

Math 1371 – Lecture 10

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1 Nuts and bolts

1. Graded exams will be returned Thursday in workshop.
2. Read the following sections before workshop tomorrow:
 - 6345 Related rates
 - 6353 Increasing and decreasing for functions (read with care, as described later)
3. Office hours this week: MW 11-12, and Thursday 1-2.

2 What's happening today

1. Related rates
2. Increasing and decreasing functions

3 Related rates

Recall this example from our discussion on differentials.

Example 0. You have a spherical balloon of radius 10 cm. Use a differential to estimate how much more surface area the balloon has when you blow up the balloon further so that its radius has increased by 1 cm.

We viewed the surface area as **a function of the radius**, and we used differentials in order to estimate how the surface area changed when the radius changed.

Now we view both quantities as **functions of time**.

By differentiating the equation that relates these quantities, we produce an equation that relates their rates of change.

Example 1. You have a spherical balloon that you are blowing up so that the radius is growing at a constant rate of 3 cm per second. At what rate is the surface area growing when the radius is 10 cm?

A checklist for related rates problems

1. Use the context of the problem to identify an equation that relates two quantities.
2. View the two quantities in the equation as functions of another variable (time, usually).
3. Differentiate the equation with respect to time to produce an equation that *relates* the *rates* of change of the two quantities.
4. Use the context of the problem to fill in the known quantities and identify the unknown quantities.

Example 2. A small town lies at the intersection of a north-south highway and an east-west highway. A red car starts at noon, 20 miles north of town, and travels at a constant speed of 60 mph south. A blue car starts at noon, 15 miles west of town, and travels at a constant speed of 50 mph east.

1. When does each car reach the center of town?
2. How far apart (as the crow flies) are the cars at 12:15 p.m.?
3. What is the rate of change of their distance apart at 12:15 p.m.?
4. When are the cars closest (as the crow flies) to one another?

The last part of the previous problem suggests that we need to understand the relationship between the increasing and decreasing behavior of a function and its first derivative.

4 Increasing and decreasing functions

Recall what we mean by an *interval*: When we write

$$(-3, 5)$$

for example, we mean the set of all x such that $-3 < x < 5$.

We say that a function $f(x)$ is *increasing* on an interval if

$$f(x_1) < f(x_2)$$

for all $x_1 < x_2$ in the interval.

We say that a function $f(x)$ is *decreasing* on an interval if

$$f(x_1) > f(x_2)$$

for all $x_1 < x_2$ in the interval.

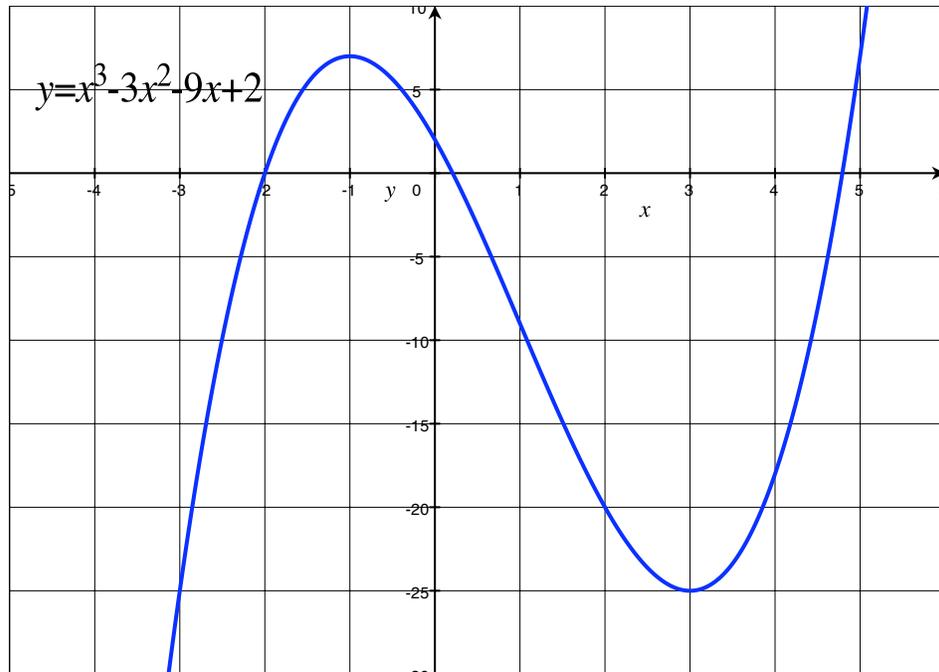
Theorem.

