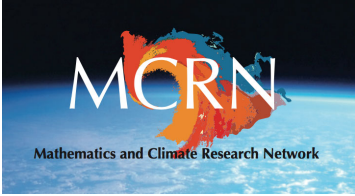


Introduction to Energy Balance

Richard McGehee
School of Mathematics
University of Minnesota
Mathematics of Climate Seminar
September 9, 2025



<https://sites.google.com/view/math-climate>

1



Energy Balance

Is the climate changing?



Mathematics of Climate Seminar 9/9/2025

2



Energy Balance

Is the climate changing?

Floods
Droughts
Fires
Heat



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3



Energy Balance

Climate Emergency?

floods

Central Texas, July 2025



https://en.wikipedia.org/wiki/July_2025_Central_Texas_floods



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4



Energy Balance

Climate Emergency?

floods



SCIENTIFIC AMERICAN

SEPTEMBER 30, 2024 | 6 MIN READ

Hurricane Helene's Devastation Shows No Region Is Safe from Climate-Fueled Disaster

Hurricane Helene fueled catastrophic flooding from Florida to Appalachia, leaving millions without power.

The swath of devastation, which cut hundreds of miles inland from Florida's Gulf Coast into southern Appalachia, is a stark reminder of the worsening impacts of climate change. Rising temperatures are fueling stronger, deadlier hurricanes with impacts that can ripple across the country.

It's also an indicator that no regions are immune to the dangers of climate-fueled disasters. Asheville, North Carolina — which experienced some of the worst hurricane-related flooding in the nation — has previously been described as a "climate haven."



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Energy Balance

Climate Emergency?

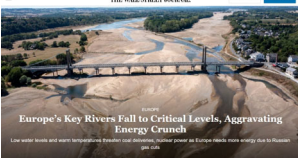
droughts

Hoover Dam, USA



www.escentral.com/story/opinion/op-ed/joannalhands/2021/05/20/lake-mead-liberty-lies-2-shortage-2023-impact-wisconsin/5183361001/

Loire River, France



THE WALL STREET JOURNAL

Europe's Key Rivers Fall to Critical Levels, Aggravating Energy Crunch

Low water levels and rising temperatures threaten to exacerbate power and energy needs across Europe, due to Russia's gas cut.

www.wsj.com/articles/europe-key-rivers-fall-to-critical-levels-aggravating-energy-crunch-11660216554



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Energy Balance

Climate Emergency?

fires

Los Angeles burning



The LA county wildfires could be the costliest in US history, early estimates say

January 11, 2025

AP

<https://www.scientificamerican.com/article/destructive-los-angeles-fires-explained-in-photos/>

MCRNN

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Energy Balance

heat

The New York Times

Phoenix's Month in Hell: A 31-Day Streak of Record Heat Ends

A continuous stretch of days reaching or exceeding 110 degrees has filled emergency rooms. On Monday, the city hit 106 degrees, breaking the run, but setting a new, brutal record.



<https://www.abcnews.com/news/5878441/abc-news-phoenix-heat-wave-could-be-the-most-100-days-on-record-for-the-city-1562782>

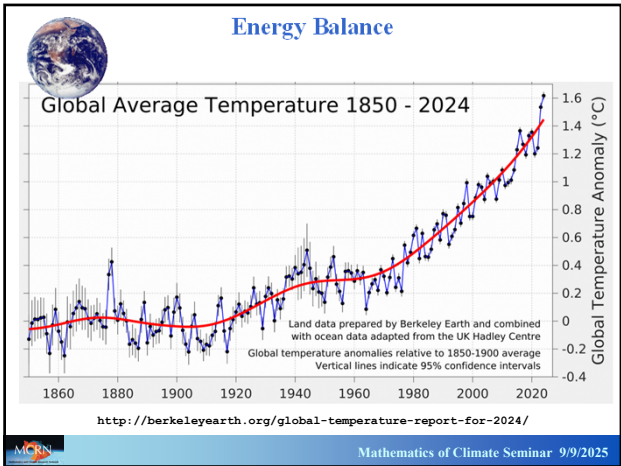


<https://abcapitolnews.com/news/2023/06/06/a-study-phoenixs-face-health-or-risk-if-heatwaves-blackout-hit-at-same-time/>

