Math 5490
Topics in Applied Mathematics
Introduction to the Mathematics of Climate

Fall 2023
1:25 - 3:20 Tuesdays and Thursdays
Amundson Hall 162

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course website
www-users.cse.umn.edu/~mcgehee/teaching/Math5490/

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Paleoclimate

Earth's climate has changed many times in the past.
Why do we think humans are responsible now?

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Humans didn't cause the climate changes on Mars.

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Logical Falacy

Climate changes occurred when humans didn't exist
implies that
humans are not the cause of climate change.

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Reasonable Conclusion

Climate changes occurred when humans didn't exist
suggests that
we should try to understand why the climate changed in the past.

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Earth's climate has changed many times in the past.

Paleoclimate

Paleoclimatology is the scientific study of climates predating the invention of meteorological instruments, when no direct measurement data were available. As instrumental records only span a tiny part of Earth's history, the reconstruction of ancient climate is important to understand natural variation and the evolution of the current climate.

`https://en.wikipedia.org/wiki/Paleoclimatology`

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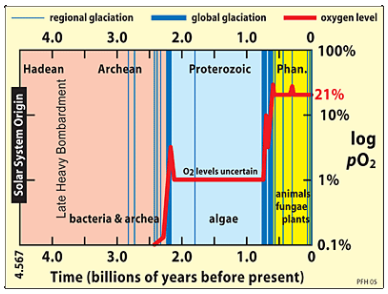
Earth's climate has changed many times in the past.



`http://www.snowballearth.org/when.html`

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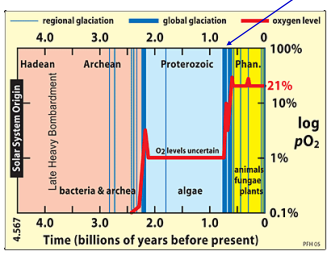

Humans didn't cause the Earth to become a snowball. Complex life did not exist then.

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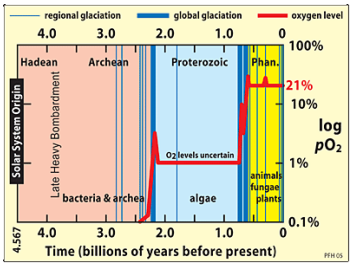

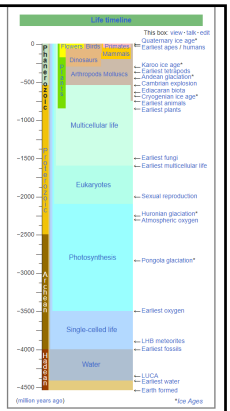

`https://en.wikipedia.org/wiki/Phanerozoic`

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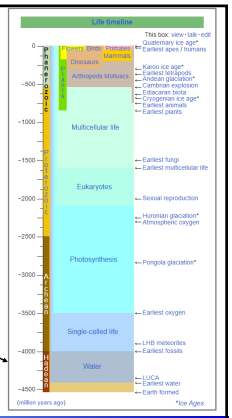
Four Geologic Eons

- Phanerozoic
- Proterozoic
- Archean
- Hadean



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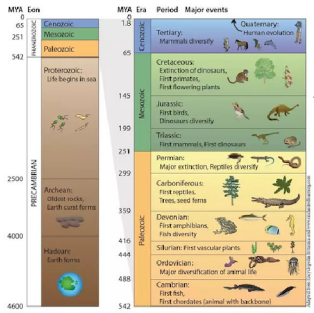

