The Case for Conceptual Climate Models

Richard McGehee
School of Mathematics
University of Minnesota

Mathematics of Climate Seminar
September 2, 2025

MCRN
Mathematics and Climate Research Network

https://sites.google.com/view/math-climate

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Conceptual Climate Models

What is a climate model?

Model		Hardware	Use
General Circulation Model	GCM	Supercomputer Cluster	Prediction
Earth-system Model of Intermediate Complexity	EMIC	Desktop	Prediction
Conceptual Climate Model	CCM	Pencil and Paper + Laptop	Understanding

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Conceptual Climate Models

Hardware Requirements

General Circulation Models

1976

https://en.wikipedia.org/wiki/Cray-1

2024

https://en.wikipedia.org/wiki/NCAR-Wyoming_Supercomputing_Center

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Conceptual Climate Models

Hardware Requirements

Earth-system Model of Intermediate Complexity

1989

https://en.wikipedia.org/wiki/SPARCstation_1

2025

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Conceptual Climate Models

Hardware Requirements

Conceptual Models

2013

2023

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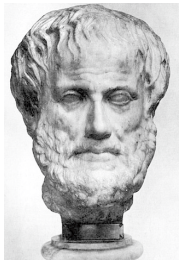
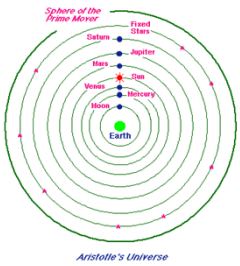
Conceptual Climate Models

Example of a Conceptual Model



Aristotle
484-422 BC



Aristotle's Universe

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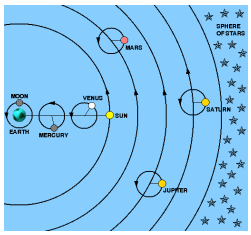
Conceptual Climate Models

Example of a Predictive Model

Add epicycles.

Ptolemy
~100-170 CE



Ptolemy's Universe

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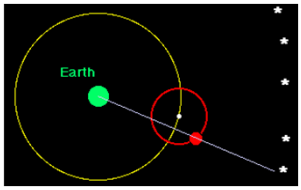
Conceptual Climate Models

Epicycles are Predictive

With the data available to the Greeks, epicycles could predict planetary motions.
In particular, the model explains retrograde motion.



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Conceptual Climate Models

Enhanced Predictive Model
Epicycles upon Epicycles

Ibn al-Shatir (1304 – 1375)

He was a muslim astronomer from Damascus who worked as timekeeper in the Umayyad Mosque, and constructed a magnificent sundial for its minaret in 1371/72.

https://alchetron.com/Ibn-al-Shatir

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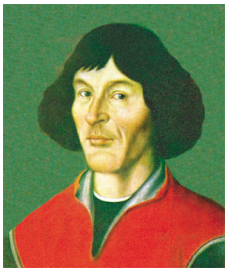
Conceptual Climate Models

Conceptual Models with Fewer Epicycles
Heliocentric



Nicolaus Copernicus
1473 - 1543

https://en.wikipedia.org/wiki/Copernican_heliocentrism

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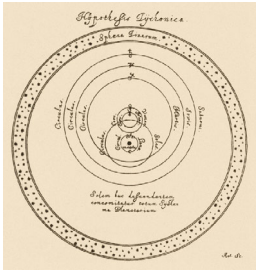
Conceptual Climate Models

Conceptual Models with Fewer Epicycles
Geo-heliocentric

Tycho Brahe
1546 – 1601



https://en.wikipedia.org/wiki/Tychonic_system

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Conceptual Climate Models

Epicycles

Epicycles ⟷ Trigonometric Polynomials

Advantages:
Data-driven
Accurate predictions
Consistent with a variety of conceptual models

Disadvantage:
Associated conceptual models become ridiculously complicated.

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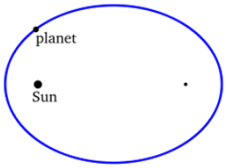
Conceptual Climate Models

Kepler's Conceptual Model

Johannes Kepler
1571-1630



Kepler's 1st Law

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Conceptual Climate Models

Kepler's Conceptual Model

Kepler's Laws

1. Planets move on ellipses with the sun at a focus.
2. Planets sweep out equal areas in equal time.
3. The square of the period is proportional to the cube of the semimajor axis.

