Budyko's Model and Snowball Earth

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Mathematics of Climate Seminar
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Mathematics and Climate Research Network

https://sites.google.com/view/math-climate

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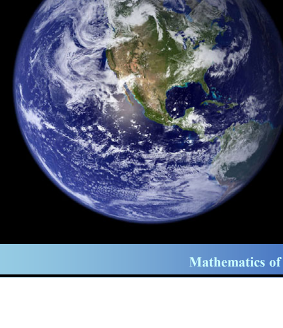
Budyko's Model



$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\bar{T} - T)$$

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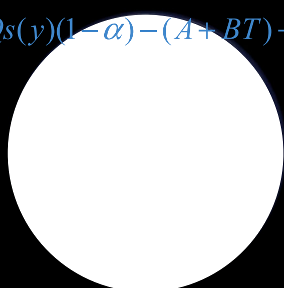
Budyko's Snowball

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\bar{T} - T)$$



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Snowball Earth

Earth's climate has changed many times in the past.

Paleoclimate

Paleoclimatology is the scientific study of climates predating the invention of meteorological instruments, when no direct measurement data were available. As instrumental records only span a tiny part of Earth's history, the reconstruction of ancient climate is important to understand natural variation and the evolution of the current climate.

https://en.wikipedia.org/wiki/Paleoclimatology

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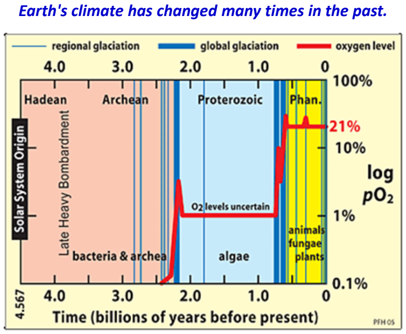
Snowball Earth

Earth's climate has changed many times in the past.



http://www.snowballearth.org/when.html

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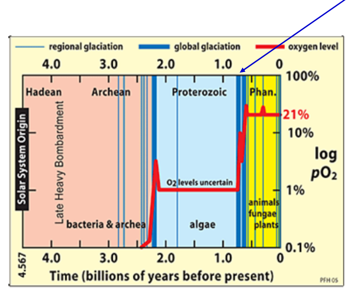
Snowball Earth



Snowball Earth is not in our immediate future, but it is an interesting phenomenon and can be studied with conceptual models, *e.g.*, Budko's model.

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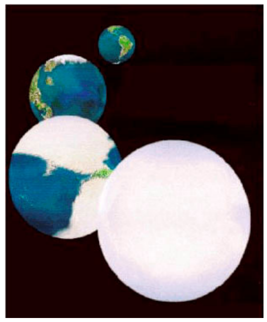
Snowball Earth

Is it possible for Earth to become completely covered in ice?
(Snowball Earth)

Did it ever happen?



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Snowball Earth

Latitude Dependence

$$R\frac{\partial T(y,t)}{\partial t} = Qs(y)(1-\alpha) - (A + BT(y,t))$$

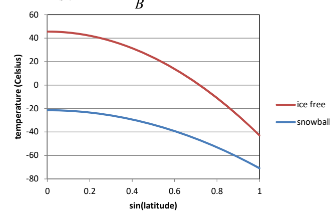
Equilibrium Solution

$$T^*(y) = \frac{Qs(y)(1-\alpha) - A}{B}$$

Tung*

α = 0.32: ice free
α = 0.62: snowball

Note that α = 0.32 is in the range of current Earth.



* K.K. Tung, *Topics in Mathematical Modeling*, Princeton U. Press, 2007

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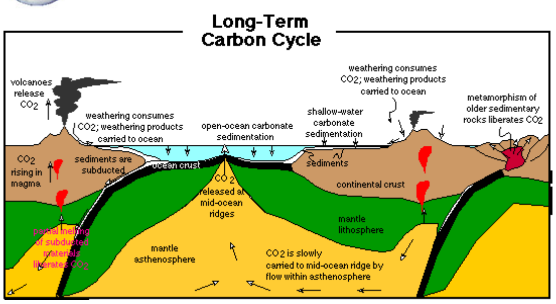
Snowball Earth

Earth's Carbon Cycle



http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm

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Snowball Earth

Earth's Carbon Cycle

Volcanos emit CO_2.

