

Math 1371 – Lecture 12

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1 Nuts and bolts

1. Read the following section before workshop tomorrow:
 - 6357 Concavity
2. Office hours this week: MW 11-12, and Thursday 10-11.

2 The main point from Lecture 11 on Wednesday

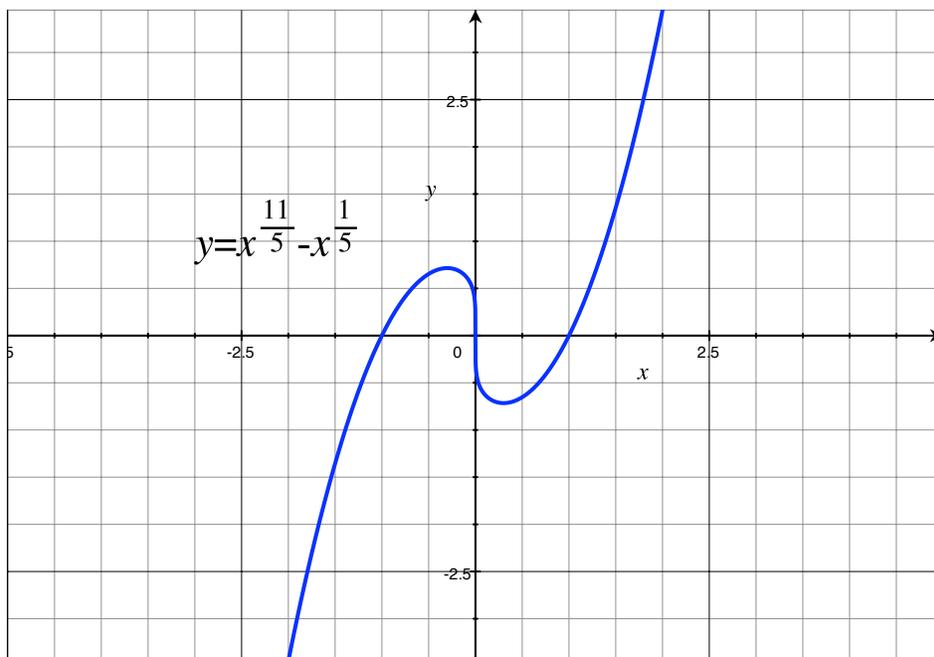
Critical points, places where the derivative is zero or undefined, are candidates for local extrema. The first derivative test identifies local extrema according to how the sign of the derivative changes near the critical point.

3 What's happening today

1. Related rates problem from Wednesday
2. More first derivative test
3. Concavity, inflection points, second derivative test

Example 1. Dave walks in a straight path at a speed of 4 feet per second. HAL's camera is 10 feet from the path and is focused on Dave. At what rate is the HAL's gaze rotating when Dave is 10 feet from the point on his path closest to HAL's camera?

Example 2. Let $f(x) = x^{11/5} - x^{1/5}$. Find all critical points, local extrema, and all intervals on which $f(x)$ is increasing or decreasing.



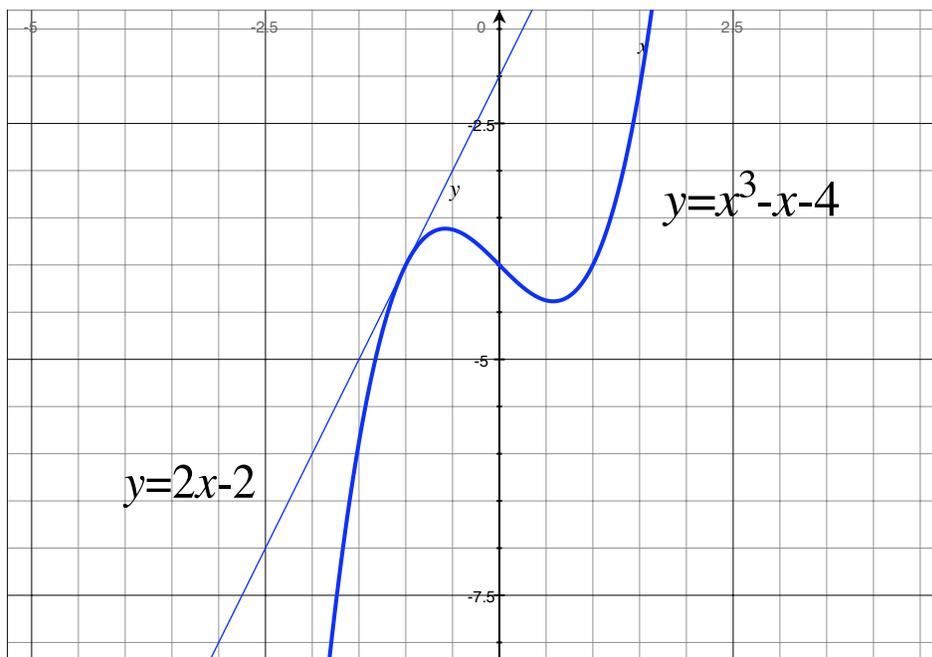
We draw further analogies between the shape of the graph of a function and its derivative(s).

A function $f(x)$ is *concave up* at $x = c$ if the graph of $f(x)$ is above its tangent line at $x = c$ for all x near c .

A function $f(x)$ is *concave down* at $x = c$ if the graph of $f(x)$ is below its tangent line at $x = c$ for all x near c .

We can determine concavity directly in some cases:

Example 3. Let $f(x) = x^3 - x - 4$. Determine directly whether $f(x)$ is concave up or concave down at $x = -1$.



Notice that in the previous example, the values of the derivative were decreasing near $x = -1$. That is, the derivative of the derivative is negative at $x = -1$. In other words, $f''(-1) < 0$.

This relationship is formalized in the following theorem:

Theorem.

- If $f''(c) > 0$, then $f(x)$ is concave up at $x = c$.
- If $f''(c) < 0$, then $f(x)$ is concave down at $x = c$.

Back to example 3:

Let $f(x) = x^3 - x - 4$.

Find all critical points, local extrema, intervals on which $f(x)$ is increasing or decreasing, and intervals on which $f(x)$ is concave up or concave down.