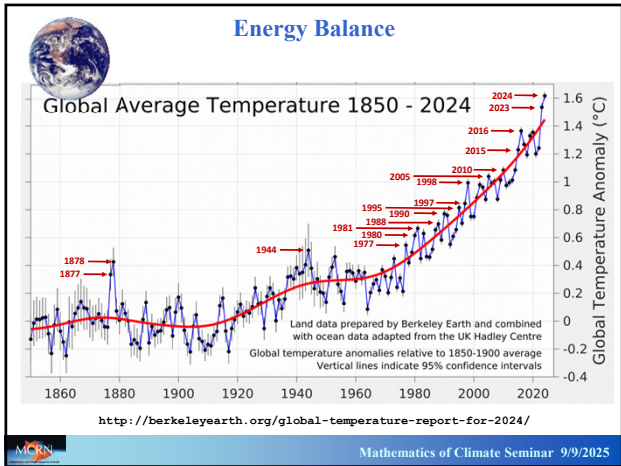
MCRNN

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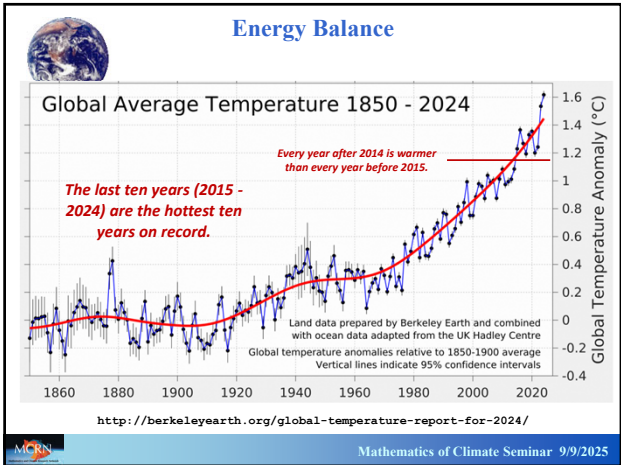
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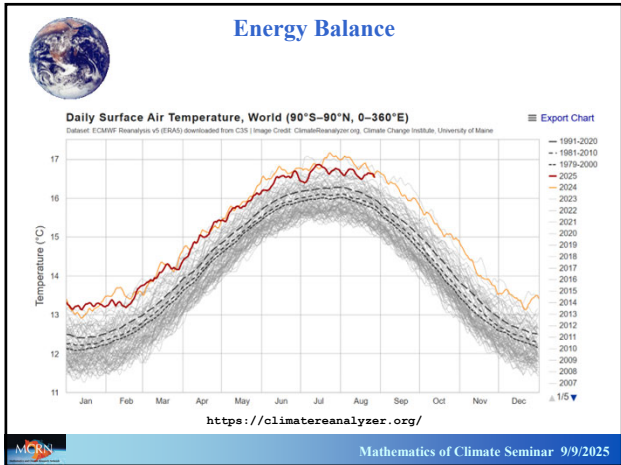
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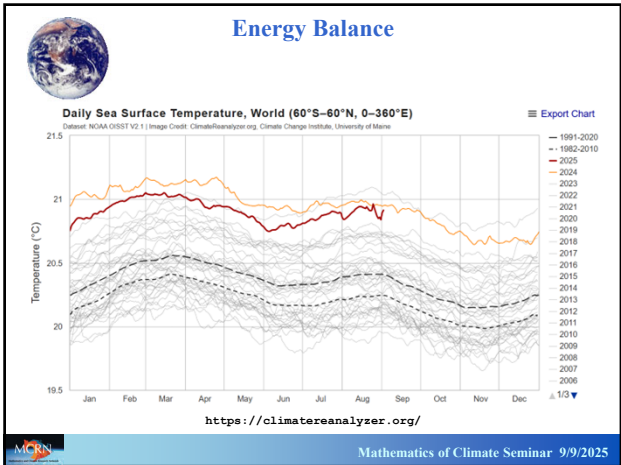
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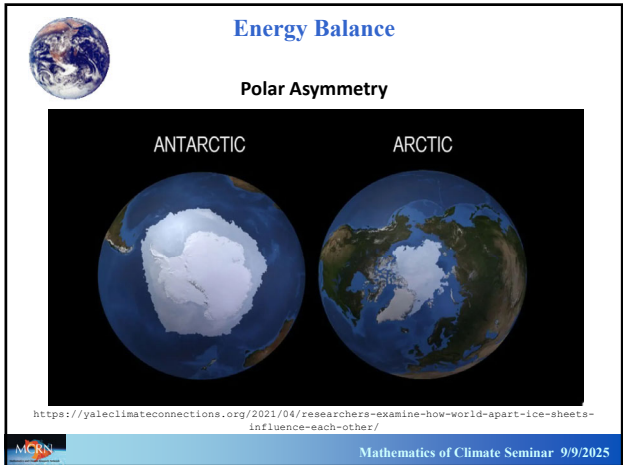
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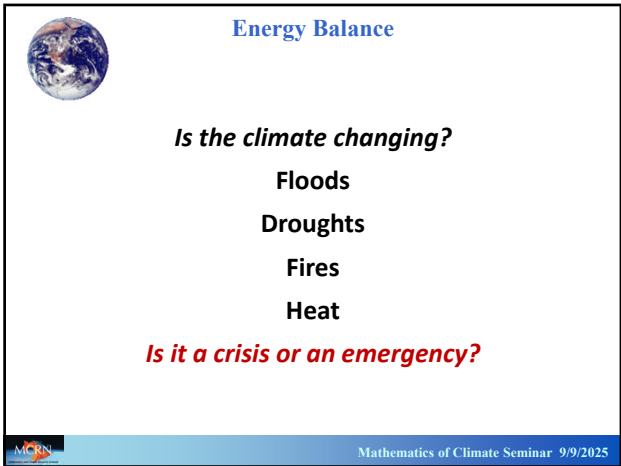
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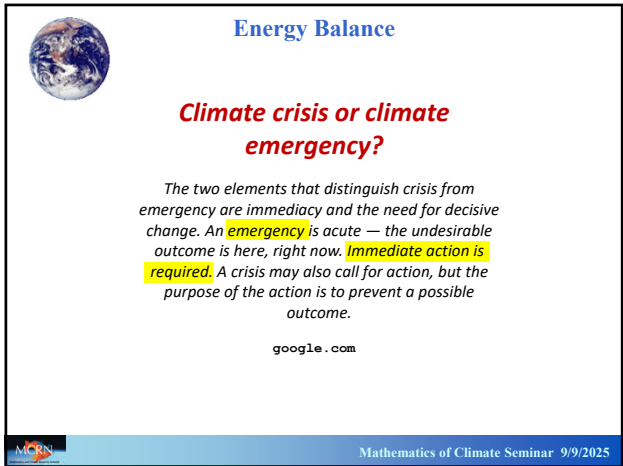
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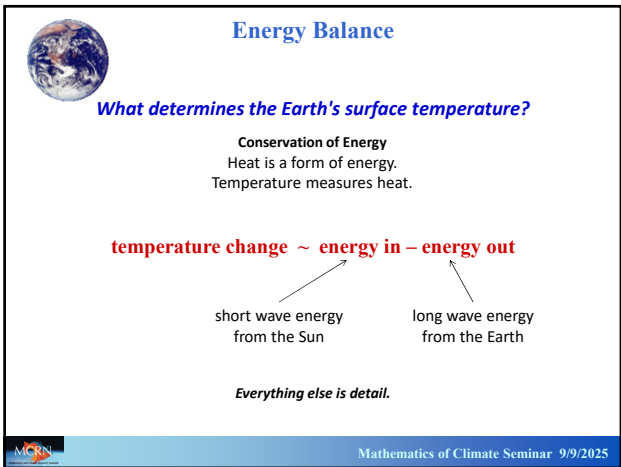
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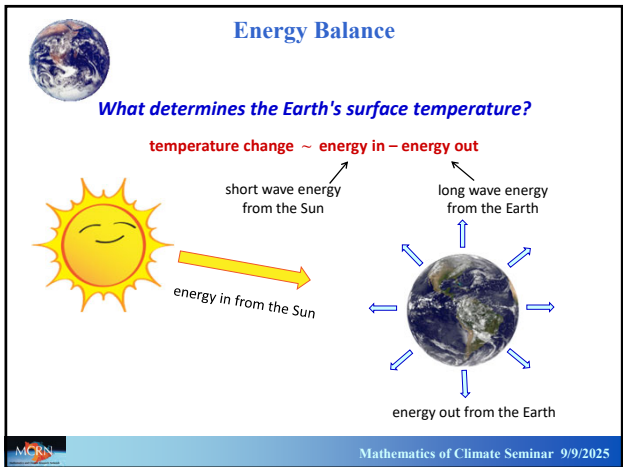
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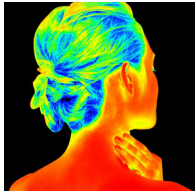


