Math 5490
Topics in Applied Mathematics
# Introduction to the Mathematics of Climate

**Fall 2023**
**1:25 - 3:20 Tuesdays and Thursdays**
**Amundson Hall 162**

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course website
www-users.cse.umn.edu/~mcgehee/teaching/Math5490/



## Math 5490
**Discussion**

***An Inconvient Truth***

**What did the film get right scientifically?**

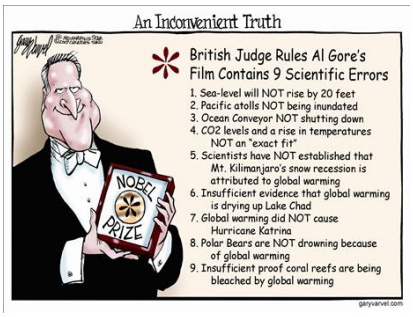
**What did it get wrong?**

**What was the political effect?**




## Math 5490
***An Inconvient Truth***

**What did it get wrong?**






## Math 5490
***An Inconvient Truth***

**What was the political effect?**


https://www.wndu.com/content/news/Trump-global-warming-announcement-out-Thursday-425665043.html

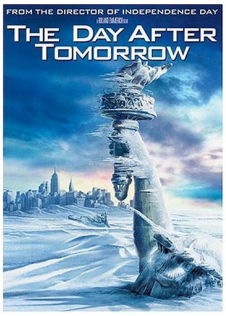


## Math 5490
***The Day After Tomorrow***

**What did the film get right scientifically?**

**What did it get wrong?**

**Did the portrayal of politics ring true or false?**

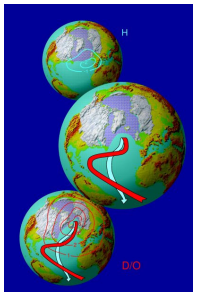





## Math 5490
***The Day After Tomorrow***

**Heinrich and Dansgaard-Oeschger events**

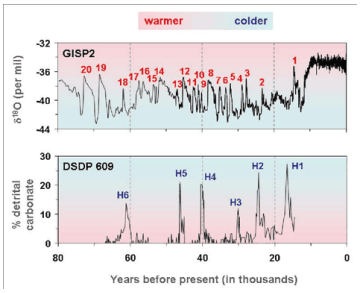
**What did the film get right scientifically?**




http://www.pik-potsdam.de/~stefan/sampleimages.html

## Math 5490
*The Day After Tomorrow*

Heinrich and Dansgaard-Oeschger events

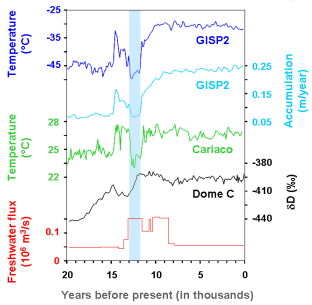



http://www.ncdc.noaa.gov/paleo/abrupt/data3.html



## Math 5490
*The Day After Tomorrow*

The Younger Dryas




Mountain Avens
(*Dryas octopetala*)

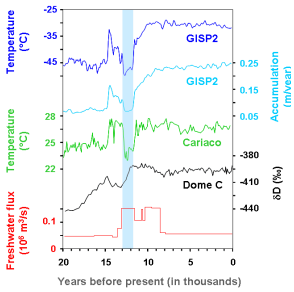
https://www.ncdc.noaa.gov/abrupt-climate-change/The%20Younger%20Dryas

http://www.ncdc.noaa.gov/paleo/abrupt/data4.html

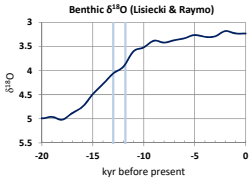


## Math 5490
*The Day After Tomorrow*

The Younger Dryas




Only a minor impact on ice volume.



