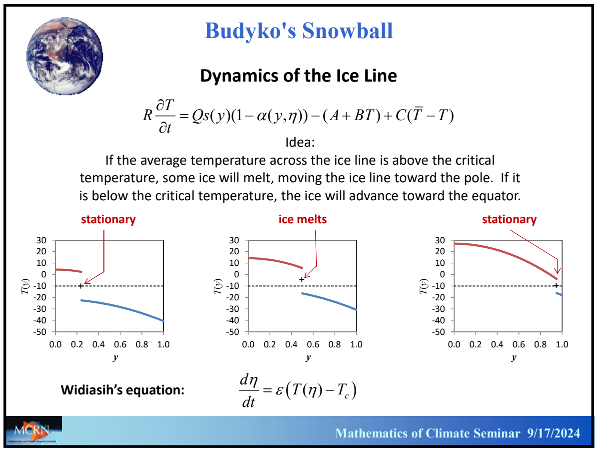

Budyko's Snowball

Dynamics of the Ice Line

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\overline{T}-T)$$

Idea:
If the average temperature across the ice line is above the critical temperature, some ice will melt, moving the ice line toward the pole. If it is below the critical temperature, the ice will advance toward the equator.

Widiasih's equation: $\frac{d\eta}{dt} = \varepsilon\left(T(\eta) - T_c\right)$

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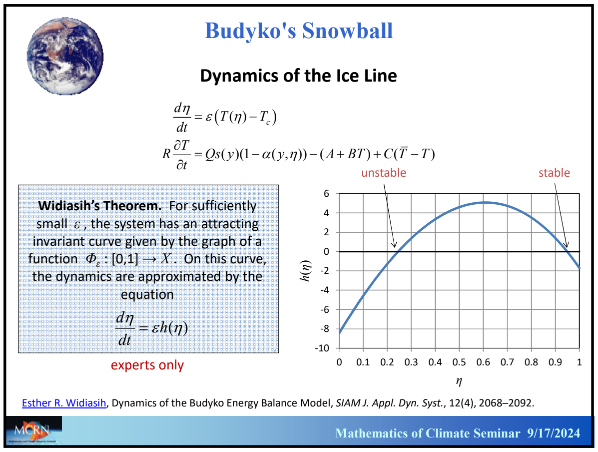

Budyko's Snowball

Dynamics of the Ice Line

$$\frac{d\eta}{dt} = \varepsilon\left(T(\eta) - T_c\right)$$

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\overline{T}-T)$$

Widiasih's Theorem. For sufficiently small ε, the system has an attracting invariant curve given by the graph of a function $\Phi_\varepsilon : [0,1] \to X$. On this curve, the dynamics are approximated by the equation

$$\frac{d\eta}{dt} = \varepsilon h(\eta)$$

experts only

Esther R. Widiasih, Dynamics of the Budyko Energy Balance Model, *SIAM J. Appl. Dyn. Syst.*, 12(4), 2068–2092.

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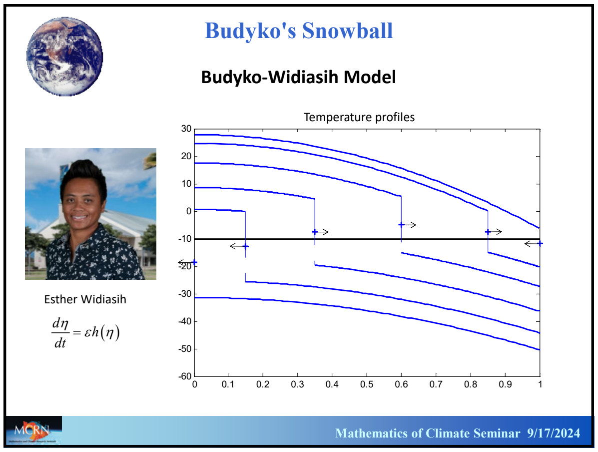