Energy Balance

Black-Body Radiation

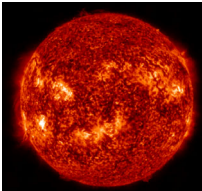
Every object emits electromagnetic radiation according to its temperature. The intensity of the radiation (power flux) is approximated by a theoretical object called a *perfect black body*.

human



<https://letstalkscience.ca/educational-resources/backgrounders/thermal-imaging>

sun



<https://solarsystem.nasa.gov/solar-system/sun/overview/>

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Energy Balance

Black-Body Radiation

Stefan-Boltzmann Law

$$F = \sigma T^4$$

power flux (W/m²) ← temperature (K)

Stefan-Boltzmann constant
 $\sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

watts per square meter
power flux (W/m²)
watt = joule per second = "power"

kelvin
0 K = -273°C = "absolute zero"

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Energy Balance

Black-Body Radiation

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watts per square meter
power flux (W/m²)
watt = joule per second = "power"

kelvin
0 K = -273°C = "absolute zero"

Reasonable approximation:
Every body in the solar system radiates energy according to this law.

Let's try the Sun.

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Energy Balance

Stefan-Boltzmann Law

$$F = \sigma T^4$$

power flux (W/m²) ← temperature (K)

Stefan-Boltzmann constant
 $\sigma \approx 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

Example
surface temperature of the Sun: 5780K
power flux: $5.67 \times 10^{-8} \times (5780)^4 = 6.33 \times 10^7 \text{ W/m}^2$
total solar power output: $6.33 \times 10^7 \times 4\pi(r_s)^2$, where r_s = radius of the sun = $6.96 \times 10^8 \text{ m}$
total solar output: $3.85 \times 10^{26} \text{ W}$
230 nanoseconds = time it takes for the Sun to produce the equivalent of the annual global electricity production.



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Energy Balance

Insolation

(Incoming solar Radiation)

How much energy from the Sun is hitting the Earth?

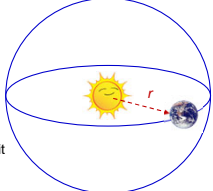
Solar flux at a distance r from the sun:

$$F = \frac{6.33 \times 10^7 4\pi r_s^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_s}{r}\right)^2 \text{ W/m}^2$$

$r_s = 6.96 \times 10^8 \text{ m}$
 $r = 1.5 \times 10^{11} \text{ m}$

$$F = 1368 \text{ W/m}^2$$

← solar flux at Earth's orbit



Average surface flux: $\frac{F \times \pi r_E^2}{4\pi r_E^2} = \frac{F}{4} = 342 \text{ W/m}^2$

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Energy Balance

Insolation

(Incoming solar Radiation)

Solar flux at a distance r from the sun:

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$$F = 1368 \text{ W/m}^2$$

← solar flux at Earth's orbit

Power intercepted by the Earth: $F \times \pi r_E^2 \text{ W}$
Earth's surface area: $4\pi r_E^2 \text{ m}^2$
Average surface flux: $\frac{F \times \pi r_E^2}{4\pi r_E^2} = \frac{F}{4} = 342 \text{ W/m}^2$

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Energy Balance

Insolation

Solar flux at a distance r from the sun:

$$F = \frac{6.33 \times 10^{27} 4\pi r_s^2}{4\pi r^2} = 6.33 \times 10^7 \left(\frac{r_s}{r}\right)^2 \text{ W/m}^2$$