Three Eras of the Phanerozic

- Cenozoic
- Mesozoic
- Paleozoic

https://study.com/learn/lesson/phanerozoic-eon-era-facts.html

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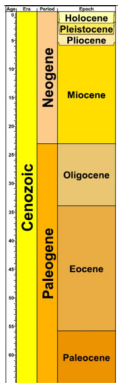
dinosaurs gone
mammals dominate

last ice age to now
last 12,000 years

glacial cycles starting
2.5 million years ago

Seven Epochs of the Cenzoic Era

- Holocene
- Pleistocene
- Pliocene
- Miocene
- Oligocene
- Eocene
- Paleocene



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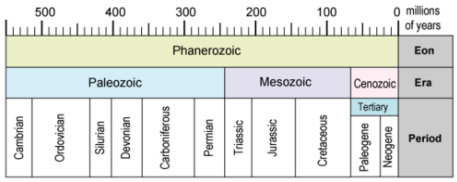
The Phanerozoic Eon

Geologic Timeline after Pre-Cambrian Era



https://qph.cf2.quoracdn.net/main-qimg-e15ce901c6d376a29276d19835778883

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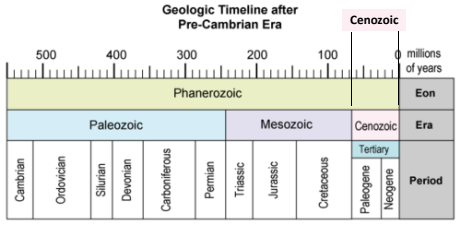
The Phanerozoic Eon

Geologic Timeline after Pre-Cambrian Era

Cenozoic



https://qph.cf2.quoracdn.net/main-qimg-e15ce901c6d376a29276d19835778883

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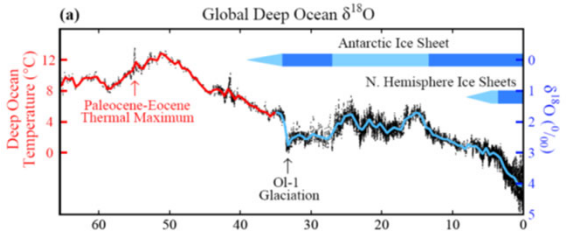
Temperatures in the Cenozoic Era
65 Myr - present



Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

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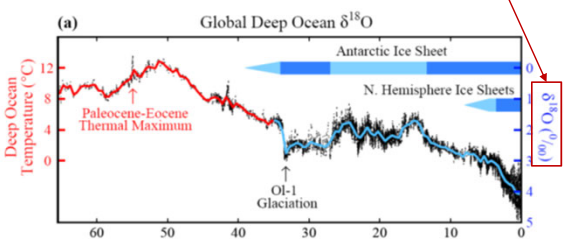
What is this? $\delta^{18}O$ (‰)



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Ocean Sediment Cores

http://eo.ucar.edu/staff/rrussell/climate/paleoclimate/sediment_proxy_records.html

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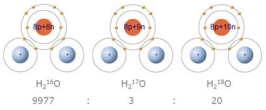
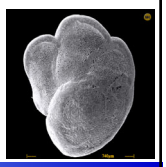
^{18}O as a Climate Proxy



The isotope ^{16}O preferentially evaporates from the ocean and is sequestered in glaciers, leaving the heavier isotope ^{18}O more highly concentrated in the ocean. Thus oceanic concentration of the isotope ^{18}O is higher during glacial periods.

Foraminifera absorb more ^{18}O into their skeletons when the water temperature is lower and when more ^{18}O is in the water.

Thus higher concentrations of ^{18}O in foraminifera fossils indicate lower ocean temperatures and higher glacier volume.



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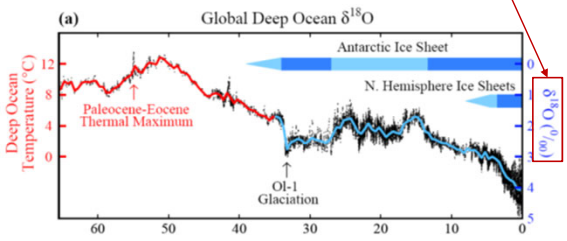

Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

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What is this? $\delta^{18}O$ (‰)

‰ : "per mil," "per thousand"
1000‰ = 100% = 1
10‰ = 1% = 0.01
1‰ = 0.1% = 0.001

^{18}O: Oxygen 18: 8 protons 8 electrons 10 neutrons
^{17}O: Oxygen 17: 8 protons 8 electrons 9 neutrons
^{16}O: Oxygen 16: 8 protons 8 electrons 8 neutrons
Most of the oxygen atoms on Earth are ^{16}O.
About 1 in 500 atoms is ^{18}O. About 1 in 2500 is ^{17}O.
There are other oxygen isotopes, but they are unstable.