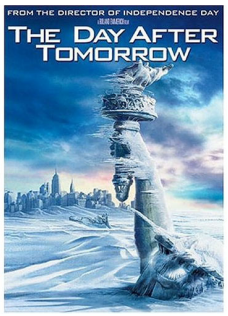



## Math 5490
*The Day After Tomorrow*

**What did the film get wrong scientifically?**

**Thermodynamics**

**Time Scale**

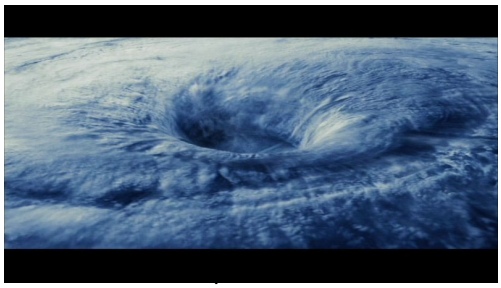





## Math 5490
*The Day After Tomorrow*

**What did the film get wrong scientifically?**

**Thermodynamics Violated**



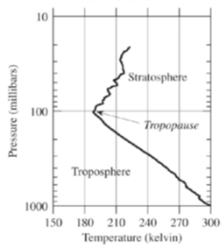


## Math 5490
*The Day After Tomorrow*

**Thermodynamics Violated**

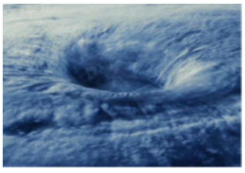
**"It's drawing −150° air down from the upper troposphere."**

March 15 1993, 12Z
2.00° S 169.02° W




Pierrehumbert, *Principles of Planetary Climate*

*The Day After Tomorrow*



Upper troposphere: ~190 K

*The Day After Tomorrow:*

-150 °F = −101 °C = 172 K

***Only a slight exaggeration.***

## Math 5490

*The Day After Tomorrow*

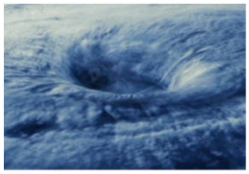
**Thermodynamics Violated**

**Professor Hall:**
**"It's drawing air −150° air down from the upper troposphere."**

**Professor Rapson:**
**"Wouldn't it heat up before it reached the surface?"**

**Professor Hall:**
**"No, it's descending too fast."**

**Kate Meyer:**
**"Wouldn't that violate entropy?"**



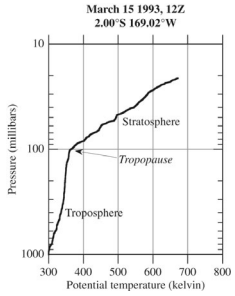
*The Day After Tomorrow*



## Math 5490

*The Day After Tomorrow*

**Thermodynamics Violated**




Pierrehumbert, *Principles of Planetary Climate*

Bringing the air down from the upper troposphere involves increasing the pressure from 0.1 atmosphere to 1 atmosphere, thereby heating it.

Potential temperature: The temperature the air would be if compressed to 1 atmosphere.

Potential temperature of the upper troposphere:

350 K = 77 °C = 171 °F

***Definitely would not freeze the fuel lines of RAF helicopters.***



## Math 5490

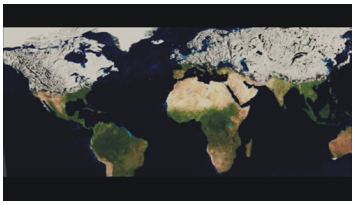
*The Day After Tomorrow*

**What did the film get wrong scientifically?**

**Time Scale Violated**

Most of the northern hemisphere land covered with 30 feet of snow, converted to about 1 meter of water.

Let's say half of all the land, or 15% of the Earth's surface.



70% of the surface is ocean, so about 15/70 or 0.2 meters of ocean evaporated and turned to snow in a few days.

***Where did the energy go? How fast could that happen?***



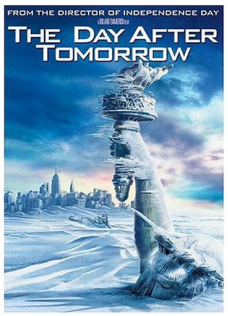
## Math 5490

*The Day After Tomorrow*

**What did the film get right scientifically?**

**What did it get wrong?**

**Did the portrayal of politics ring true or false?**

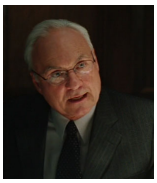





## Math 5490

*The Day After Tomorrow*

**Did the portrayal of politics ring true or false?**



**Vice President: *"Maybe you should stick to science and leave policy to us."***

**Scientist: *"Well, we tried that approach. You didn't want to hear about the science when it could have made a difference."***



Math 5490
Topics in Applied Mathematics
**Introduction to the Mathematics of Climate**

## Math 5490
Earth's Carbon Cycle

### Earth's Carbon Cycle

Earth's climate has changed many times in the past. Why do we think humans are responsible now?

Why do we think that atmospheric $CO_2$ has anything to do with climate change?

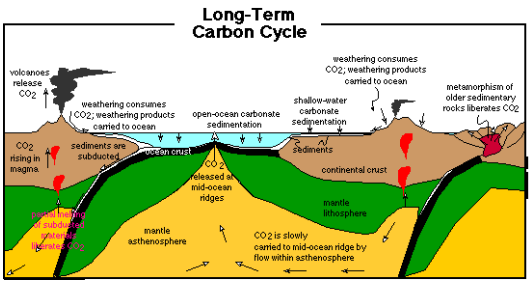
Why do we think that the increase in atmospheric $CO_2$ has anything to do with human activity?