Budyko's Snowball

Budyko-Widiasih Model

Esther Widiasih

$$\frac{d\eta}{dt} = \varepsilon h(\eta)$$

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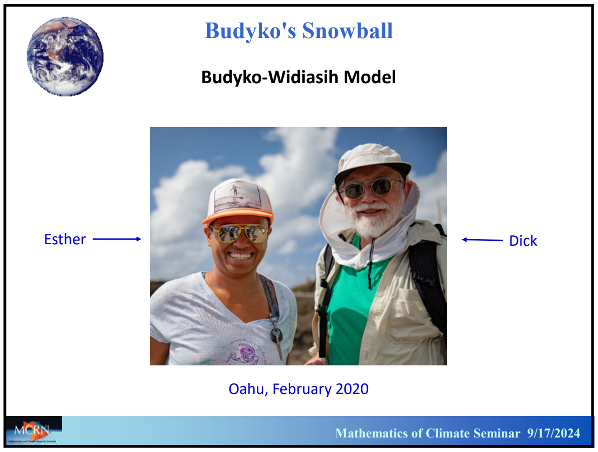

Budyko's Snowball

Budyko-Widiasih Model

Oahu, February 2020

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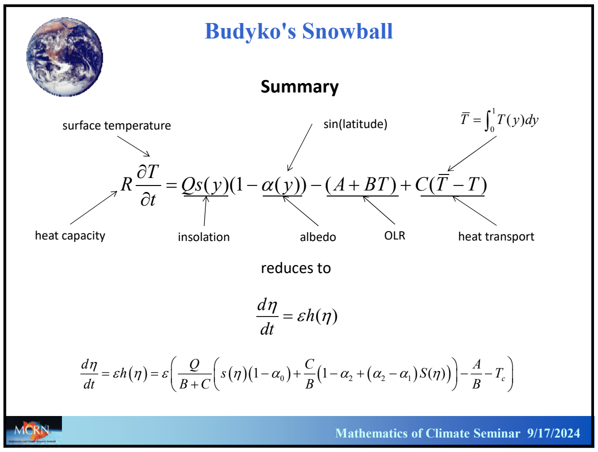

Budyko's Snowball

Summary

$$\overline{T} = \int_0^1 T(y)dy$$

$$R\frac{\partial T}{\partial t} = \underline{Qs(y)}(1-\underline{\alpha(y)}) - \underline{(A+BT)} + \underline{C(\overline{T}-T)}$$

reduces to

$$\frac{d\eta}{dt} = \varepsilon h(\eta)$$

$$\frac{d\eta}{dt} = \varepsilon h(\eta) = \varepsilon\left(\frac{Q}{B+C}\left(s(\eta)(1-\alpha_0) + \frac{C}{B}\left(1-\alpha_2 + (\alpha_2-\alpha_1)S(\eta)\right)\right) - \frac{A}{B} - T_c\right)$$

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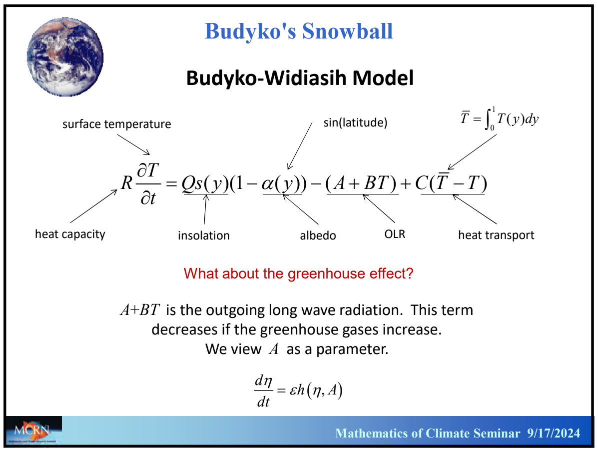

Budyko's Snowball

Budyko-Widiasih Model

$$\overline{T} = \int_0^1 T(y)dy$$

$$R\frac{\partial T}{\partial t} = \underline{Qs(y)}(1-\underline{\alpha(y)}) - \underline{(A+BT)} + \underline{C(\overline{T}-T)}$$

What about the greenhouse effect?