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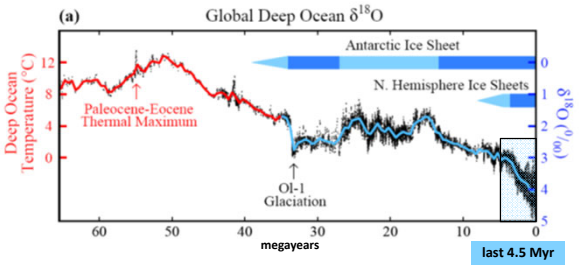
Temperatures in the Cenozoic Era
65 Myr - present



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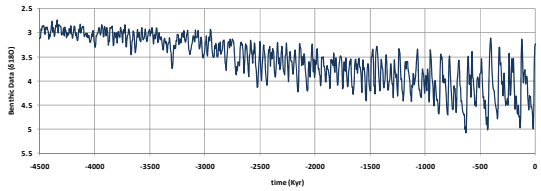
^{18}O in Foraminifera Fossils During the Past 4.5 Myr



Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

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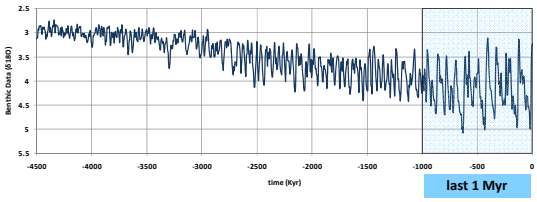

Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

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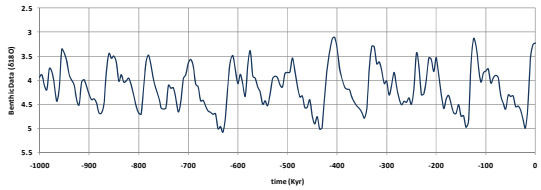
^{18}O in Foraminifera Fossils During the Past 1.0 Myr



Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

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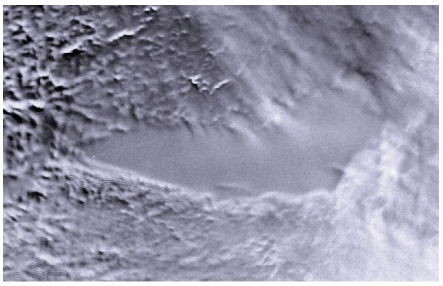
Lake Vostok



http://svs.gsfc.nasa.gov/vis/a000000/a000900/a000996/index.html

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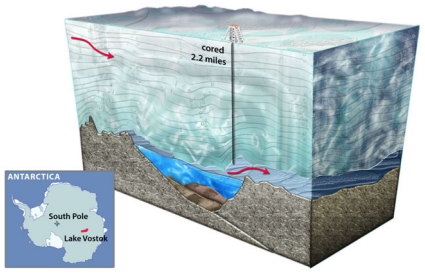
Lake Vostok



http://www.nsf.gov/news/mmg/media/images/lake_vostok_nsf_h.jpg

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Core Samples
Photographs copyright Reto Stöckli, NASA GSFC

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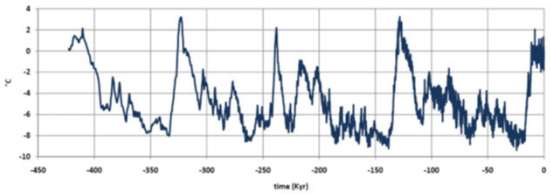
Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data



J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* 399, 429-436.

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What Causes Glacial Cycles?

Widely Accepted Hypothesis

The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms and/or triggering mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work are not fully understood.