Silicate weathering carries carbon to the ocean.

Carbon sinks to the bottom of the ocean by the "biological pump" and by the precipitation of calcium carbonate.

The carbon is captured in the sediment.

The sediment is subducted beneath the continental crust by plate tectonics.

Volcanic activity released carbon from the carbonate rocks in the form of CO_2.

Repeat.

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Snowball Earth

Earth's Carbon Cycle

Silicate Weathering

Rainwater containing dissolved CO_2 falling on silicate rocks replaces a silicon atom with a carbon atom, ultimately producing calcium carbonate (limestone) and silicon dioxide (quartz). For example, calcium silicate (Wollastonite)**:**

$$CaSiO_3 + CO_2 \rightarrow CaCO_3 + SiO_2$$

Under volcanic conditions, the carbon atom is replaced by a silicon atom, completing the long-term carbon cycle.

$$CaCO_3 + SiO_2 \rightarrow CaSiO_3 + CO_2$$

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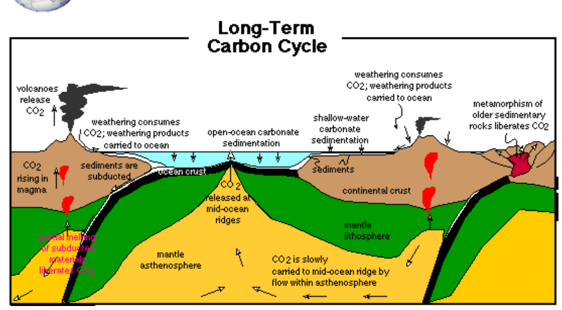
Earth's Carbon Cycle



http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm

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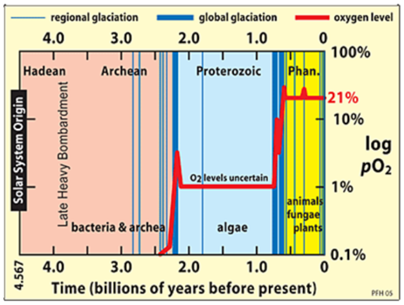
Snowball Earth

There is evidence that Snowball Earth has occurred, the last time about 600 million years ago.



http://www.snowballearth.org/when.html

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Snowball Earth

The continents were clustered near the equator.

Hoffman & Schrag, *Snowball Earth*, Scientific American, January 2000, 68-75

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Snowball Earth

“Ice-rafted debris” occurred in ocean sediments near the equator, indicating large equatorial glaciers calving icebergs.

Hoffman & Schrag, *Snowball Earth*, Scientific American, January 2000, 68-75

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Snowball Earth

Large limestone deposits “cap carbonates” are found immediately above the glacial debris, indicating a rapid warming period following the snowball.

Hoffman & Schrag, *Snowball Earth*, Scientific American, January 2000, 68-75

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Snowball Earth

If Earth ever was a snowball, how did we escape?

Idea:

When the Earth is ice-covered, silicate weathering almost disappears, but volcanic activity continues, allowing for a build-up of CO_2 in the atmosphere.

If the ice starts to melt, the ocean opens up and starts to reabsorb the CO_2, which warms the planet, causing more ice to melt, causing more CO_2 to be absorbed in the ocean, *etc.* Rapid positive feedback leads to an ice-free Earth.

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Snowball Earth

Budyko’s Equation

M. I. Budyko, "The effect of solar radiation variations on the climate of the Earth," Tellus XXI, 611-619 , 1969.

surface temperature sin(latitude) $\bar{T} = \int_0^1 T(y)dy$

$$R\frac{\partial T}{\partial t} = \underline{Qs(y)}\underline{(1-\alpha)} - \underline{(A+BT)} + \underline{C(\bar{T}-T)}$$

heat capacity insolation albedo OLR heat transport

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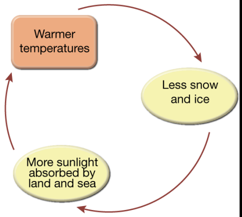
Snowball Earth

Ice-albedo Feedback

temperature warms
ice melts
albedo decreases
more sunlight absorbed
temperature warms
REPEAT

Why would it stop?



http://www.i-fink.com/melting-polar-ice/

Where is this feedback in Budko's equaton?