Why do we think that atmospheric $CO_2$ has anything to do with surface temperature?



## Math 5490
Earth's Carbon Cycle

### Long-Term Carbon Cycle




http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm



## Math 5490
Earth's Carbon Cycle

### Long-Term Carbon Cycle

Volcanos emit $CO_2$.

Silicate weathering carries carbon to the ocean.

Carbon sinks to the bottom of the ocean by the "biological pump" and by the precipitation of calcium carbonate.

The carbon is captured in the sediment.

The sediment is subducted beneath the continental crust by plate tectonics.

Volcanic activity released carbon from the carbonate rocks in the form of $CO_2$.

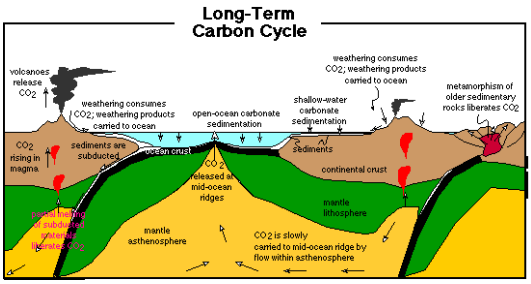
***Repeat.***

And much more.



## Math 5490
Earth's Carbon Cycle

### Long-Term Carbon Cycle




http://www.carleton.edu/departments/geol/DaveSTELLA/Carbon/long_term_carbon.htm



## Math 5490
Earth's Carbon Cycle

### Silicate Weathering

Rainwater containing dissolved $CO_2$ falling on silicate rocks replaces a silicon atom with a carbon atom, ultimately producing calcium carbonate (limestone) and silicon dioxide (quartz). For example, calcium silicate (Wollastonite):

$$CaSiO_3 + CO_2 \rightarrow CaCO_3 + SiO_2$$

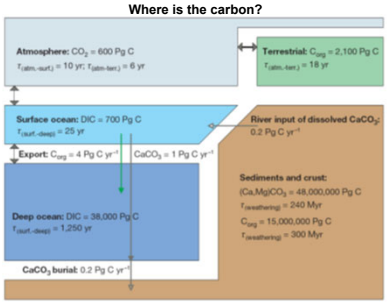
Under volcanic conditions, the carbon atom is replaced by a silicon atom, completing the long term carbon cycle.

$$CaCO_3 + SiO_2 \rightarrow CaSiO_3 + CO_2$$



## Math 5490
Earth's Carbon Cycle

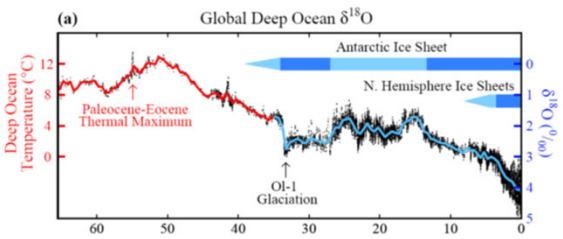
### Where is the carbon?




Sigman & Boyle, *Nature* **207** (2000), p.860

## Math 5490
### Earth's Carbon Cycle

**Temperatures in the Cenozoic Era**
**65 Myr - present**




Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)



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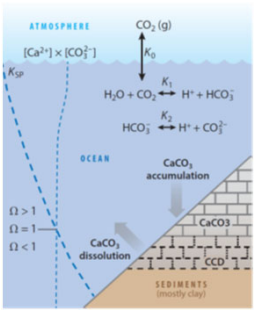
### Earth's Carbon Cycle

**Atmosphere Ocean Sediment Interactions**




*Annu. Rev. Earth Planet. Sci.* 2021. **49**:609–41



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### Earth's Carbon Cycle

**Ocean Sediment Cores**

http://eo.ucar.edu/staff/rrussell/climate/paleoclimate/sediment_proxy_records.html



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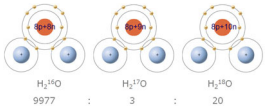
### Earth's Carbon Cycle

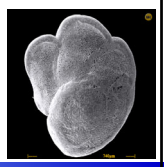
**$^{18}O$ as a Climate Proxy**




The isotope $^{16}O$ preferentially evaporates from the ocean and is sequestered in glaciers, leaving the heavier isotope $^{18}O$ more highly concentrated in the ocean. Thus oceanic concentration of the isotope $^{18}O$ is higher during glacial periods.