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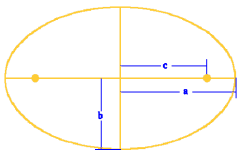
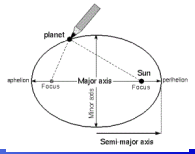
Earth's Orbit

Kepler's First Law: The orbit of every planet is an ellipse with the Sun at one of the two foci.



Eccentricity = c/a

Johannes Kepler (1571-1630)



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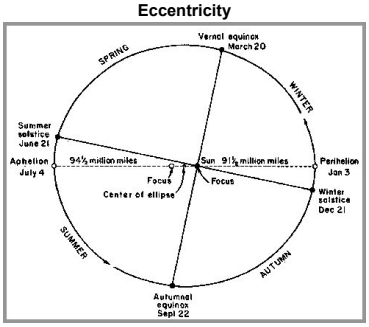
Eccentricity



John Imbrie & Katherine Palmer Imbrie, *Ice Ages: Solving the Mystery,* Harvard Univ. Press, 1979.

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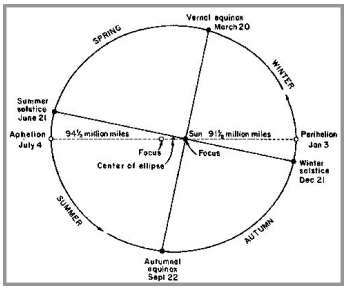
Eccentricity

Perihelion: 91.5×10^6 mi
Aphelion: 94.5×10^6 mi

Semimajor axis: 93×10^6 mi

Eccentricity: 1.5/93 = 0.016



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Eccentricity

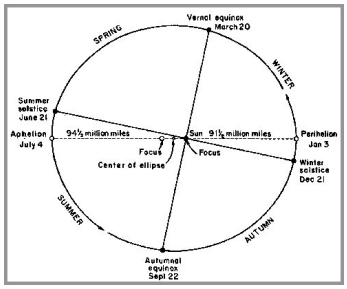
Perihelion: 91.5
Aphelion: 94.5

Change in radius:
3/93 = 3.2%

Change in insolation:
6.4%

Six percent less insolation in the southern winter than the northern winter.

6.4% of 342 W/m^2 =
22 W/m^2



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Global Annual Average Insolation

Solar output: $K \approx 4\times10^{26}$ Watts

Solar intensity at distance r from the sun:

$$Q(t) = \frac{K}{4\pi r(t)^2} \quad \mathrm{Wm}^{-2}$$

Cross section of Earth: πr_E^2 m^2

Global solar input: $\frac{Kr_E^2}{4r(t)^2}$ W

Total annual solar input (P = one year (in seconds)):

$$\int_0^P \frac{Kr_E^2}{4r(t)^2}dt = \frac{Kr_E^2}{4}\int_0^P \frac{dt}{r(t)^2} \quad \text{Joules}$$

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Global Annual Average Insolation

Specific angular momentum (angular momentum per unit mass):

$$\Omega = r^2\dot{\theta} \quad \mathrm{m}^2\mathrm{s}^{-1}$$

Total annual solar input:

$$\frac{Kr_E^2}{4}\int_0^P \frac{dt}{r(t)^2} = \frac{Kr_E^2}{4}\int_0^P \frac{\dot{\theta}dt}{\Omega} = \frac{Kr_E^2}{4\Omega}\int_0^{2\pi} d\theta = \frac{\pi Kr_E^2}{2\Omega} \quad \text{Joules}$$

Mean annual solar input: $\frac{\pi Kr_E^2}{2P\Omega}$ Watts

Mean annual solar intensity on the Earth's surface:

$$\frac{\pi Kr_E^2}{2P\Omega}\cdot\frac{1}{4\pi r_E^2} = \frac{K}{8P\Omega} \quad \mathrm{Wm}^{-2}$$

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Global Annual Average Insolation

