Midterm Exam

October 9, 2008

Closed book exam. Books, notes, and electronic devices may not be used.

(36) **1.** Define each of the following statements or notation.

(4) **a.** $\mathcal{M}$ is a σ-algebra on the set X.

(4) **b.** μ is a measure on the measurable space $(X, \mathcal{M})$.

(4) **c.** $f : X \to \mathbb{R}$ is measurable.

(4) **d.** $\varphi : X \to \mathbb{R}$ is simple.

(4) **e.** $\int \varphi \, d\mu$, where φ is a nonnegative measurable simple function.

(4) **f.** $\int f \, d\mu$, where f is a nonnegative measurable function.

(4) **g.** $f \in L^1$.

(4) **h.** $f_n \to f$ almost everywhere.

(4) **i.** $f_n \to f$ in L^1.

(25) **2.**

(10) **a.** State the Monotone Convergence Theorem, and give an example to show that monotonicity is a necessary hypothesis.

(10) **b.** State Fatou's Lemma, and give an example to show that the inequality cannot be replace with equality.

(20) **3.** If $(X, \mathcal{M}, \mu)$ is a measure space, and if $\{f_n\}$ is a sequence of measurable functions on X, then $\{x : \lim f_n(x) \text{ exists}\}$ is a measurable set.

(25) **4.** Suppose that $E \subset \mathbb{R}$ has finite Lebesgue measure. Show that $m\left(E \cap [x,\infty)\right) \to 0$ as $x \to \infty$.