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\overline{T}-T)$$

α shouldn't be a constant.

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Snowball Earth

Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\overline{T}-T)$$

Where is the ice-albedo feedback?

Ice-albedo Feedback

albedo of ice: 0.62
albedo of land and water: 0.32

Assumption: there is a single boundary between ice and no ice occurring at $y=\eta$.

$$\alpha = \alpha(y,\eta) = \begin{cases} \alpha_1 = 0.32, & y<\eta \\ \alpha_2 = 0.62, & y>\eta \end{cases}$$

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Snowball Earth

Budyko's Equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha) - (A+BT) + C(\overline{T}-T)$$

Where is the ice-albedo feedback?

Ice-albedo Feedback

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$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\overline{T}-T)$$

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Snowball Earth

Ice-albedo Feedback

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$$\alpha = \alpha(y,\eta) = \begin{cases} \alpha_1 = 0.32, & y<\eta \\ \alpha_2 = 0.62, & y>\eta \end{cases}$$

What determines the ice line η?

Is it a dynamic variable or a parameter?

For now, we consider it to be a parameter.

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\overline{T}-T)$$

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Snowball Earth

Budyko's Equilibrium

$$R\frac{\partial T}{\partial t}(y,t) = Qs(y)\big(1-\alpha(y,\eta)\big) - \big(A+BT(y,t)\big) + C\big(\overline{T}(t)-T(y,t)\big)$$

Equilibrium:

$$Qs(y)\big(1-\alpha(y,\eta)\big) - \big(A+BT^*(y)\big) + C\big(\overline{T}^*-T^*(y)\big) = 0$$

Assume we know the global mean temperature.

Solve for $T^*(y)$

$$Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^* - \big(BT^*(y)\big) + C\big(-T^*(y)\big) = 0,$$
$$(B+C)T^*(y) = Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*$$

$$\boxed{T^*(y) = \frac{Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\overline{T}^*}{B+C}}$$

Now we can compute the global mean temperature!

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We need to compute this.

Budyko's Equilibrium

$$T^*(y) = \frac{Qs(y)\big(1-\alpha(y,\eta)\big) - A + C\boxed{\overline{T}^*}}{B+C}$$

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Budyko's Equilibrium

We need to compute this.

$$T^*(y)=\frac{Qs(y)(1-\alpha(y,\eta))-A+C\boxed{\overline{T}^*}}{B+C}$$

$$(B+C)T^*(y)=Qs(y)(1-\alpha(y,\eta))-A+C\overline{T}^*$$

Integrate:

$$\int_0^1(B+C)T^*(y)dy=\int_0^1\left(Qs(y)(1-\alpha(y,\eta))-A+C\overline{T}^*\right)dy$$

$$(B+C)\underbrace{\int_0^1 T^*(y)dy}_{\overline{T}^*}=Q\left(\underbrace{\int_0^1 s(y)dy}_{1}-\underbrace{\int_0^1\alpha(y,\eta)s(y)dy}_{\overline{\alpha}(\eta)}\right)-A\underbrace{\int_0^1 dy}_{1}+C\overline{T}^*\underbrace{\int_0^1 dy}_{1}$$

$$(B+\cancel{C})\overline{T}^*=Q(1-\overline{\alpha}(\eta))-A+\cancel{C\overline{T}^*}$$

$$\overline{T}^*=\frac{Q(1-\overline{\alpha}(\eta))-A}{B}$$

Success!