Kepler's Third Law: $P \sim a^{3/2}$ a = semimajor axis

Derived from Kepler: $\Omega^2 \sim a(1-e^2)$ e = eccentricity

Mean annual solar intensity:

$$\frac{K}{8P\Omega} = \frac{\hat{K}}{a^{3/2}a^{1/2}\sqrt{1-e^2}} = \frac{\hat{K}a^{-2}}{\sqrt{1-e^2}} \quad \mathrm{Wm}^{-2}$$

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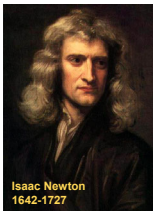
Glacial Cycles

Planetary Motion

$$m_i\frac{d^2x_i}{dt^2} = \sum_{\substack{j=1\\ j\neq i}}^{n} \frac{Gm_im_j(x_j - x_i)}{|x_j - x_i|^3}$$



Jacques Laskar (1955-)

The orbits of all the planets can be computed (both forward and backward in time) for billions of years.

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Earth's Semi-major Axis

Laskar: Global Annual Average Insolation: $\frac{\hat{K}a^{-2}}{\sqrt{1-e^2}}$

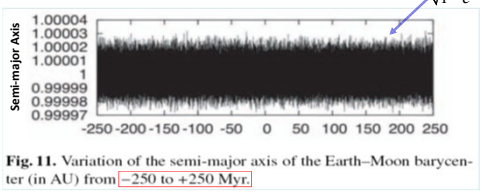

Fig. 11. Variation of the semi-major axis of the Earth–Moon barycenter (in AU) from −250 to +250 Myr.

Semi major axis does not change much:
0.005% corresponding to .01% change in global average insolation

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

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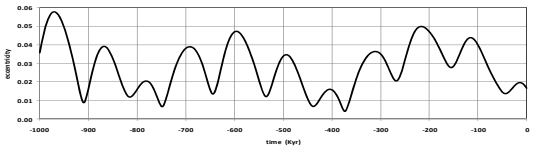
Earth's Eccentricity



Note periods of about 100 kyr and 400 kyr.

The effect due to eccentricity is more significant:

As e varies between 0 and 0.06, $(1\text{-}e^2)^{-1/2}$ varies between 1 and 1.0018, or about 0.2%. (Twenty times the effect due to a.)

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

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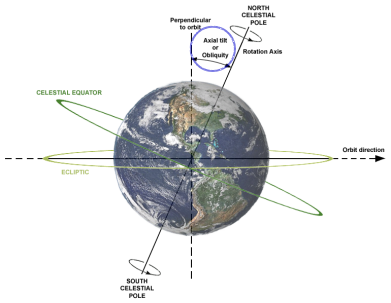
Earth's Obliquity



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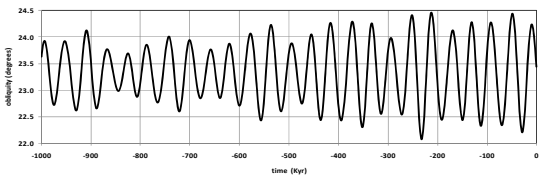
Earth's Obliquity



Note period of about 41 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

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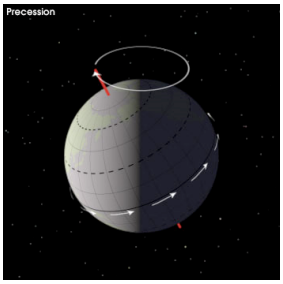
Precession



`http://earthobservatory.nasa.gov/Library/Giants/Milankovitch/milankovitch_2.html`

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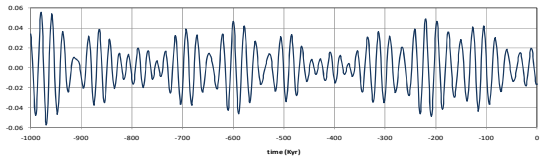
Earth's Precession Index



index $= e \sin\rho$, where e = eccentricity and ρ = precession angle (measured from spring equinox)

