Math 5490
Topics in Applied Mathematics
Introduction to the Mathematics of Climate

Fall 2023
1:25 - 3:20 Tuesdays and Thursdays
Amundson Hall 162

Richard McGehee, Instructor
458 Vincent Hall
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www-users.cse.umn.edu/~mcgehee/

course website
www-users.cse.umn.edu/~mcgehee/teaching/Math5490/

Math 5490
The Future

Can We Predict the Future?

More relevant question:

What Part of the Future Can't We Predict?

Solar System?
Statistical Mechanics?
Weather?
Climate?

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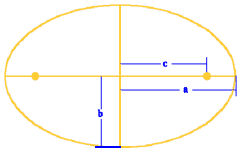
Solar System

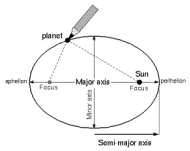
Planetary Orbits

Kepler's First Law: The orbit of every planet is an ellipse with the Sun at one of the two foci.



Eccentricity = c/a

Johannes Kepler (1571-1630)



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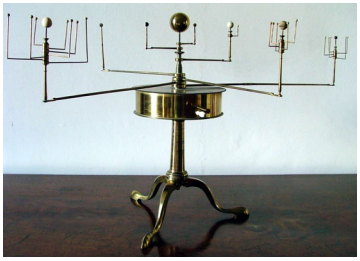
Solar System



Orrery (circa 1820)

http://star.arm.ac.uk/orrery/mdpopescu_armobs_orrery.jpg

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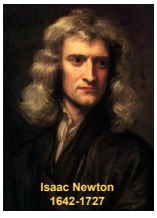
Solar System

$$m_i \frac{d^2 x_i}{dt^2} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{G m_i m_j \left(x_j - x_i \right)}{\left| x_j - x_i \right|^3}$$



Isaac Newton 1642-1727

Jacques Laskar (1955-)

The orbits of all the planets can be computed (both forward and backward in time) for billions of years.

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Solar System

Earth's Semi-major Axis

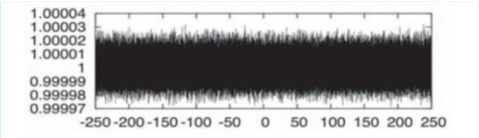

Fig. 11. Variation of the semi-major axis of the Earth–Moon barycenter (in AU) from −250 to +250 Myr.

Semi major axis does not change much and can be computed accurately for hundreds of millions of years.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

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Orbits of comets can be chaotic.

Jurgen Moser, *Stable and Random Motions in Dynamical Systems*, Princeton U. Press, 1973.

http://spaceplace.nasa.gov/comet-quest/en/

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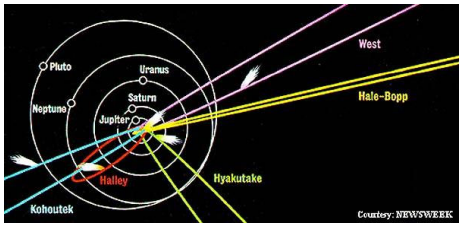
Solar System

Orbits of comets can be chaotic.

timescale: millennia



http://www.enterprisemission.com/comets.html

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Solar System

Orbits of the inner planets are chaotic.

timescale: gigayears

Jacques Laskar (1955-)

J. Laskar & M. Gastineau, Existence of collisional trajectories of Mercury, Mars and Venus with the Earth, *Nature* **459**, 817-819, 2009.

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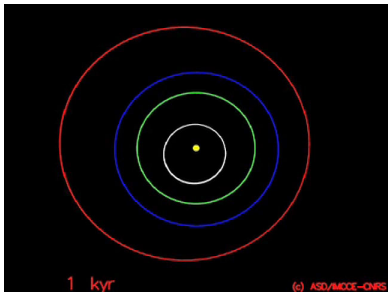
Solar System

Orbits of the inner planets are chaotic.

timescale: gigayears



http://www.imcce.fr/Equipes/ASD/person/Laskar/jxl_collision.html

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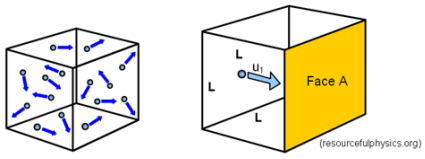
Orbits of molecules in a gas are chaotic.

timescale: microseconds

individual particles chaotic ➡ **statistical mechanics** ➡ **thermodynamics predictable**



http://tap.iop.org/energy/kinetic/603/page_47443.html

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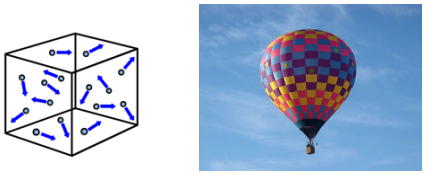

http://commons.wikimedia.org/wiki/File:Balloon_free_image.jpg

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Analogy with Climate

individual particles chaotic → statistical mechanics → thermodynamics predictable

weather chaotic → ?? → climate predictable?

http://agwx.soils.wisc.edu/

http://crooksandliars.com/2014/03/out-10855-peer-reviewed-articles-climate

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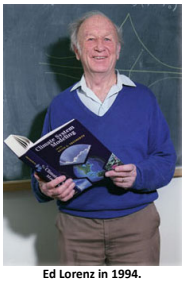
Weather is chaotic.

timescale: weeks



Ed Lorenz in 1994.
(©UCAR, Photo by Curt Zukosky.)