$r_s = 6.96 \times 10^8 \text{ m}$
 $r = 1.5 \times 10^{11} \text{ m}$
 $F = 1368 \text{ W/m}^2$

Power intercepted by the Earth:
 $F \times \pi r_E^2 \text{ W}$, $r_E = \text{radius of Earth} = 6.37 \times 10^6 \text{ m}$
Power intercepted: $1.74 \times 10^{17} \text{ W}$

Biologically Stored Energy
total coal reserves: 10^{15} kg
energy content: $3 \times 10^7 \text{ J/kg}$
total energy in coal reserves: $3 \times 10^{22} \text{ J}$
= 2 days of insolation



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Energy Balance

Insolation

Global Average Insolation

intercepted flux: $F = 1368 \text{ W/m}^2$
Earth cross-section: πr_E^2
surface area: $4\pi r_E^2$
average flux: $1368/4 = 342 \text{ W/m}^2 = Q$

Simple Model

Assume that Earth is a perfectly thermally conducting black body.

$$Q = \sigma T^4$$
$$T = (Q/\sigma)^{1/4} = (342/5.67 \times 10^{-8})^{1/4}$$
$$= 279\text{K} = 6^\circ\text{C} = 43^\circ\text{F}$$

Dynamics
 $R \frac{dT}{dt} = Q - \sigma T^4$

heat capacity $\rightarrow$ $R \frac{dT}{dt}$ $\rightarrow$ stable equilibrium

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Energy Balance

Insolation

Global Average Insolation

intercepted flux: $F = 1368 \text{ W/m}^2$
Earth cross-section: πr_E^2
surface area: $4\pi r_E^2$
average flux: $1368/4 = 342 \text{ W/m}^2 = Q$

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heat imbalance

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Energy Balance

What determines the Earth's surface temperature?

Conservation of Energy
Heat is a form of energy.
Temperature measures heat.

heat imbalance

temperature change ~ energy in – energy out

short wave energy from the Sun $\rightarrow$ energy in
long wave energy from the Earth $\rightarrow$ energy out

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Energy Balance

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Can we measure heat imbalance?

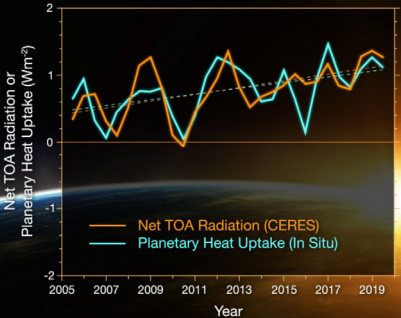
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Energy Balance

Earth's Heat Imbalance



Net TOA Radiation or Planetary Heat Uptake (Wm^{-2})

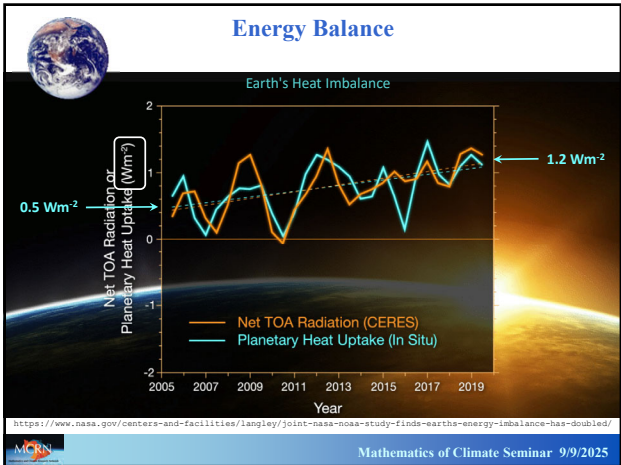
Year

— Net TOA Radiation (CERES)
— Planetary Heat Uptake (In Situ)

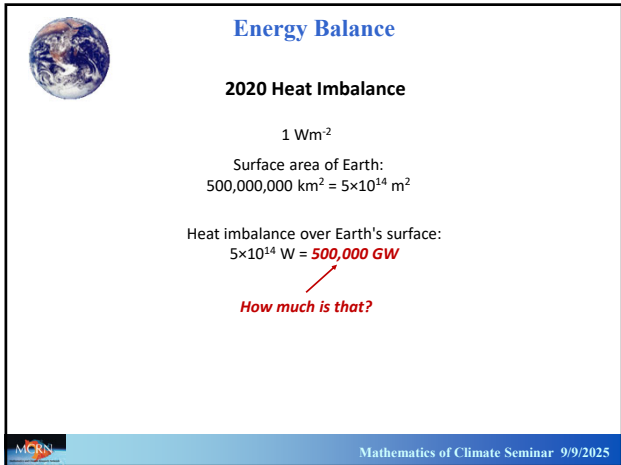
<https://www.nasa.gov/centers-and-facilities/langley/joint-nasa-noaa-study-finds-earths-energy-imbalance-has-doubled/>