Note period of about 23 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

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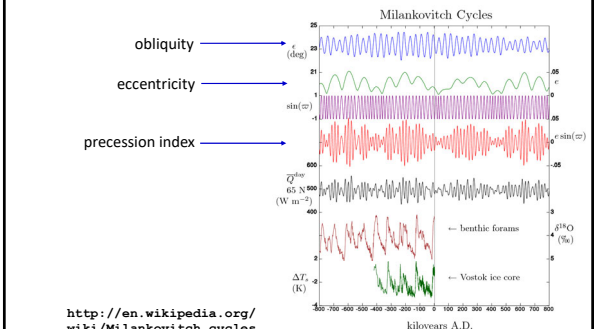

`http://en.wikipedia.org/wiki/Milankovitch_cycles`

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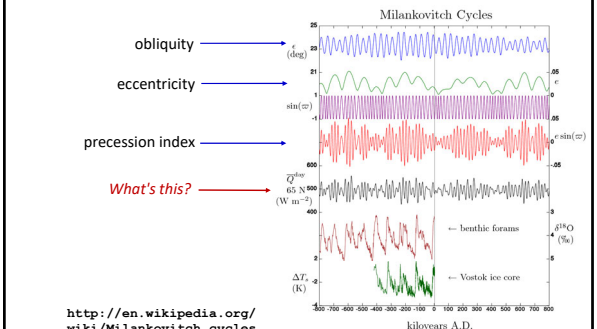

`http://en.wikipedia.org/wiki/Milankovitch_cycles`

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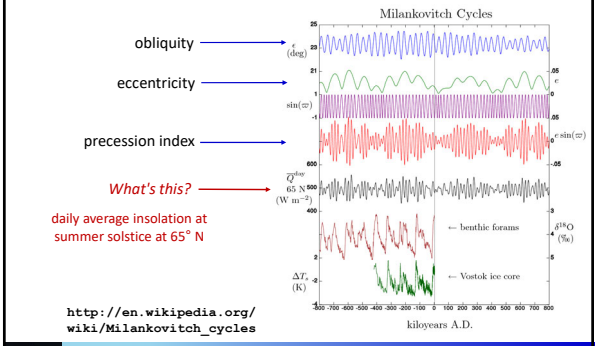

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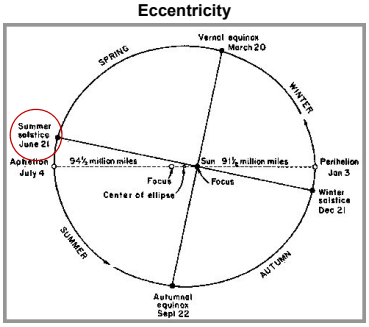
Eccentricity

summer solstice



John Imbrie & Katherine Palmer Imbrie, *Ice Ages: Solving the Mystery,* Harvard Univ. Press, 1979.

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Daily Average Insolation at Summer Solstice at 65° N

Insolation at a point on the Earth's surface

$$I(\beta,\rho,r,\theta,\varphi,\gamma)=\frac{K}{4\pi r^2}\left[-\cos\varphi\left(\cos\beta\cos(\theta-\rho)\cos\gamma+\sin(\theta-\rho)\sin\gamma\right)-\sin\varphi\sin\beta\cos(\theta-\rho)\right]^+$$

(φ,γ) = (latitude, longitude)
(r,θ) = position of Earth in orbital plane
β = obliquity angle
ρ = precession angle

Daily average insolation at latitude φ at summer solstice

$$\bar{I}(e,\beta,\rho',\varphi)=Q\frac{(1-e\sin\rho')^2}{(1-e^2)^2}\frac{1}{2\pi}\int_0^{2\pi}\left[\cos\varphi\cos\beta\cos\gamma+\sin\varphi\sin\beta\right]^+ d\gamma$$