Advantages:

Elegant (Occam's razor)
Predictive
No epicycles

Disadvantage:

Still doesn't explain everything.

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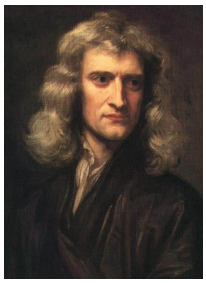
Conceptual Climate Models

Newton's Conceptual Model



Sir Isaac Newton
1643-1727

F = ma

Equations of Planetary Motion

$$m_j \frac{d^2 x_j}{dt^2} = -\sum_{i \neq j} \frac{G m_i m_j}{|x_i - x_j|^3}\left(x_j - x_i\right)$$

$$x_j \in \mathbb{R}^3, \quad j = 1, \ldots n$$

No Epicycles!
No words!
Only equations.

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Conceptual Climate Models

Newton's Predictive Model

Jacques Laskar
Observatoire de Paris

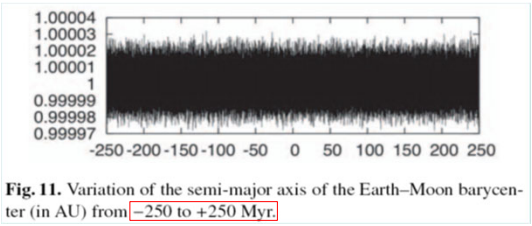

Fig. 11. Variation of the semi-major axis of the Earth–Moon barycenter (in AU) from −250 to +250 Myr.

Jacques Lakar, et al, 2004, *Astronomy and Astrophysics* **428**, 261-285.

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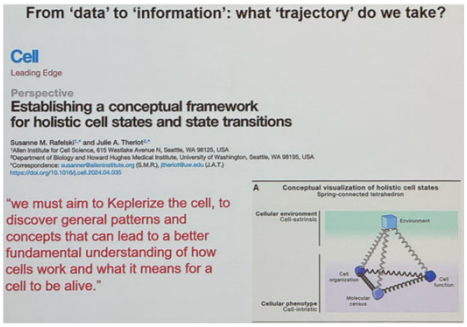
Conceptual Climate Models

"Keplerize"



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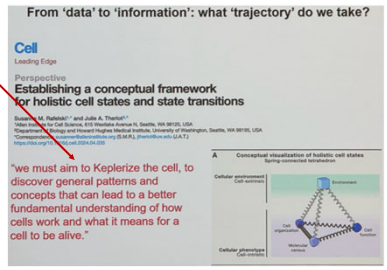
Conceptual Climate Models

"Keplerize"

Should we aim to Keplerize the climate?



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Conceptual Climate Models

Should we aim to Keplerize the climate?

Opinion of a Twentieth Century Mathematician (me)

Climate science (like most science) was Keplerized from its inception until the 21st century.

The advent of huge computing power favored prediction over understanding.

Understanding is important.

Mathematics is a good tool for understanding.

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Conceptual Climate Models

Understanding is important.

Wisconsin Assembly Speaker Robin Vos says he doesn't know if climate change is real, but it "probably" is.

"I don't know if climate change is real," Vos said. "I think it probably is. I have no idea why it is occurring nor do most people on the planet."

https://www.wpr.org/environment/vos-climate-change-probably-real

https://legis.wisconsin.gov/assembly/33/vos/

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Conceptual Climate Models

Milankovitch Cycles

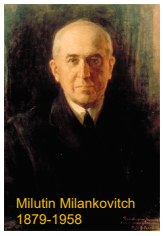
A Mathematical Contribution to Climate Science

Who was Milankovitch?

Milutin Milankovitch was a Serbian mathematician and professor at the University of Belgrade.

In 1920 he published his seminal work on the relation between incoming solar radiation and the Earth's orbital parameters.

In 1941 he published a book explaining his entire theory.



Milutin Milankovitch 1879-1958

His work was not fully accepted until 1976.