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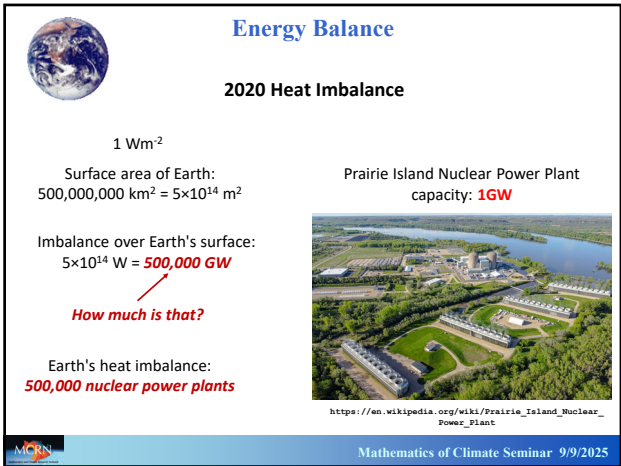
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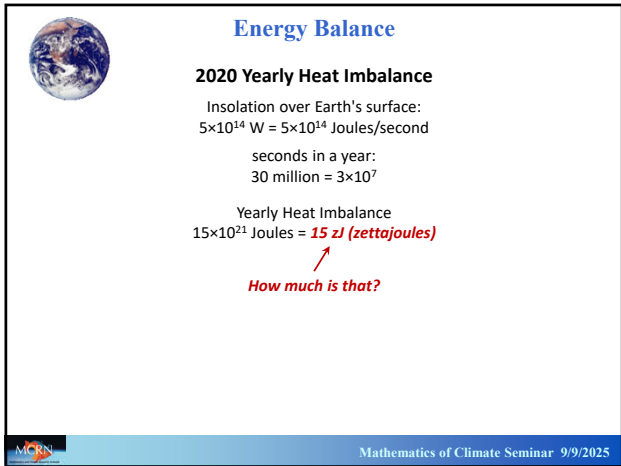
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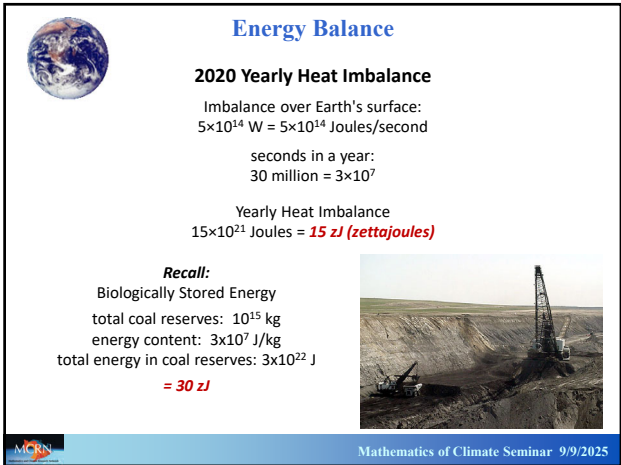
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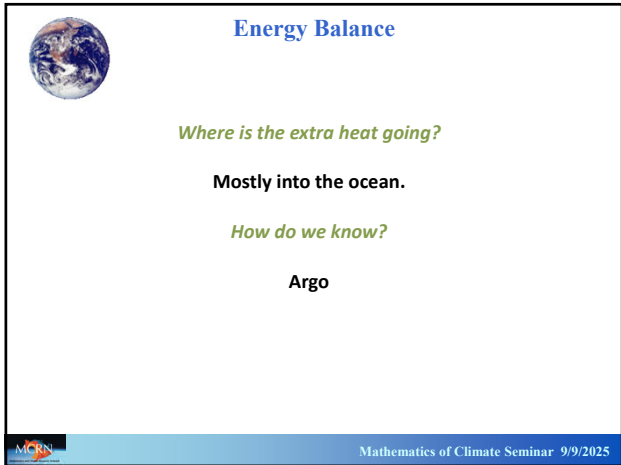
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34



35



36

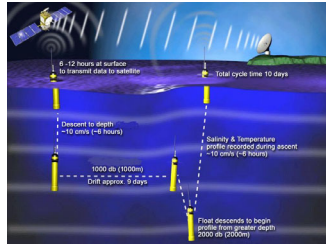


Energy Balance

Ocean Heat Content

Argo

part of the integrated global observation strategy



<http://www.argo.ucsd.edu/>

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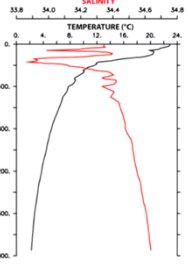
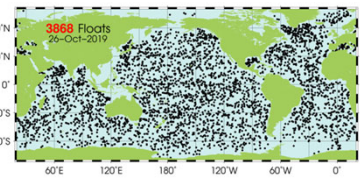


Energy Balance

Ocean Heat Content

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<http://www.argo.ucsd.edu/>

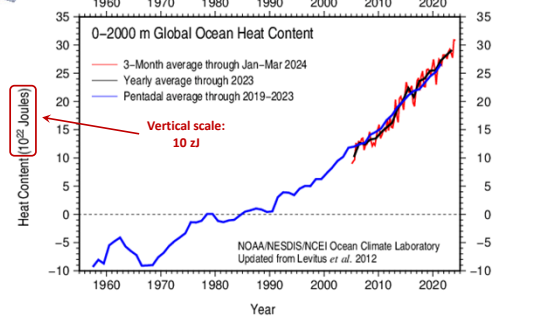
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Energy Balance

Ocean Heat Content



Heat Content (10^{21} Joules)

Vertical scale: 10 zJ

NOAA/NESDIS/NCEI Ocean Climate Laboratory
Updated from Levitus *et al.* 2012

<https://www.ncei.noaa.gov/access/global-ocean-heat-content/>

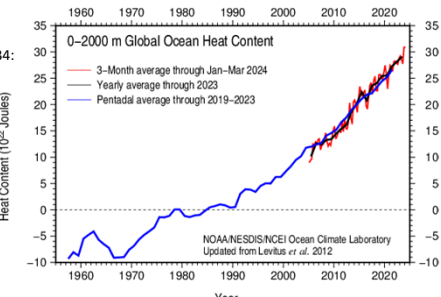
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Energy Balance

Ocean Heat Content



Heat Content (10^{21} Joules)

Energy added 1990 - 2034:
310 zJ
coal reserves: 30 zJ
cat 5 hurricane: 0.0001 zJ
megaton bomb: 0.000005 zJ

What's causing the heat imbalance?