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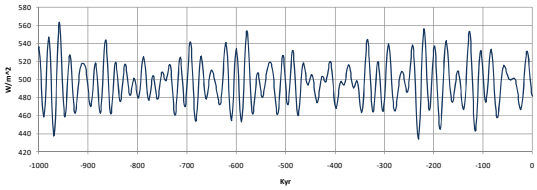
Daily Average Insolation at Summer Solstice at 65° N



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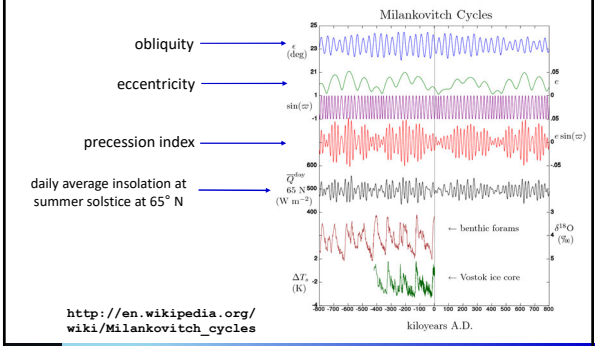

http://en.wikipedia.org/wiki/Milankovitch_cycles

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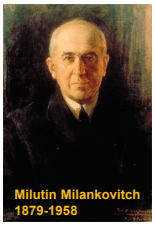
Who was Milankovitch?

Milutin Milankovitch was a Serbian mathematician and professor at the University of Belgrade.

In 1920 he published his seminal work on the relation between insolation and the Earth's orbital parameters.

In 1941 he published a book explaining his entire theory.

His work was not fully accepted until 1976.



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What happened in 1976?


James D. Hays

Hays, Imbrie, and Shackleton, "Variations in the Earth's Orbit: Pacemaker of the Ice Ages," *Science* **194**, 10 December 1976.


John Imbrie

"It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages."


Nicholas Shackleton

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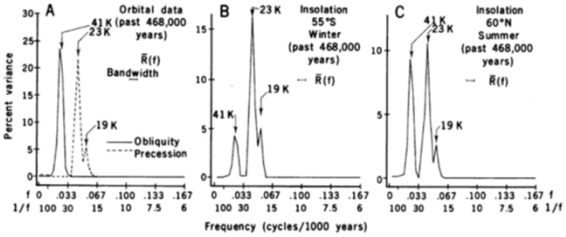
Solar Forcing (Hays, et al)



Hays, *et al*, *Science* **194** (1976), p. 1125

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Hays, et al, Summary

1) Three indices of global climate have been monitored in the record of the past 450,000 years in Southern Hemisphere ocean-floor sediments.

2) ... climatic variance of these records is concentrated in three discrete spectral peaks at periods of 23,000, 42,000, and approximately 100,000 years. These peaks correspond to the dominant periods of the earth's solar orbit, and contain respectively about 10, 25, and 50 percent of the climatic variance.

Hays, *et al*, *Science* **194** (1976), p. 1125

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Hays, et al, Summary

3) The 42,000-year climatic component has the same period as variations in the obliquity of the earth's axis and retains a constant phase relationship with it.

4) The 23,000-year portion of the variance displays the same periods (about 23,000 and 19,000 years) as the quasiperiodic precession index.

5) The dominant, 100,000-year climatic component has an average period close to, and is in phase with, orbital eccentricity. Unlike the correlations between climate and the higher-frequency orbital variations (which can be explained on the assumption that the climate system responds linearly to orbital forcing), an explanation of the correlation between climate and eccentricity probably requires an assumption of nonlinearity.

Hays, *et al*, *Science* **194** (1976), p. 1125

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Hays, et al, Summary

6) It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages.

7) A model of future climate based on the observed orbital-climate relationships, ***but ignoring anthropogenic effects***, predicts that the long-term trend over the next seven thousand years is toward extensive Northern Hemisphere glaciation*.