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Conceptual Climate Models

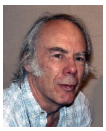
What happened in 1976?

John Imbrie

Hays, Imbrie, and Shackleton, "Variations in the Earth's Orbit: Pacemaker of the Ice Ages," *Science* 194, 10 December 1976.

James D. Hays

"It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages."



Nicholas Shackleton

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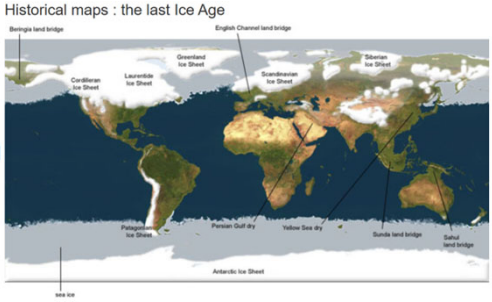
Conceptual Climate Models

succession of Quaternary ice ages?

Historical maps : the last Ice Age



https://www.planetaryvisions.com/Project.php?pid=2226

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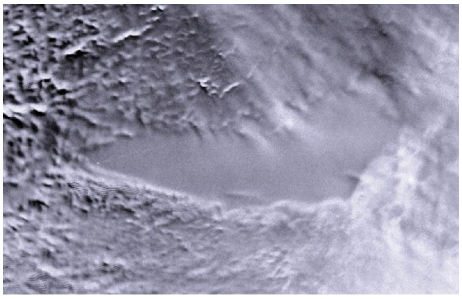
Conceptual Climate Models

succession of Quaternary ice ages?

Lake Vostok



http://svs.gsfc.nasa.gov/vis/a000000/a000900/a000996/index.html

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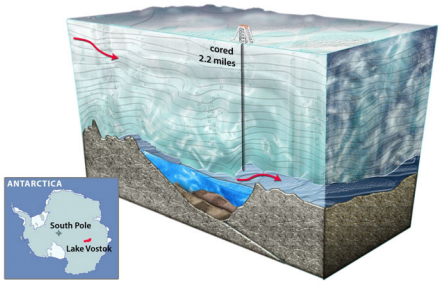
Conceptual Climate Models

succession of Quaternary ice ages?

Lake Vostok



http://www.nsf.gov/news/mmg/media/images/lake_vostok_nsf_h.jpg

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Conceptual Climate Models

succession of Quaternary ice ages?

Core Samples

Photographs copyright Reto Stöckli, NASA GSFC

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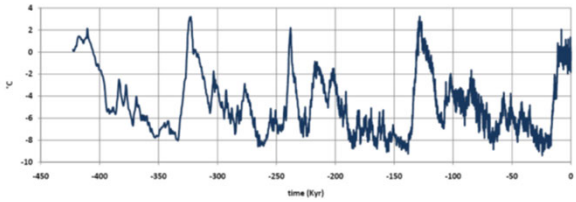
Conceptual Climate Models

succession of Quaternary ice ages?

Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data



J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* 399, 429-436.

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Conceptual Climate Models

What Causes Glacial Cycles?

Widely Accepted Hypothesis

The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms and/or triggering mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work are not fully understood.

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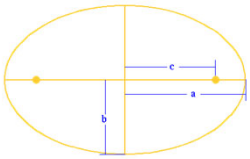
Conceptual Climate Models

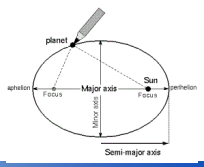
Earth's Orbit

Kepler's First Law: The orbit of every planet is an ellipse with the Sun at one of the two foci.



Eccentricity = c/a

Johannes Kepler
(1571-1630)



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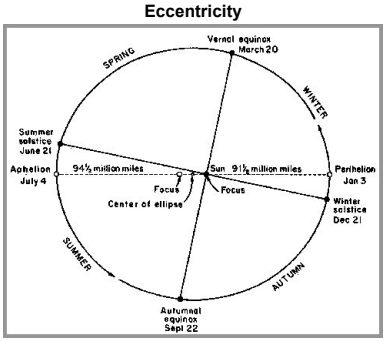
Conceptual Climate Models

Eccentricity



John Imbrie & Katherine Palmer Imbrie, *Ice Ages: Solving the Mystery,* Harvard Univ. Press, 1979.