E.N. Lorenz, Deterministic nonperiodic flow, *Journal of the Atmospheric Sciences* **20** (1963), 261-285.

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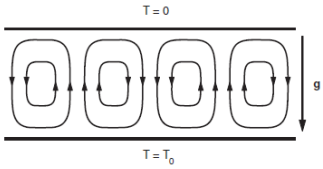
The Lorenz Equations



$$\dot{x} = -\sigma x + \sigma y$$
$$\dot{y} = \rho x - y - xz$$
$$\dot{z} = -\beta z + xy$$

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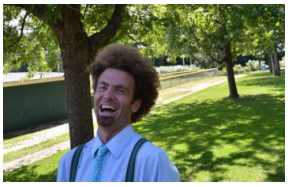
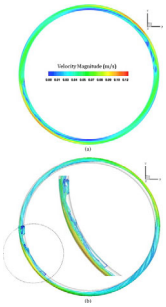
Chris Danforth's Hula Hoop

E.-H. Ridouane, C. M. Danforth, D. L. Hitt, A Numerical Investigation of 3-D Flow Regimes in a Toroidal Natural Convection Loop, *International Journal of Heat and Mass Transfer* **54** (2011), 5253-5261.



http://www.uvm.edu/~cdanfort/main/home.html



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Chris Danforth's Hula Hoop

https://www.youtube.com/watch?v=Vbni-7veJ-c#t=41

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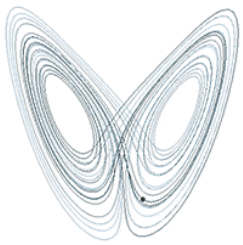
The Lorenz Attractor



http://en.wikipedia.org/wiki/Lorenz_system

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The Lorenz Attractor

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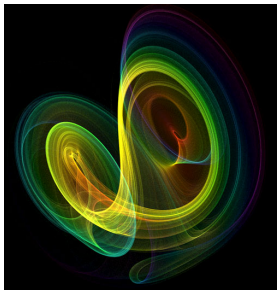

http://www.edc.ncl.ac.uk/highlight/rhnovember2006g02.php

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The Lorenz Attractor

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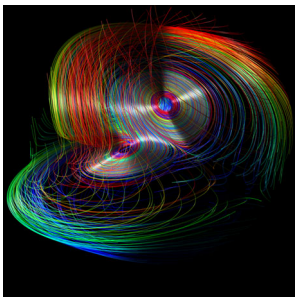

http://www.cs.swan.ac.uk/~cstony/research/star/

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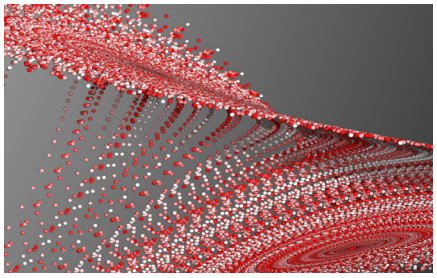
The Lorenz Attractor



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http://safepit.deviantart.com/art/Lorenz-Attractor-2-107809554

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The Lorenz Attractor

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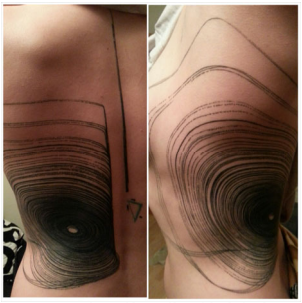

http://pikdit.com/i/lorenz-attractor-by-xoil-needlesside-loic-lavenu/

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The Lorenz Attractor

$$\dot{x} = -\sigma x + \sigma y$$
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http://www.spreadshirt.com/chaotic-good-lorenz-attractor-C3376A8822413

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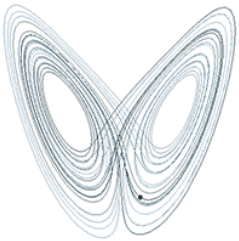
The Lorenz Attractor



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The Lorenz attractor has a positive Lyapunov exponent. *I.e.*, nearby solutions diverge exponentially.

http://en.wikipedia.org/wiki/Lorenz_system

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The Lorenz Attractor

https://www.youtube.com/watch?v=FYE4JKAXSfY

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The Lorenz Attractor

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You can download a MATLAB function to make your own Lorenz attractor images:

http://www.mathworks.com/matlabcentral/fileexchange/30066-lorenz-attaractor-plot

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Climate vs Weather

Meteorology
The climate is the 30 year average of the weather.

Dynamical Systems
The climate is the attractor; the weather is the flow on the attractor.

Stochastic Systems
The climate is the probability distribution of the weather.

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Climate vs Weather

If we can't predict the weather, how can we predict the climate?

Perhaps statistical mechanics provides hope.

individual particles (chaotic) → statistical mechanics → thermodynamics (predictable)

weather (chaotic) → ?? → climate (predictable ??)

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Which aspects of the climate are predictable?

Perhaps Predictable	Perhaps Not
Global Mean Surface Temperature	Rainfall
Ocean Temperature	Droughts
Sea Ice Extent	Floods
Glacier and Ice Sheet Retreat	Biological Pump
Permafrost Melt	Ecosystems
	Political/Economic Response

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Next Week

Student Presentations

Wrap up

Course Evaluation