*Quoted by George Will, Washington Post, February 5, 2009

Hays, *et al*, *Science* **194** (1976), p. 1125

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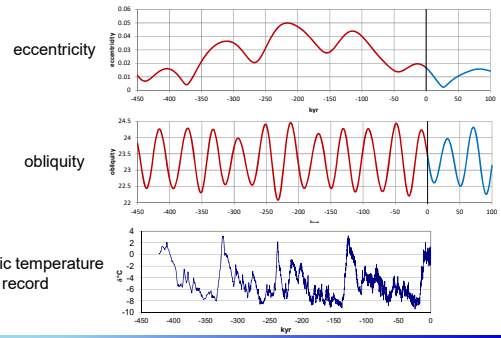
The Coming Ice Age



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Earth's climate has changed many times in the past.
Why do we think humans are responsible now?

Here's a reason.

The paleoclimate evidence points to the conclusion that the Earth should be entering a new ice age.

But we're not.

Instead, the Earth is warming and the ice sheets are melting.

The climate is not following the patterns of the last million years.

Something has changed.

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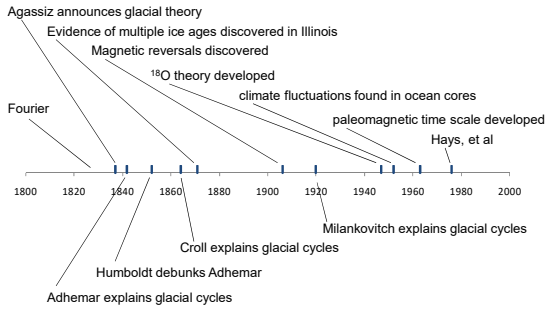
History of Discovery



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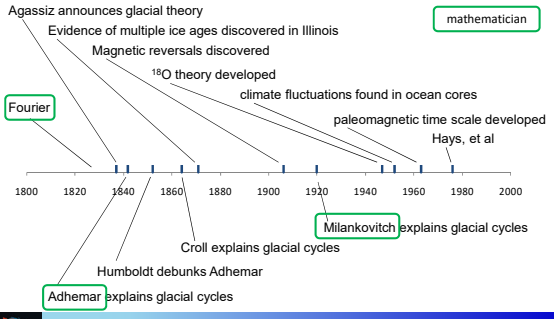
History of Discovery



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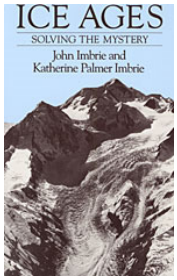
Suggested Reading



John Imbrie & Katherine Palmer Imbrie, *Ice Ages: Solving the Mystery*, HARVARD UNIVERSITY PRESS, 1979

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Church of Saint Sulpice, Paris

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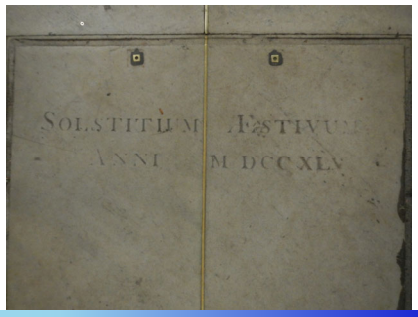
Church of Saint Sulpice, Paris



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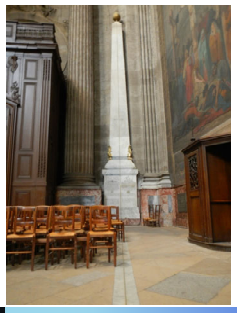
Church of Saint Sulpice, Paris

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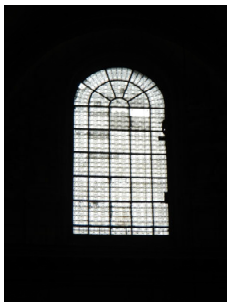

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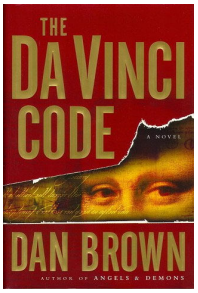

Dan Brown, *The Da Vinci Code*, ANCHOR BOOKS, 2003