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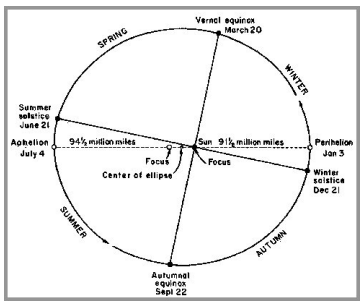
Conceptual Climate Models

Eccentricity

Perihelion: 91.5x10^6 mi
Aphelion: 94.5x10^6 mi

Semimajor axis: 93x10^6 mi

Eccentricity: 1.5/93 = 0.016



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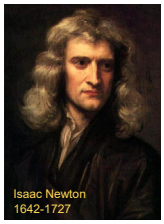
Conceptual Climate Models

Planetary Motion

$$m_i \frac{d^2 x_i}{dt^2} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{G m_i m_j \left(x_j - x_i\right)}{\left|x_j - x_i\right|^3}$$



Jacques Laskar (1955-)

The orbits of all the planets can be computed (both forward and backward in time) for billions of years.

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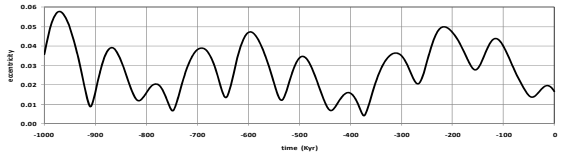
Conceptual Climate Models

Earth's Eccentricity



Note periods of about 100 kyr and 400 kyr.

As e varies between 0 and 0.06, the total amount of solar energy hitting Earth varies by about 0.2%, not much, but enough to affect the climate.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* 428, 261–285.

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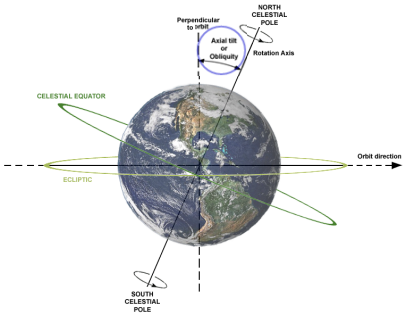
Conceptual Climate Models

Earth's Obliquity



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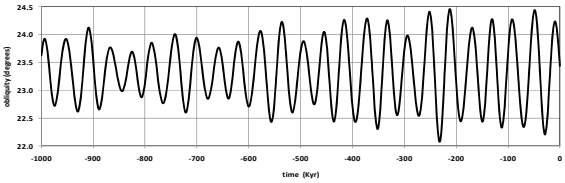
Conceptual Climate Models

Earth's Obliquity



Note period of about 41 Kyr.

Changes in obliquity do not change the total amount of solar energy hitting Earth, but they do change its distribution across latitudes.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* 428, 261–285.

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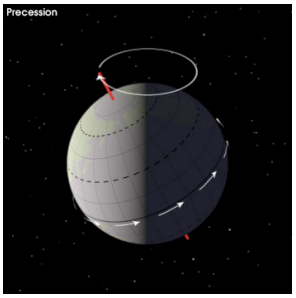
Conceptual Climate Models

Precession



http://earthobservatory.nasa.gov/Library/Giants/Milankovitch/milankovitch_2.html

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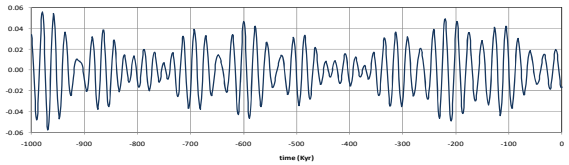
Conceptual Climate Models

Earth's Precession Index



index = $e \sin\rho$, where e = eccentricity and ρ = precession angle (measured from spring equinox)

Note period of about 23 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* 428, 261–285.