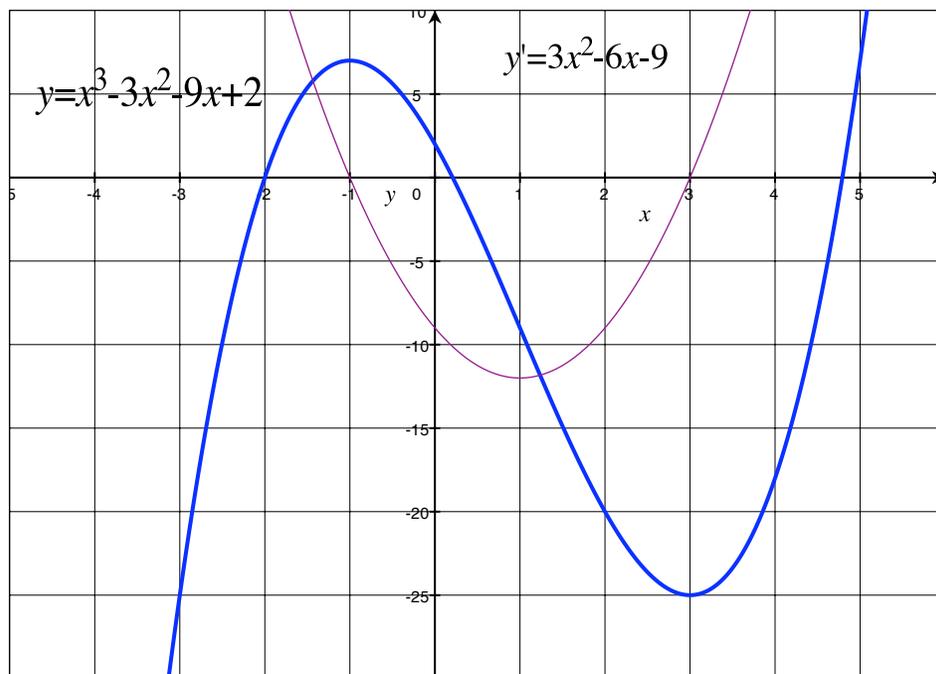
1. If $f'(x) > 0$ on an interval, then $f(x)$ is increasing on that interval.
2. If $f'(x) < 0$ on an interval, then $f(x)$ is decreasing on that interval.

Example 3. On what intervals is the function

$$f(x) = x^3 - 3x^2 - 9x + 2$$

increasing and decreasing?