$A+BT$ is the outgoing long wave radiation. This term decreases if the greenhouse gases increase.
We view A as a parameter.

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A)$$

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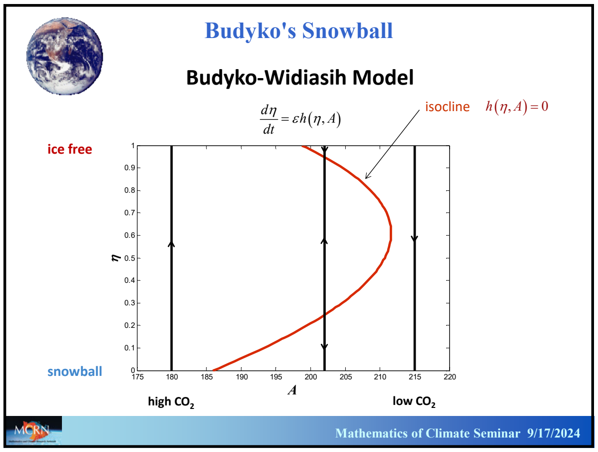


Budyko's Snowball

Budyko-Widiasih Model

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A)$$

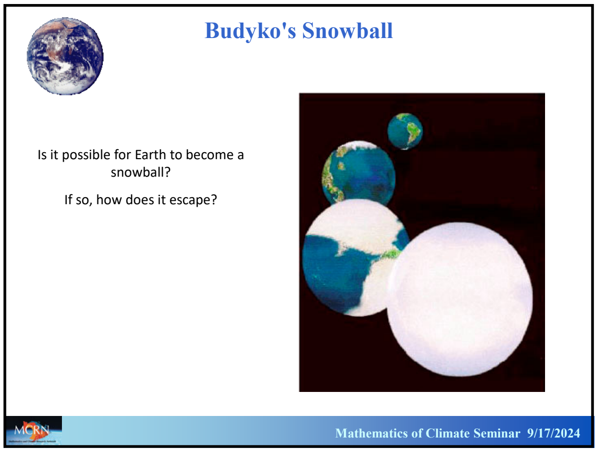
isocline $h(\eta, A) = 0$

ice free

snowball

high CO_2

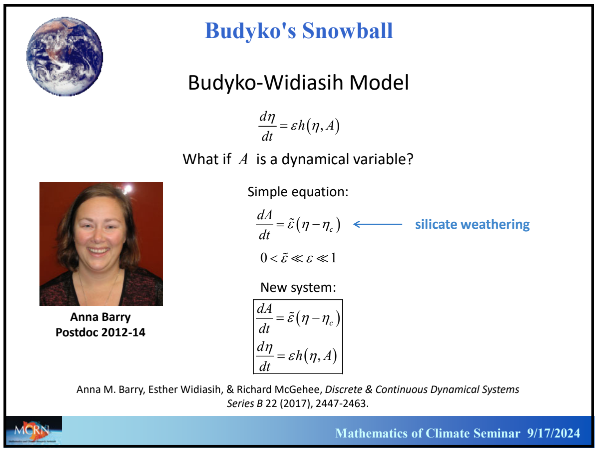
low CO_2



Budyko's Snowball

Is it possible for Earth to become a snowball?

If so, how does it escape?



Budyko's Snowball

Budyko-Widiasih Model

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A)$$

What if A is a dynamical variable?