NOAA/NESDIS/NCEI Ocean Climate Laboratory
Updated from Levitus *et al.* 2012

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Energy Balance

What's causing the heat imbalance?

The Greenhouse Effect

Greenhouse gases (CO_2 , H_2O , CH_4) are transparent to visible light, but opaque to infrared light. The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.



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Energy Balance

What's causing the heat imbalance?

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Who discovered the greenhouse effect?



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Energy Balance

What's causing the heat imbalance?

The Greenhouse Effect

Greenhouse gases (CO_2 , H_2O , CH_4) are transparent to visible light, but opaque to infrared light. The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.

Who discovered the greenhouse effect?

A mathematician!

Joseph Fourier (1827), *Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires*, *Mémoires de l'Académie Royale des Sciences*, t. vii., p. 569.



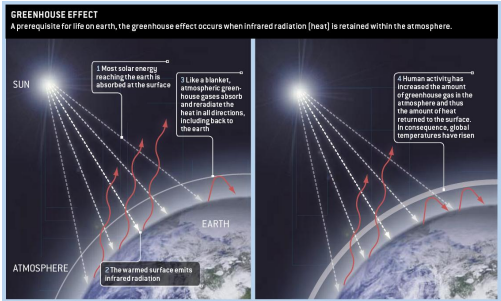
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Energy Balance

The Greenhouse Effect



GREENHOUSE EFFECT
A prerequisite for life on earth, the greenhouse effect occurs when infrared radiation (heat) is retained within the atmosphere.

1 Non solar energy reaching the earth is absorbed at the surface
2 The warmed surface emits infrared radiation
3 Like a blanket, atmospheric greenhouse gases absorb and re-radiate the heat in all directions, including back to the earth
4 Human activity has increased the amount of greenhouse gas in the atmosphere and thus the amount of heat returned to the surface. In consequence, global temperatures have risen

Gary Stix, *Scientific American* September 2006, pp.46-49

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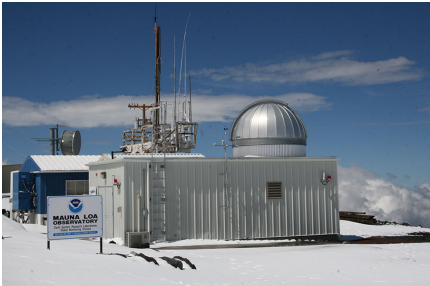
44



Energy Balance

Can we measure greenhouse gasses?

Mauna Loa Observatory



https://research.noaa.gov/Portals/0/EasyDNNews/1502/2000600p587EDNmain10061mlo_sign_miller.jpg

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Energy Balance

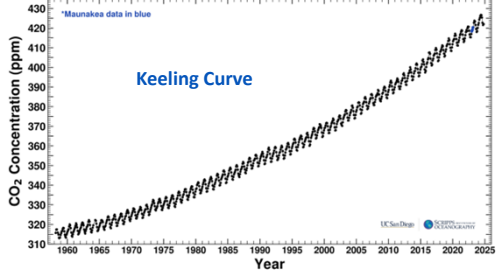
Can we measure greenhouse gasses?

Mauna Loa Observatory, Hawaii*

Monthly Average Carbon Dioxide Concentration

Data from Scripps CO₂ Program Last updated December 2024

*Maunakea data in blue



CO₂ Concentration (ppm)

Year

Keeling Curve

<https://keelingcurve.ucsd.edu/>

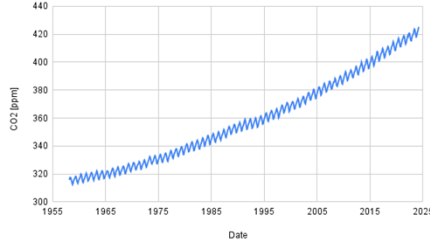
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Energy Balance

Mauna Loa Data



CO₂ [ppm]

Date

Data downloaded from https://scrippsco2.ucsd.edu/data/atmospheric_co2/primary_mlo_co2_record.html

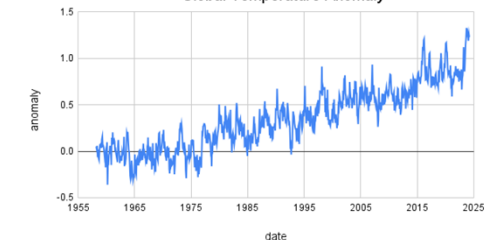
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Energy Balance