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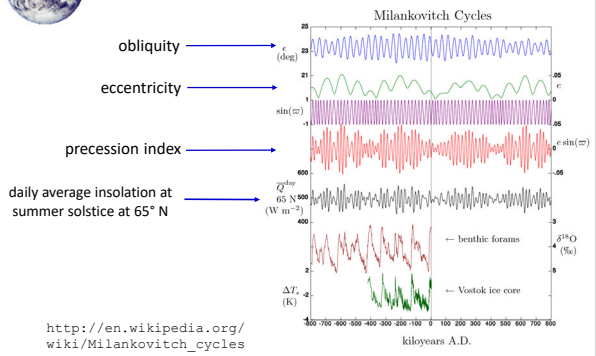
Conceptual Climate Models



http://en.wikipedia.org/wiki/Milankovitch_cycles

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Conceptual Climate Models

Milankovitch Cycles

A Mathematical Contribution to Climate Science

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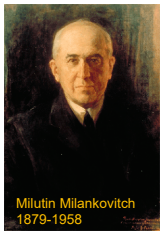
His work was not fully accepted until 1976.



Milutin Milankovitch 1879-1958

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Conceptual Climate Models

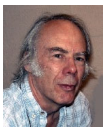
What happened in 1976?

John Imbrie

Hays, Imbrie, and Shackleton, "Variations in the Earth's Orbit: Pacemaker of the Ice Ages," *Science* 194, 10 December 1976.

James D. Hays

"It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages."



Nicholas Shackleton

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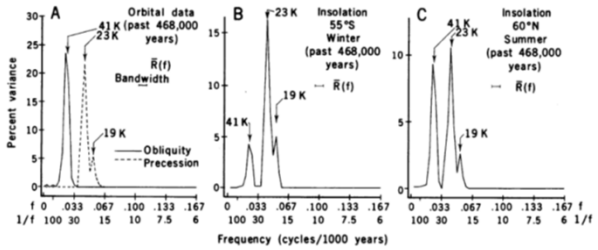
Conceptual Climate Models

Solar Forcing (Hays, et al)



Hays, *et al*, *Science* 194 (1976), p. 1125

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Conceptual Climate Models

Hays, et al, Summary

1) Three indices of global climate have been monitored in the record of the past 450,000 years in Southern Hemisphere ocean-floor sediments.

2) ... climatic variance of these records is concentrated in three discrete spectral peaks at periods of 23,000, 42,000, and approximately 100,000 years. These peaks correspond to the dominant periods of the earth's solar orbit, and contain respectively about 10, 25, and 50 percent of the climatic variance.

Hays, *et al*, *Science* 194 (1976), p. 1125

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Conceptual Climate Models

Hays, et al, Summary

3) The 42,000-year climatic component has the same period as variations in the obliquity of the earth's axis and retains a constant phase relationship with it.

4) The 23,000-year portion of the variance displays the same periods (about 23,000 and 19,000 years) as the quasiperiodic precession index.

5) The dominant, 100,000-year climatic component has an average period close to, and is in phase with, orbital eccentricity. Unlike the correlations between climate and the higher-frequency orbital variations (which can be explained on the assumption that the climate system responds linearly to orbital forcing), an explanation of the correlation between climate and eccentricity probably requires an assumption of nonlinearity.

Hays, *et al*, *Science* 194 (1976), p. 1125

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Conceptual Climate Models

Hays, et al, Summary

6) It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages.

7) A model of future climate based on the observed orbital-climate relationships, *but ignoring anthropogenic effects*, predicts that the long-term trend over the next seven thousand years is toward extensive Northern Hemisphere glaciation*.

*Quoted by George Will, Washington Post, February 5, 2009

Hays, *et al*, *Science* 194 (1976), p. 1125

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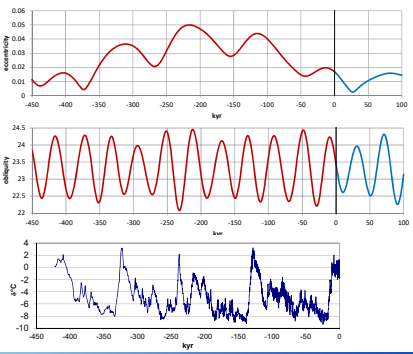
Conceptual Climate Models