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Snowball Earth

Budyko's Equilibrium

$$T^*(y)=\frac{Qs(y)(1-\alpha(y,\eta))-A+C\overline{T}^*}{B+C}$$

$$(B+C)T^*(y)=Qs(y)(1-\alpha(y,\eta))-A+C\overline{T}^*$$

Integrate:

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$$(B+\cancel{C})\overline{T}^*=Q(1-\overline{\alpha}(\eta))-A+\cancel{C\overline{T}^*}$$

$$\boxed{\overline{T}^*=\frac{Q(1-\overline{\alpha}(\eta))-A}{B}}$$

Note the dependence on the parameter η.

$$\boxed{T^*(y,\eta)=\frac{Qs(y)(1-\overline{\alpha}(\eta))-A+C\overline{T}^*}{B+C},\quad\text{where}\quad \overline{T}^*(\eta)=\frac{Q(1-\overline{\alpha}(\eta))-A}{B}}$$

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Snowball Earth

Budyko's Equilibrium

$$T^*(y,\eta)=\frac{Qs(y)(1-\overline{\alpha}(\eta))-A+C\overline{T}^*}{B+C},\quad\text{where}\quad \overline{T}^*(\eta)=\frac{Q(1-\overline{\alpha}(\eta))-A}{B}$$

Can we compute the global albedo?

recall: $\alpha(y,\eta)=\begin{cases}\alpha_1=0.32, & y<\eta\\ \alpha_2=0.62, & y>\eta\end{cases}$

global albedo $\overline{\alpha}(\eta)=\int_0^1\alpha(y,\eta)s(y)dy=\int_0^\eta\alpha_1 s(y)dy+\int_\eta^1\alpha_2 s(y)dy$

$$=\alpha_1 S(\eta)+\alpha_2(1-S(\eta))$$

where $S(\eta)=\int_0^\eta s(y)dy=\int_0^\eta\underbrace{\left(1-0.241\left(3y^2-1\right)\right)}_{\text{Chylek \& Coakley}}dy=\eta-0.241\left(\eta^3-\eta\right)$

Time to summarize.

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Snowball Earth

Budyko's Equilibrium

$$R\frac{\partial T}{\partial t}(y,t)=Qs(y)(1-\alpha(y,\eta))-(A+BT(y,t))+C\left(\overline{T}(t)-T(y,t)\right)$$

Equilibrium temperature distribution:

$$T^*(y,\eta)=\frac{Qs(y)(1-\overline{\alpha}(\eta))-A+C\overline{T}^*(\eta)}{B+C}$$

$$\overline{T}^*(\eta)=\frac{Q(1-\overline{\alpha}(\eta))-A}{B}$$

$$\overline{\alpha}(\eta)=\alpha_1 S(\eta)+\alpha_2(1-S(\eta))$$

$$s(y)=1-0.241\left(3y^2-1\right)$$

$$S(\eta)=\eta-0.241\left(\eta^3-\eta\right)$$

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Snowball Earth

Budyko's Equilibria

For each fixed η, there is an equilibrium solution for Budyko's equation.

$$T_\eta^*(y)=\frac{1}{B+C}\left(Qs(y)(1-\alpha(y,\eta))-A+C\overline{T}_\eta^*\right)$$

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Snowball Earth

Budyko's Equilibria

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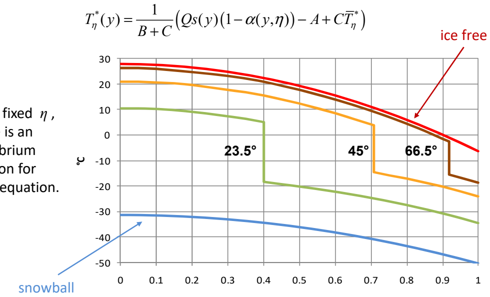

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Snowball Earth

Stability of Equilibria

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\bar{T}-T)$$

Let X be the space of functions where T lives. (*e.g.* $L^1([0,1])$)
Let

$$L: X \to X : LT = C\bar{T} - (B+C)T,$$
$$f(y) = Qs(y)\left(1-\alpha(y)\right) - A$$

Budyko's equation can be written as a linear vector field on X .

$$R\frac{dT}{dt} = f + LT$$

The operator L has only point spectrum, with all eigenvalues negative.
Therefore, *all solutions are stable*.
True for any albedo function.

experts only

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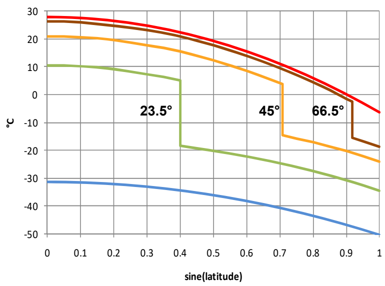
Snowball Earth

Stability of Equilibria

$$R\frac{\partial T}{\partial t}(y,t) = Qs(y)\left(1-\alpha(y,\eta)\right) - \left(A+BT(y,t)\right) + C\left(\bar{T}(t)-T(y,t)\right)$$

For each fixed η , there is a globally stable equilibrium solution for Budyko's equation.