Global Temperature Anomaly



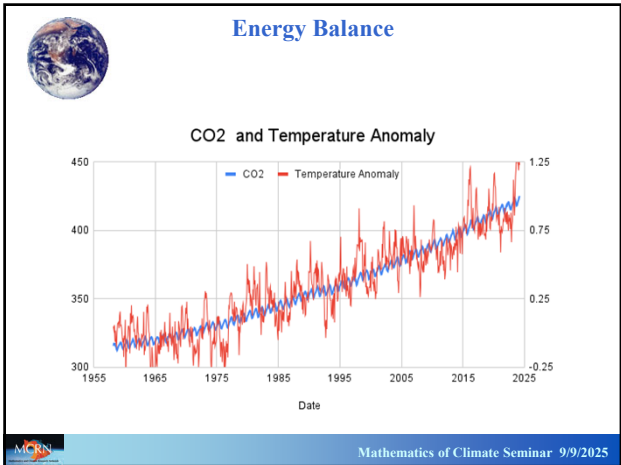
anomaly

date

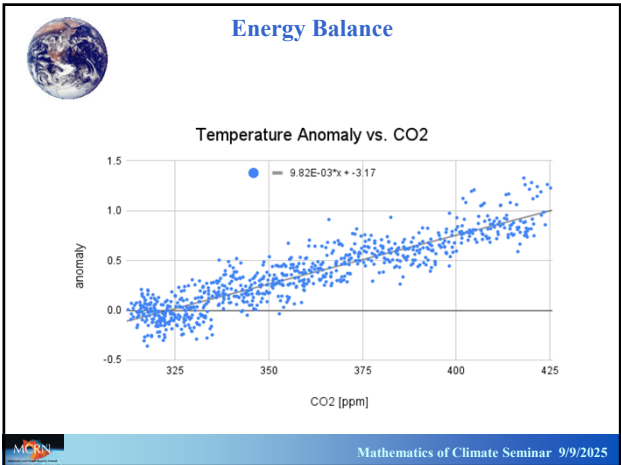
Data downloaded from <https://berkeleyearth.org/data/>