The Coming Ice Age

eccentricity

obliquity

Antarctic temperature record



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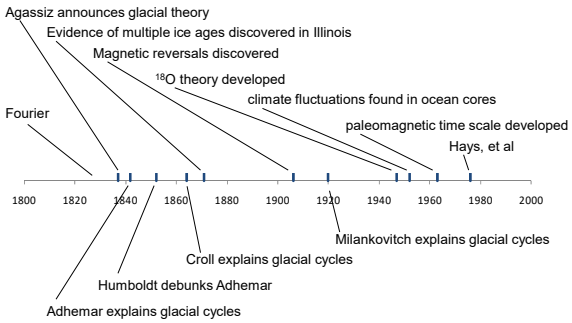
Conceptual Climate Models

Example: Milankovitch Cycles

History of Discovery



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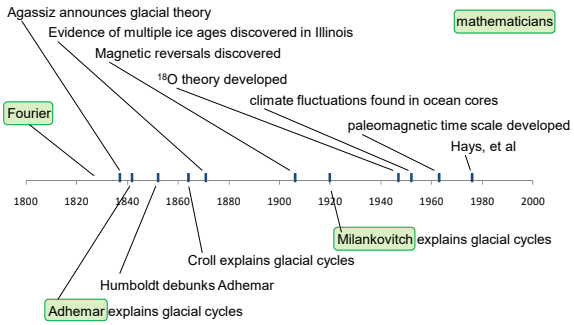
Conceptual Climate Models

Example: Milankovitch Cycles

History of Discovery



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Conceptual Climate Models

Moral

Mathematicians have always played a role in climate science.

We still can!

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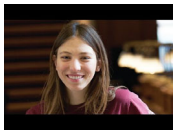
Conceptual Climate Models

What can mathematicians do?

Understand and predict permafrost melt.

Kaitlin Hill
Postdoc 2017-19



Aileen Zebrowski
BS 2016

Maria Sanchez Muniz
PhD 2025

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Conceptual Climate Models

What can mathematicians do?

Understand and predict permafrost melt.

The amount of carbon stored in permafrost is comparable to the cumulative emissions from fossil fuels.

How fast will it melt?

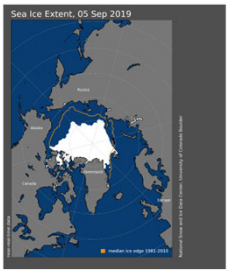
Tools: Dynamical Systems, PDE

50

Conceptual Climate Models

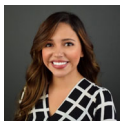
What can mathematicians do?

Understand and predict sea ice melt



Somyi Baek
PhD 2020

Wen Xing
MS 2015



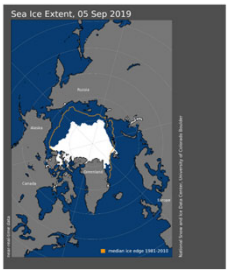
Daniela Beckelhymer,
PhD student

51

Conceptual Climate Models

What can mathematicians do?

Understand and predict sea ice melt



Bad News
As the sea ice is replaced with open water, more of the Sun's energy is absorbed, raising global temperatures.

Worse News
Open water leads to oil extraction, further increasing the fossil fuel emissions.

Tools: Dynamical Systems, Data Analysis

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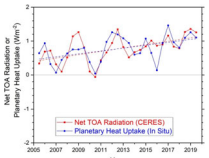
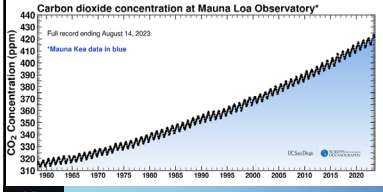
Conceptual Climate Models

What can mathematicians do?





Model heat imbalance as a function of atmospheric carbon.

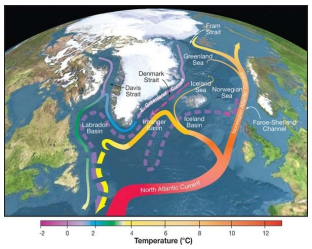
Gisell Ayala-Corral
PhD student

53

Conceptual Climate Models

What can mathematicians do?