How to pick one?



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Snowball Earth

Ice Albedo Feedback

Idea

If we artificially hold the ice line at a fixed latitude, then the surface temperature will come to an equilibrium.

However,
if the temperature at the ice line is high, we would expect ice to melt and the ice line to retreat to higher latitudes. If the temperature is low, we would expect ice to form and the ice line to advance to lower latitudes.

How to model this expectation?

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Snowball Earth

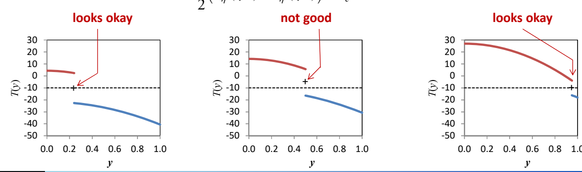
Ice Albedo Feedback

For each fixed η , there is a stable equilibrium solution for Budyko's equation.

Standard assumption: Permanent ice forms if the annual average temperature is below T_c=-10 °C and melts if the annual average temperature is above T_c.

Additional condition: The average temperature across the ice boundary must be the critical temperature T_c .

$$\frac{1}{2}\left(T_\eta^*(\eta+) + T_\eta^*(\eta-)\right) = T_c = -10$$



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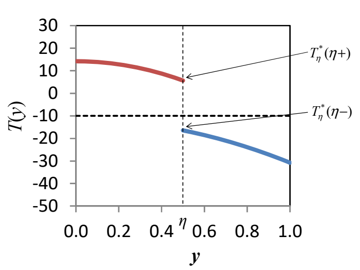
Ice Albedo Feedback

ice line condition: $\frac{1}{2}\left(T_\eta^*(\eta+) + T_\eta^*(\eta-)\right) = T_c = -10$



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Snowball Earth

Ice Albedo Feedback

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\bar{T}-T)$$

Equilibrium: $T_\eta^*(y) = \frac{1}{B+C}\left(Qs(y)\left(1-\alpha(y,\eta)\right) - A + C\bar{T}_\eta^*\right)$

Ice line condition: $\frac{1}{2}\left(T_\eta^*(\eta+) + T_\eta^*(\eta-)\right) = T_c = -10$

Albedo: $\alpha(y,\eta) = \begin{cases} \alpha_1 = 0.32, & y < \eta \\ \alpha_2 = 0.62, & y > \eta \end{cases} \qquad \alpha(\eta-,\eta) = \alpha_1, \quad \alpha(\eta+,\eta) = \alpha_2$

$$T_\eta^*(\eta+) = \frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_2\right) - A + C\bar{T}_\eta^*\right) \qquad T_\eta^*(\eta-) = \frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_1\right) - A + C\bar{T}_\eta^*\right)$$

Ice line condition:

$$\frac{1}{2}\left(T_\eta^*(\eta+) + T_\eta^*(\eta-)\right) = \frac{1}{B+C}\left(Qs(\eta)\left(1-\alpha_0\right) - A + C\bar{T}_\eta^*\right) = T_c = -10$$

where: $\alpha_0 = \frac{\alpha_1+\alpha_2}{2} = 0.47$

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Ice Albedo Feedback

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

Ice line condition: $\frac{1}{B+C}\left(Qs(\eta)(1-\alpha_0)-A+C\overline{T}_\eta^*\right)=T_c=-10$

Rewrite: $h(\eta)=\frac{1}{B+C}\left(Qs(\eta)(1-\alpha_0)-A+C\overline{T}_\eta^*\right)-T_c=0$

Recall equilibrium GMT: $\overline{T}_\eta^*=\frac{1}{B}\left(Q\left(1-\overline{\alpha}(\eta)\right)-A\right)$