Foraminifera absorb more $^{18}O$ into their skeletons when the water temperature is lower and when more $^{18}O$ is in the water.

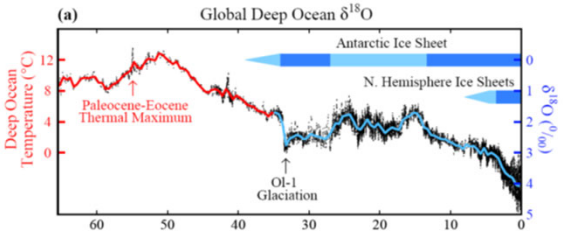
Thus higher concentrations of $^{18}O$ in foraminifera fossils indicate lower ocean temperatures and higher glacier volume.





## Math 5490
### Earth's Carbon Cycle

**Temperatures in the Cenozoic Era**
**65 Myr - present**




Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

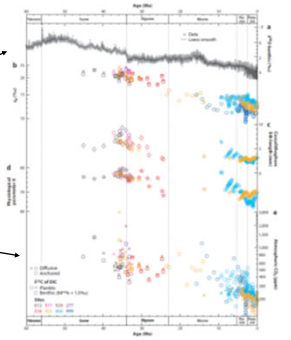


## Math 5490
### Earth's Carbon Cycle

**Cenozoic Climate**

temperature

atmospheric $CO_2$



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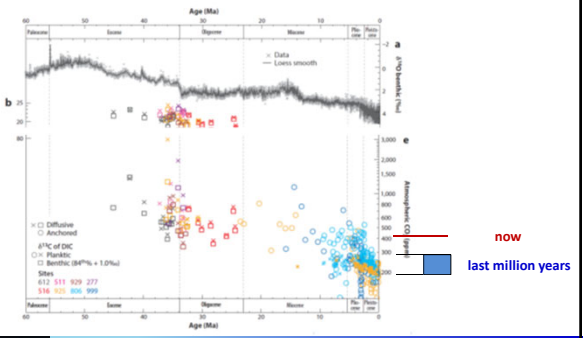
## Math 5490
Earth's Carbon Cycle



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Earth's Carbon Cycle

Those $CO_2$ data are pretty poor compared to the temperature data.

Do we have anything more compelling?

Yes!

Remember Lake Vostok in Antarctica?

The bubbles in the ice cores are samples of past atmospheres that tell us about the $CO_2$ and that temperature.



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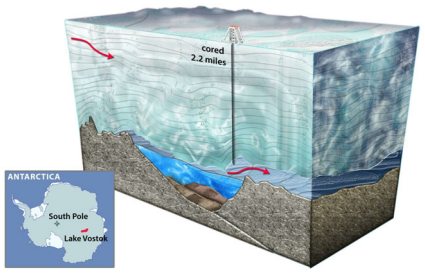
Earth's Carbon Cycle

Lake Vostok




http://www.nsf.gov/news/mmg/media/images/lake_vostok_nsf_h.jpg



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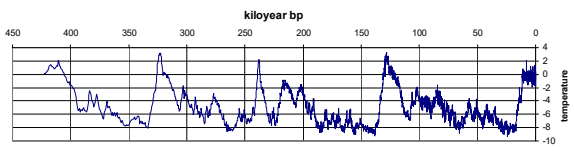
Earth's Carbon Cycle

Core Samples
Photographs copyright Reto Stöckli, NASA GSFC



## Math 5490
Earth's Carbon Cycle

Antarctic Temperature Data

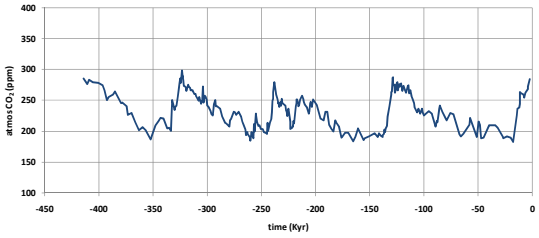



Petit, et al, *Nature* **399** (June 3 1999), pp.429-436



## Math 5490
Earth's Carbon Cycle

Atmospheric $CO_2$ (Vostok data)




Petit, et al, *Nature* **399** (June 3 1999), pp.429-436

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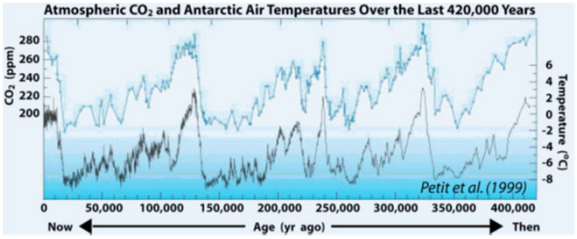
### Earth's Carbon Cycle

The Vostok data tell us that atmospheric $CO_2$ and surface temperature are highly correlated.