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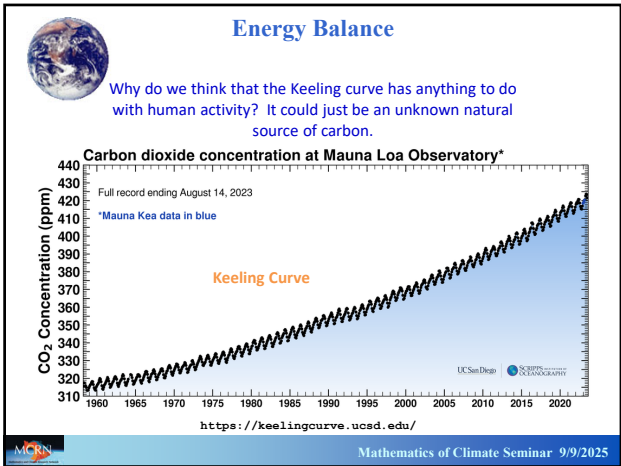
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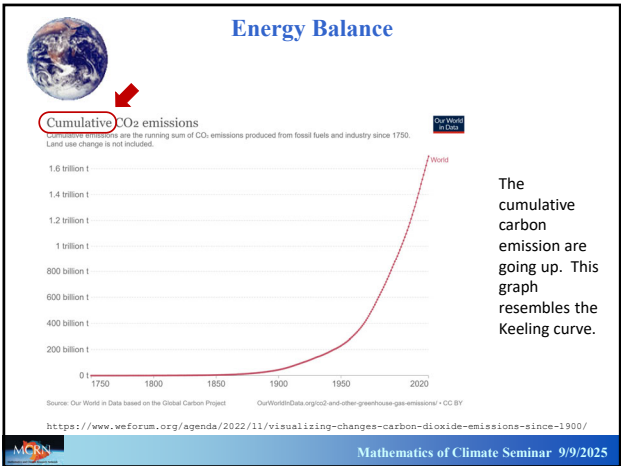
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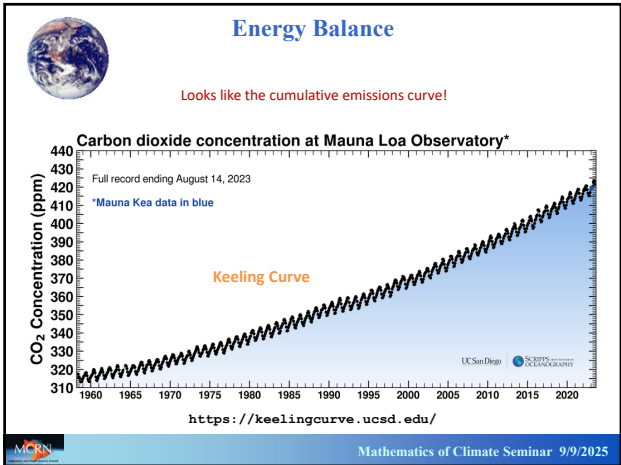
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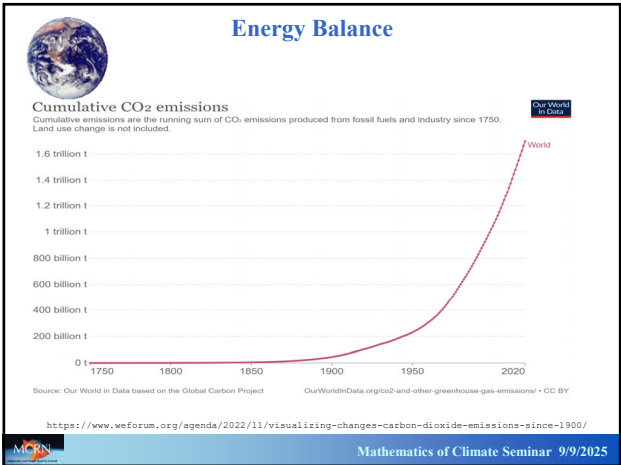
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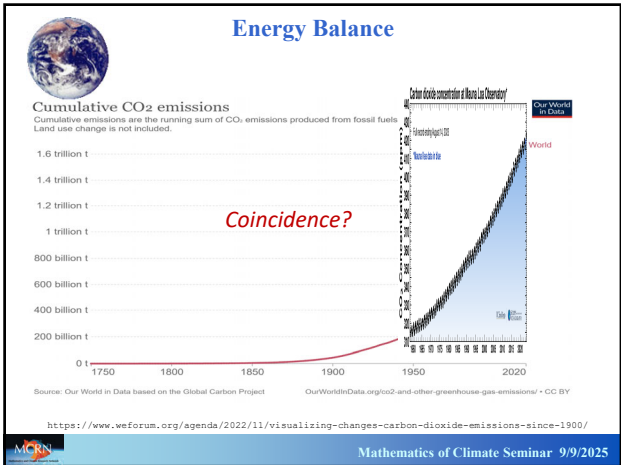
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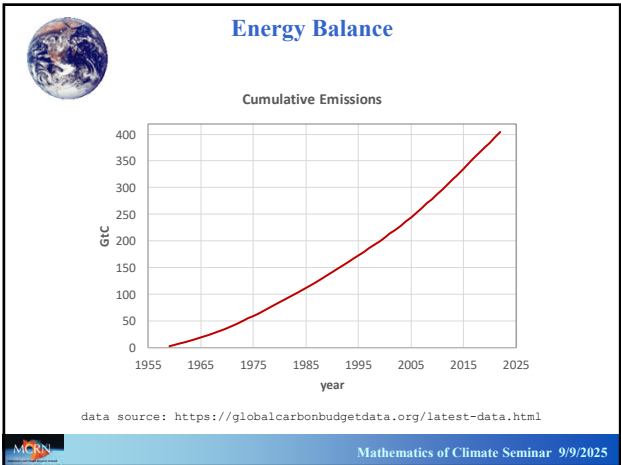
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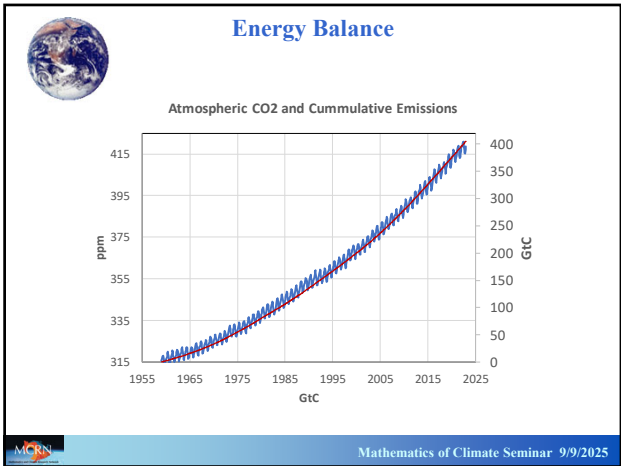
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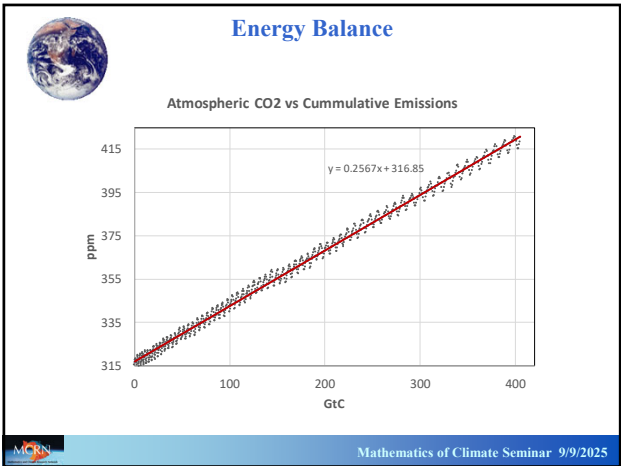
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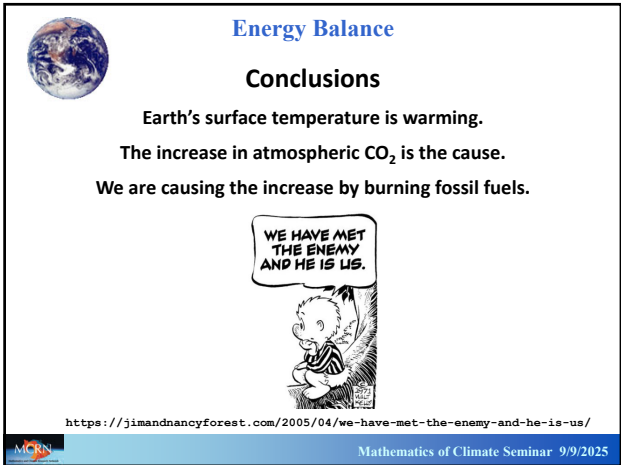
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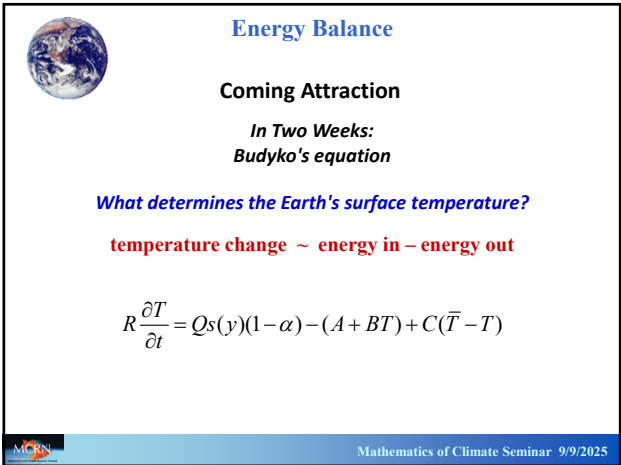
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58



59



60