Recall average albedo: $\overline{\alpha}(\eta)=\int_0^1\alpha(y,\eta)s(y)dy=\alpha_2-(\alpha_2-\alpha_1)S(\eta)=0.62-0.3S(\eta)$

where: $S(\eta)=\int_0^\eta s(y)dy=\eta-0.241\left(\eta^3-\eta\right)$

$$h(\eta)\equiv\frac{Q}{B+C}\left(s(\eta)(1-\alpha_0)+\frac{C}{B}\left(1-\alpha_2+(\alpha_2-\alpha_1)S(\eta)\right)\right)-\frac{A}{B}-T_c=0$$

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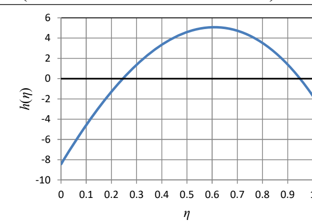
Ice Albedo Feedback

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

The additional condition: $\frac{1}{2}\left(T_\eta^*(\eta+)+T_\eta^*(\eta-)\right)=T_c=-10$

can be written: $\boxed{h(\eta)\equiv\frac{Q}{B+C}\left(s(\eta)(1-\alpha_0)+\frac{C}{B}\left(1-\alpha_2+(\alpha_2-\alpha_1)S(\eta)\right)\right)-\frac{A}{B}-T_c=0}$

Two equilibria (zeros of h) satisfy the additional condition.



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Snowball Earth

Ice Albedo Feedback

Equilibrium temperature profiles $T_\eta^*(y)=\frac{1}{B+C}\left(Qs(y)(1-\alpha(y,\eta))-A+C\overline{T}_\eta^*\right)$

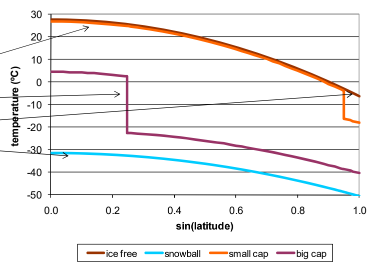
Interesting Solutions:

small cap

large cap

ice free

snowball



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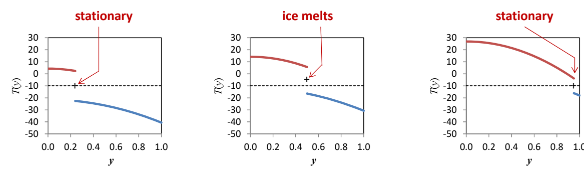
Snowball Earth

Dynamics of the Ice Line

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

Idea:

If the average temperature across the ice line is above the critical temperature, some ice will melt, moving the ice line toward the pole. If it is below the critical temperature, the ice will advance toward the equator.



Widiasih's equation: $\frac{d\eta}{dt}=\varepsilon\left(T(\eta)-T_c\right)$

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Snowball Earth

Dynamics of the Ice Line

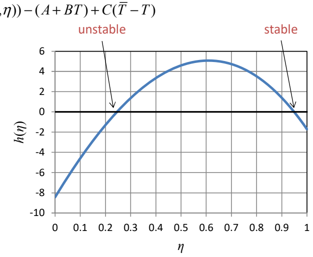
$$\frac{d\eta}{dt}=\varepsilon\left(T(\eta)-T_c\right)$$

$$R\frac{\partial T}{\partial t}=Qs(y)(1-\alpha(y,\eta))-(A+BT)+C(\overline{T}-T)$$

Widiasih's Theorem. For sufficiently small ε, the system has an attracting invariant curve given by the graph of a function $\Phi_\varepsilon:[0,1]\to X$. On this curve, the dynamics are approximated by the equation

$$\frac{d\eta}{dt}=\varepsilon h(\eta)$$

experts only



Esther R. Widiasih, Dynamics of the Budyko Energy Balance Model, *SIAM J. Appl. Dyn. Syst.*, 12(4), 2068–2092.