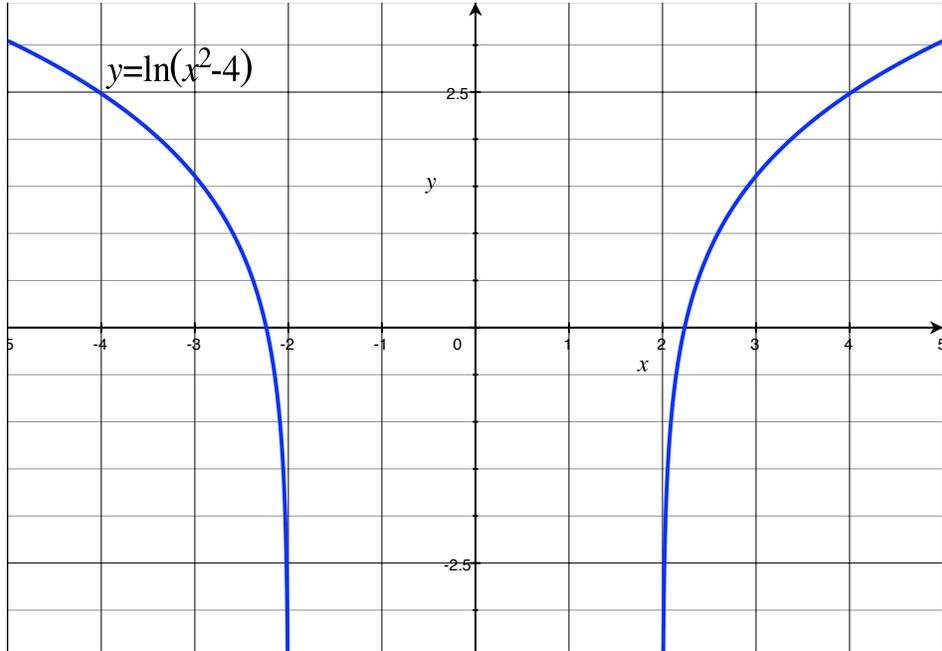


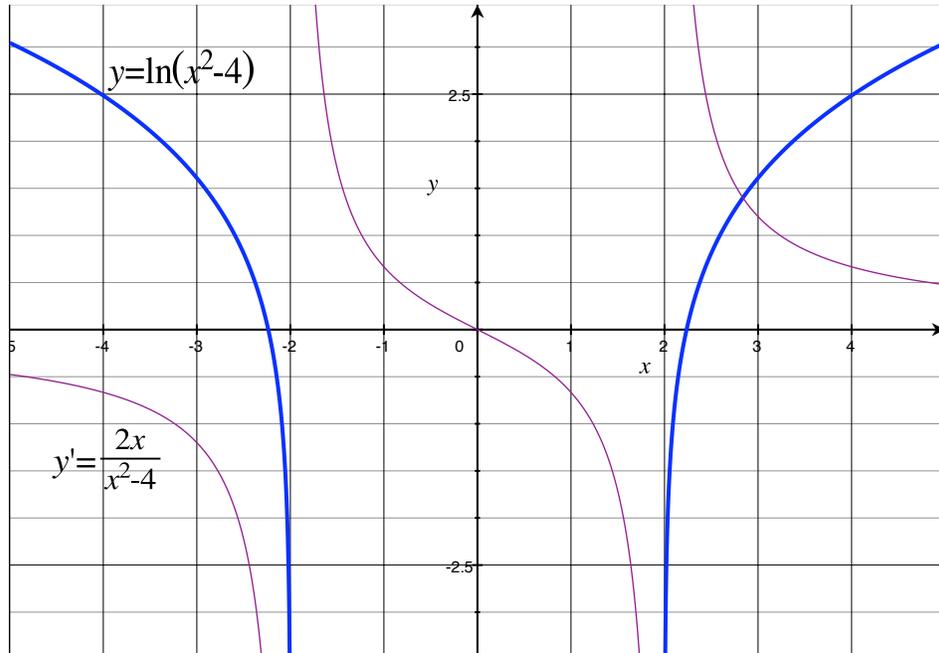


Example 4. Let

$$f(x) = \ln(x^2 - 4).$$

Find the intervals on which $f(x)$ is increasing and decreasing.





The text introduces the concept of $f(x)$ increasing/decreasing “at a point”. Our approach in these notes is to talk about functions increasing/decreasing on intervals only.

For all practical purposes, there is no difference: if you have to show that $f(x)$ is increasing at a point, you will be showing that $f(x) > 0$ at that point.

If you have to show that $f(x)$ is decreasing at a point, you will be showing that $f(x) < 0$ at that point.