Model tipping points in the Atlantic Meridional Overturning Circulation (AMOC).



https://en.wikipedia.org/wiki/Atlantic_meridional_overturning_circulation

Jasmine Noory
PhD student

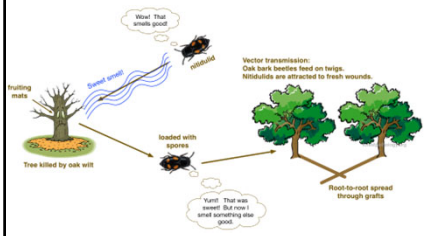
54

Conceptual Climate Models

What can mathematicians do?

Model oak wilt disease.

Yorkinoy Shermatova
PhD 2024



55

Conceptual Climate Models

What can mathematicians do?

Model the effects of climate change on boreal forests.

https://www.borealforestfacts.com/?p=234

Christine Brasic
PhD student

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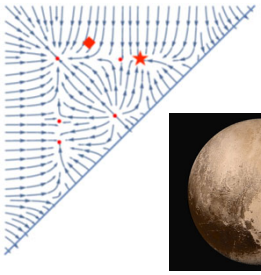
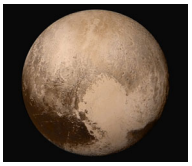
Conceptual Climate Models

What can mathematicians do?

Explain Pluto's heart.





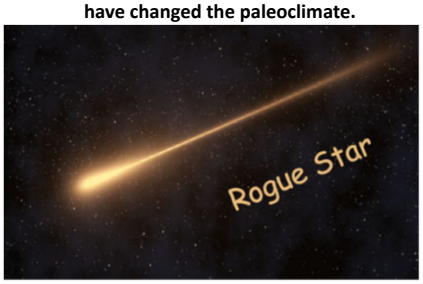
Alice Nadeau
PhD 2019

57

Conceptual Climate Models

What can mathematicians do?

Explore whether a rogue star could have changed the paleoclimate.



Harini Chandramouli,
PhD 2020

Hye Kyung Kim
PhD student

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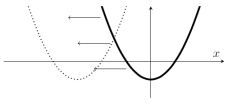
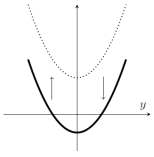
Conceptual Climate Models

What can mathematicians do?

Kate Meyer
PhD 2019

Develop a mathematical theory of resilience.





Grace Zhang
PhD 2024

59

Conceptual Climate Models

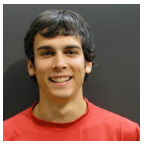
What can mathematicians do?

Develop new mathematics to study climate models.

Esther Widiasih
PhD 2010

Samantha Schumacher
PhD 2014

Julie Leifeld
PhD 2016



Jon Hahn
PhD 2017

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Conceptual Climate Models

What can mathematicians do?

Develop new mathematics to study climate models.

Shannon Negaard Paper
PhD 2019

Cameron Thieme
PhD 2021

Carlos Osco Huaricapcha
PhD student

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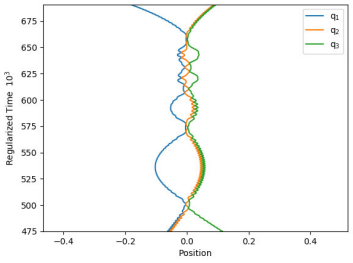
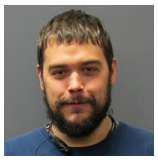
Conceptual Climate Models

What can mathematicians do?

Study whether neural networks can learn to solve ODEs.





Ty Frazier
PhD 2024

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Conceptual Climate Models

What can mathematicians do?

Join MCRN!

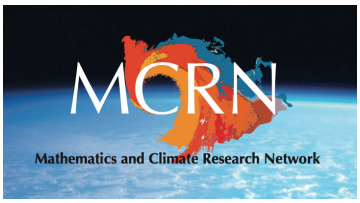
Local activists:
Dani Beckelhymer
Jasmine Noory
Yorkinoy Shermatova

63

The Case for Conceptual Climate Models

Richard McGehee
School of Mathematics
University of Minnesota

Mathematics of Climate Seminar
September 2, 2025



https://sites.google.com/view/math-climate

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