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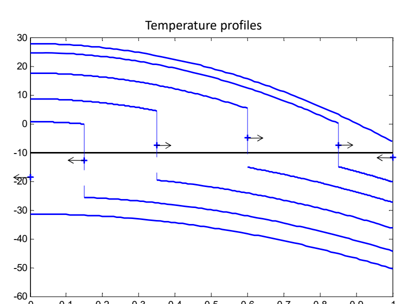
Budyko-Widiasih Model

Esther Widiasih

$$\frac{d\eta}{dt}=\varepsilon h(\eta)$$



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Snowball Earth

Budyko-Widiasih Model

Esther →  ← Dick

Oahu, February 2020

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Snowball Earth

Summary

surface temperature, sin(latitude), $\bar{T} = \int_0^1 T(y)dy$

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\bar{T}-T)$$

heat capacity, insolation, albedo, OLR, heat transport

reduces to

$$\frac{d\eta}{dt} = \varepsilon h(\eta)$$

$$\frac{d\eta}{dt} = \varepsilon h(\eta) = \varepsilon\left(\frac{Q}{B+C}\left(s(\eta)(1-\alpha_0) + \frac{C}{B}\left(1-\alpha_2+(\alpha_2-\alpha_1)S(\eta)\right)\right) - \frac{A}{B} - T_c\right)$$

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Snowball Earth

Budyko-Widiasih Model

surface temperature, sin(latitude), $\bar{T} = \int_0^1 T(y)dy$

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y)) - (A+BT) + C(\bar{T}-T)$$

heat capacity, insolation, albedo, OLR, heat transport

What about the greenhouse effect?

$A+BT$ is the outgoing long wave radiation. This term decreases if the greenhouse gases increase. We view A as a parameter.

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A)$$

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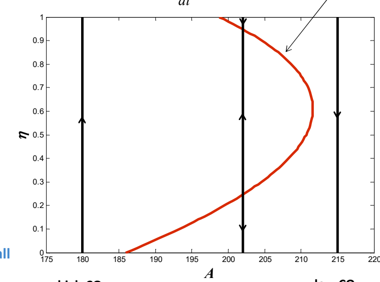
Snowball Earth

Budyko-Widiasih Model

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A)$$

isocline $h(\eta, A) = 0$



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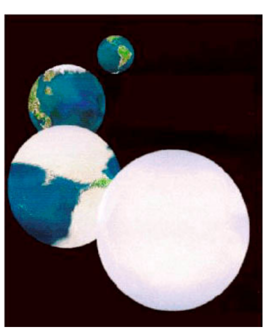
Snowball Earth

Is it possible for Earth to become a snowball?

If so, how does it escape?



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Snowball Earth

Budyko-Widiasih Model

$$\frac{d\eta}{dt} = \varepsilon h(\eta, A)$$

What if A is a dynamical variable?

Anna Barry
Postdoc 2012-14

Simple equation:

$$\frac{dA}{dt} = \tilde{\varepsilon}(\eta - \eta_c)$$ ← silicate weathering

$$0 < \tilde{\varepsilon} \ll \varepsilon \ll 1$$

New system:

$$\boxed{\begin{aligned}\frac{dA}{dt} &= \tilde{\varepsilon}(\eta - \eta_c) \\ \frac{d\eta}{dt} &= \varepsilon h(\eta, A)\end{aligned}}$$

Anna M. Barry, Esther Widiasih, & Richard McGehee, *Discrete & Continuous Dynamical Systems Series B* 22 (2017), 2447-2463.

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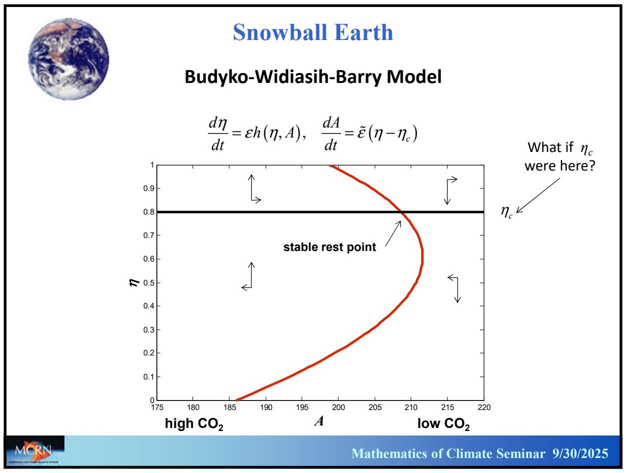