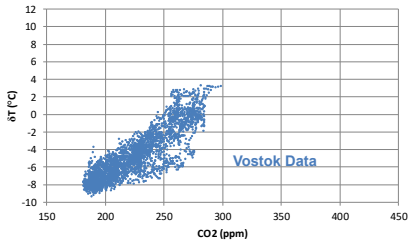



Pam Martin, University of Chicago, 2010



## Math 5490
### Earth's Carbon Cycle

A scatter plot of the Vostok data showing temperature anamoly vs. CO2 shows the correlation






## Math 5490
### Earth's Carbon Cycle

The Vostok data do not extend to the post-industrial times. Do we have anything recent?

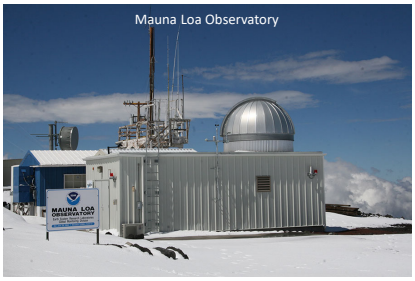
Yes!

Remember the Keeling curve?



## Math 5490
### Earth's Carbon Cycle

Remember the Keeling curve?

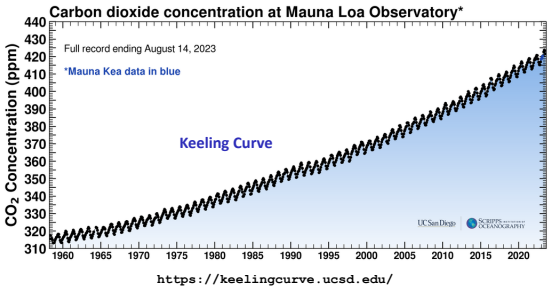



https://research.noaa.gov/Portals/0/EasyDNNnews/1502/2000600p587EDNmain10061mlo_sign_miller.jpg



## Math 5490
### Earth's Carbon Cycle

Remember the Keeling curve?




https://keelingcurve.ucsd.edu/



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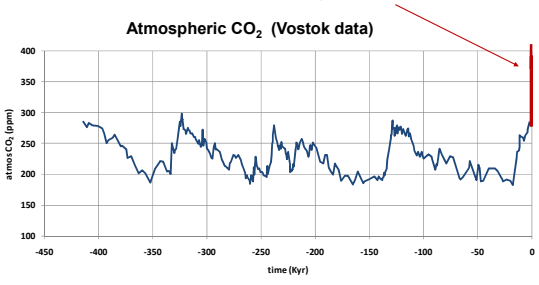
### Earth's Carbon Cycle

Keeling curve on Vostok time scale.

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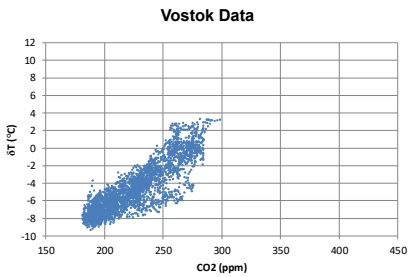
### Earth's Carbon Cycle

Current conditions are well outside the range recorded in the ice core data.






## Math 5490
### Earth's Carbon Cycle

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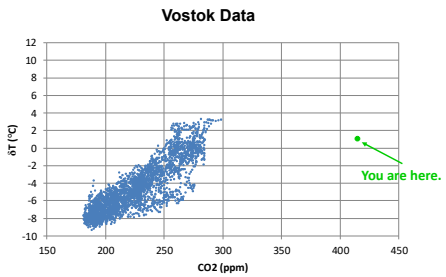




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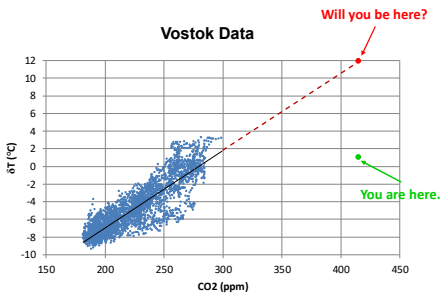
### Earth's Carbon Cycle

Extrapolate linear regression to 420 ppm CO2.






## Math 5490
### Earth's Carbon Cycle

**That's all great. The carbon in the atmosphere creates a greenhouse effect and causes the surface temperature to increase.**

**Why do we think that the Keeling curve has anything to do with human activity? It could just be an unknown natural source of carbon.**

**Stay tuned.**