Simple equation:

$$\frac{dA}{dt} = \tilde{\varepsilon}(\eta - \eta_c)$$

silicate weathering

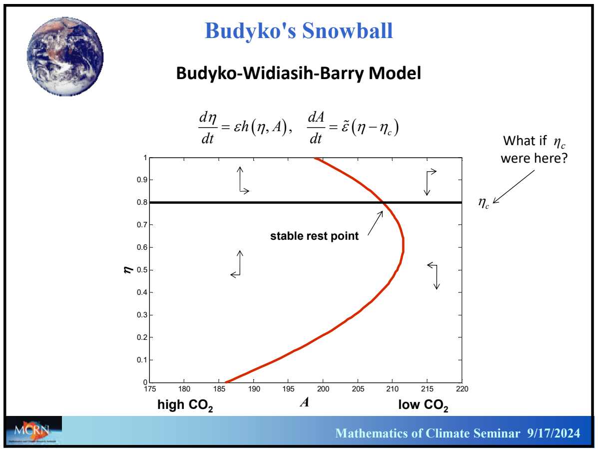
$$0 < \tilde{\varepsilon} \ll \varepsilon \ll 1$$

New system:

$$\boxed{\begin{aligned} \frac{dA}{dt} &= \tilde{\varepsilon}(\eta - \eta_c) \\ \frac{d\eta}{dt} &= \varepsilon h(\eta, A) \end{aligned}}$$

Anna Barry
Postdoc 2012-14

Anna M. Barry, Esther Widiasih, & Richard McGehee, *Discrete & Continuous Dynamical Systems Series B* 22 (2017), 2447-2463.



Budyko's Snowball

Budyko-Widiasih-Barry Model

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A), \quad \frac{dA}{dt} = \tilde{\varepsilon}(\eta - \eta_c)$$

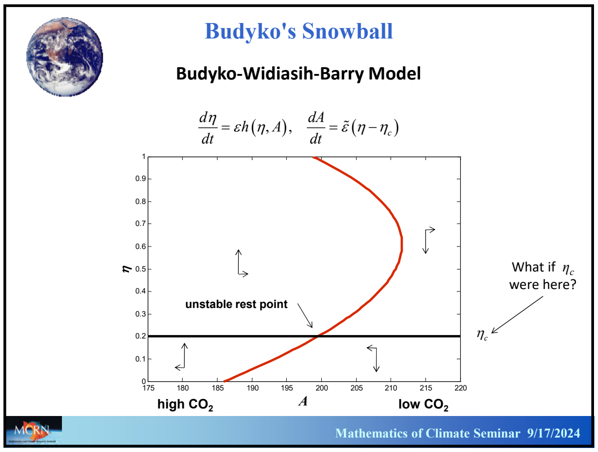
What if η_c were here?

η_c

stable rest point

high CO_2

low CO_2



Budyko's Snowball

Budyko-Widiasih-Barry Model

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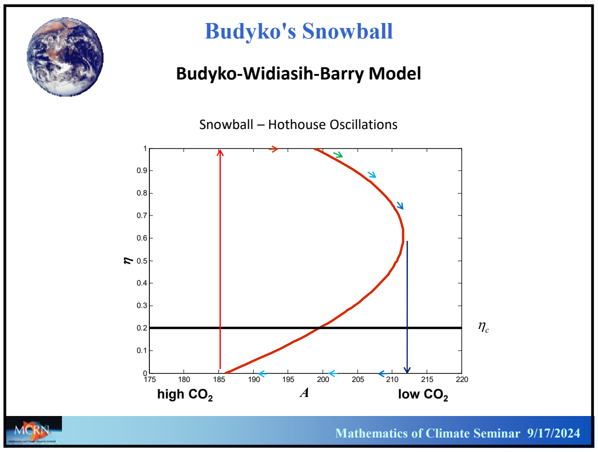