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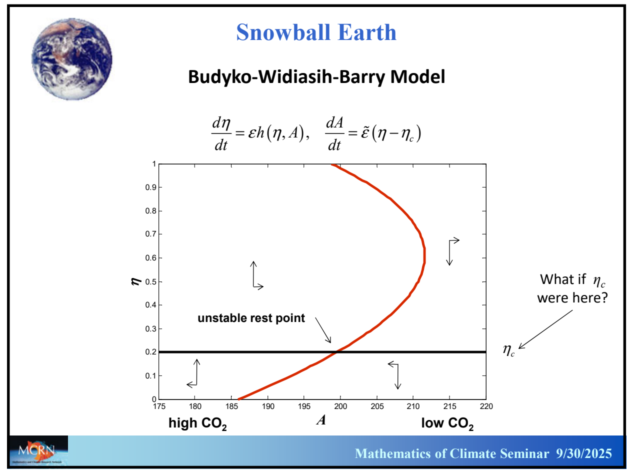

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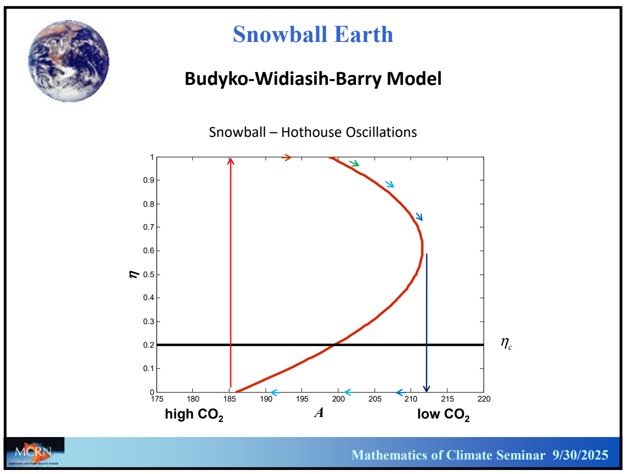

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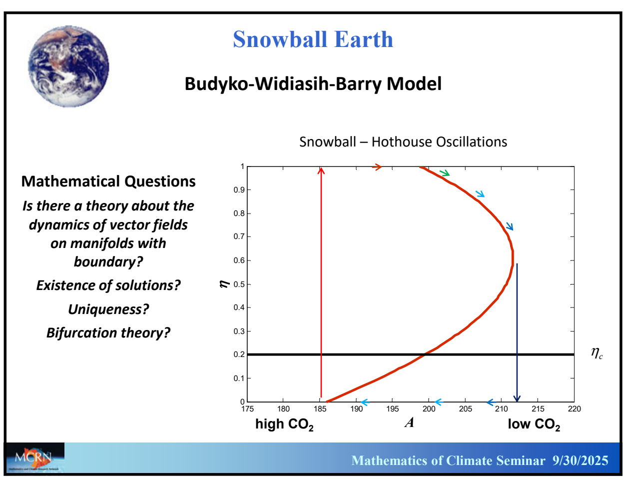

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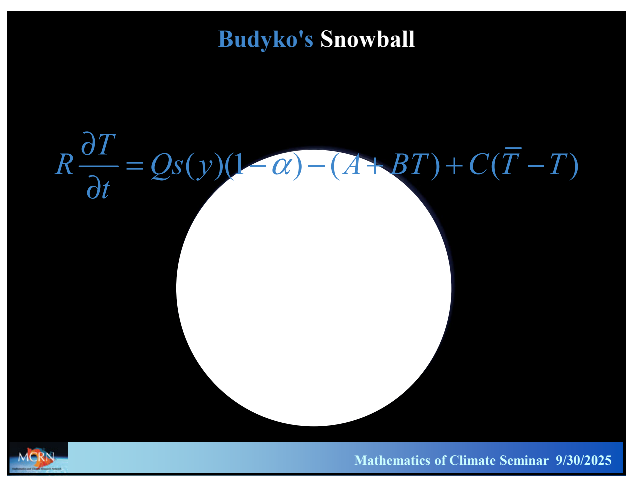

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