Set 2 Solutions

For these exercises, consider Budyko's equation

$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + C(\overline{T}-T)$$

with standard parameters $Q = 343$, $A = 202$, $B = 1.9$, and $C = 3.04$. Also, take

$$\alpha(y,\eta) = \begin{cases} \alpha_1 = 0.32 & y < \eta \\ \alpha_2 = 0.62 & y > \eta \end{cases} \quad \text{and} \quad s(y) = 1 - 0.241\left(3y^2 - 1\right).$$

1. Remove the heat transport in the model by replacing the parameter C with zero. Find the equilibrium solution for each value of η, and discuss its stability.

Solution. The equilibrium solution satisfies

$$Qs(y)(1-\alpha(y,\eta)) - (A + BT^*(y,\eta)) = 0.$$

Solving for $T^*(y,\eta)$ yields

$$\boxed{T^*(y,\eta) = \frac{1}{B}\left(Qs(y)(1-\alpha(y,\eta)) - A\right)}.$$

The equilibrium solution is stable, since $B > 0$. We can see the stability by letting

$$\tau(t,y,\eta) = T(t,y,\eta) - T^*(y,\eta).$$

Then

$$\begin{aligned} R\frac{\partial \tau}{\partial t} = R\frac{\partial T}{\partial t} &= Qs(y)(1-\alpha(y,\eta)) - (A+BT) \\ &= Qs(y)(1-\alpha(y,\eta)) - A - B(T^*(y,\eta) + \tau(t,y,\eta)) \\ &= -B\tau(t,y,\eta). \end{aligned}$$

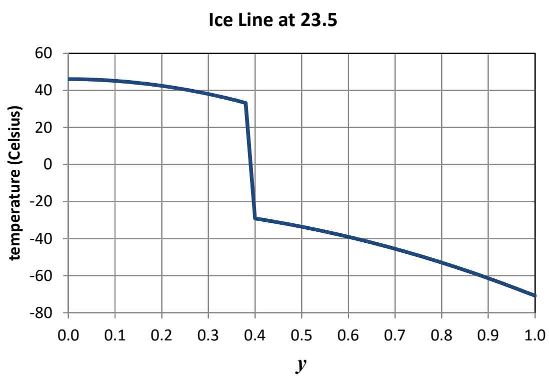
Therefore,

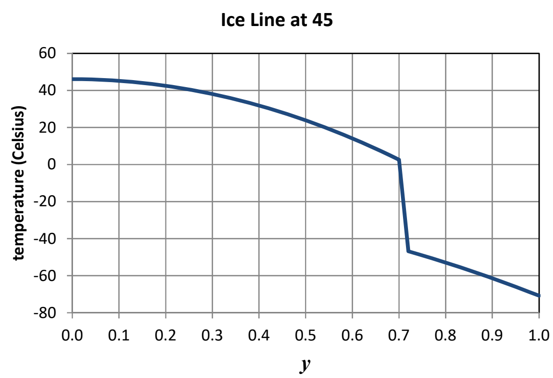
$$\tau(t,y,\eta) = \tau(0,y,\eta)e^{-Bt/R},$$

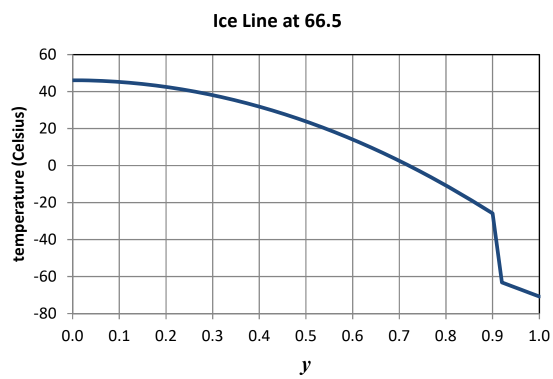
which decays exponentially to zero as $t \to \infty$. Therefore, $T(t,y,\eta)$ decays exponentially to $T^*\left(y,\eta\right)$.

2. Graph each of the equilibrium temperature distributions found in Exercise 1 for ice lines at these latitudes: 23.5º, 45º, and 66.5º. Compare the graphs to those of the equilibrium solutions for Budyko's equation with the standard parameters. Discuss the differences.

Solution.







The temperature difference between the equatorial region and the polar region is larger in the absence of heat transport.

3. Reconsider the situation in Exercise 1 (where $C = 0$). Is there a value of η where the ice line condition is met? (The ice line condition is that the average temperature across the discontinuity at the ice line is $-10^\circ\mathrm{C}$.)

Solution. We found the equilibrium temperature distribution in Exercise 1:

$$T^*(y,\eta) = \frac{1}{B}\big(Qs(y)(1-\alpha(y,\eta)) - A\big),$$

which we can write

$$T^*(y,\eta) = \begin{cases} \big(Qs(y)(1-\alpha_1) - A\big)\big/B & y < \eta, \\ \big(Qs(y)(1-\alpha_2) - A\big)\big/B & y > \eta. \end{cases}$$

The ice line condition then becomes

$$\frac{1}{2}\left(T^*(\eta-,\eta)+T^*(\eta+,\eta)\right)=\frac{1}{B}\left(Qs(\eta)(1-\alpha_0)-A\right)=T_c=-10,$$

where $\alpha_0=(\alpha_1+\alpha_2)/2=0.47$. Therefore, the ice line condition is met at a value of η satisfying

$$\frac{1}{B}\left(Q\left(1-0.241(3\eta^2-1)\right)(1-\alpha_0)-A\right)=T_c,$$

which can be written

$$\frac{0.241\cdot 3(1-\alpha_0)Q}{B}\eta^2=\frac{(1+0.241)(1-\alpha_0)Q-A}{B}-T_c.$$

Substituting the values for the constants yields $69.18\eta^2=11.42$, or

$$\boxed{\eta=0.569},$$

which is a latitude of about 35º.