Budyko's Snowball

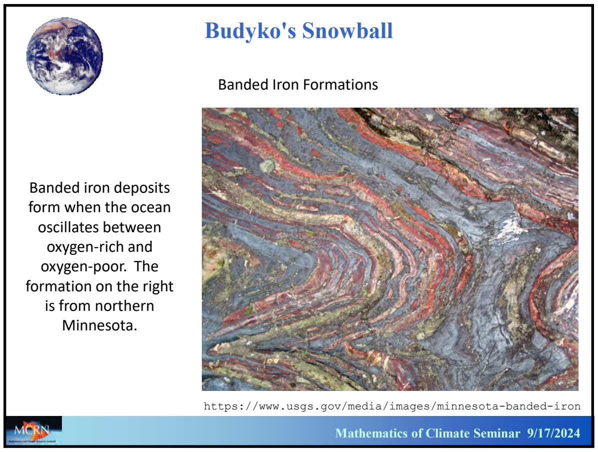
Budyko-Widiasih-Barry Model

Snowball – Hothouse Oscillations

η_c

high CO_2

low CO_2



Budyko's Snowball

Banded Iron Formations

Banded iron deposits form when the ocean oscillates between oxygen-rich and oxygen-poor. The formation on the right is from northern Minnesota.

https://www.usgs.gov/media/images/minnesota-banded-iron

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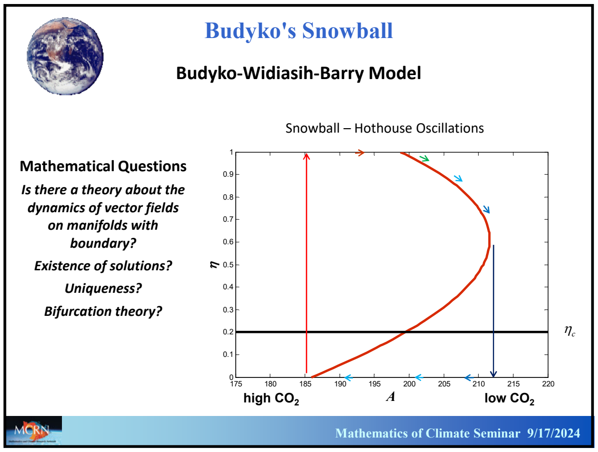

Budyko's Snowball

Budyko-Widiasih-Barry Model

Mathematical Questions

Is there a theory about the dynamics of vector fields on manifolds with boundary?

Existence of solutions?

Uniqueness?

Bifurcation theory?

Snowball – Hothouse Oscillations

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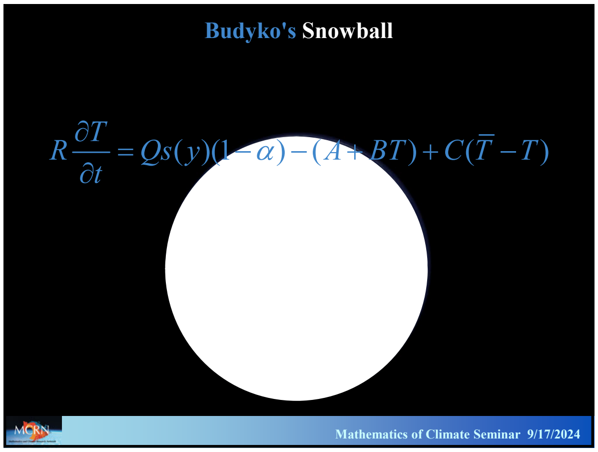

Budyko's Snowball

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha)-(A+BT)+C(\bar{T}-T)$$

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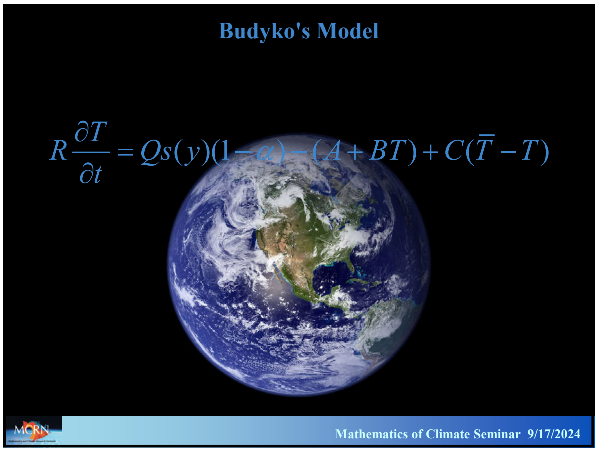

Budyko's Model

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha)-(A+BT)+C(\bar{T}-T)$$

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