Math 1371 – Lecture 17

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1 Nuts and bolts

1. Office hours this week: MW 11-12, and anytime Thursday by appointment.
2. The second exam is Thursday, either 5-6 or 6-7. **The room for this lecture is once again 250 Anderson on the West Bank.** Note: other lectures of Math 1371 have different rooms.

2 What's happening today

Review for exam

1. Graph sketching based on first and second derivatives
2. Optimization
3. Related rates
4. L'Hôpital's rule? Anything else?

3 Graph sketching

Based on the sign of the first and second deriviative,

Here are all the possible shapes near $x = a$...

For each shape, is there a critical point or inflection point? Is there a local extremum? How do you know? (first or second derivative test?)

Putting it all together:

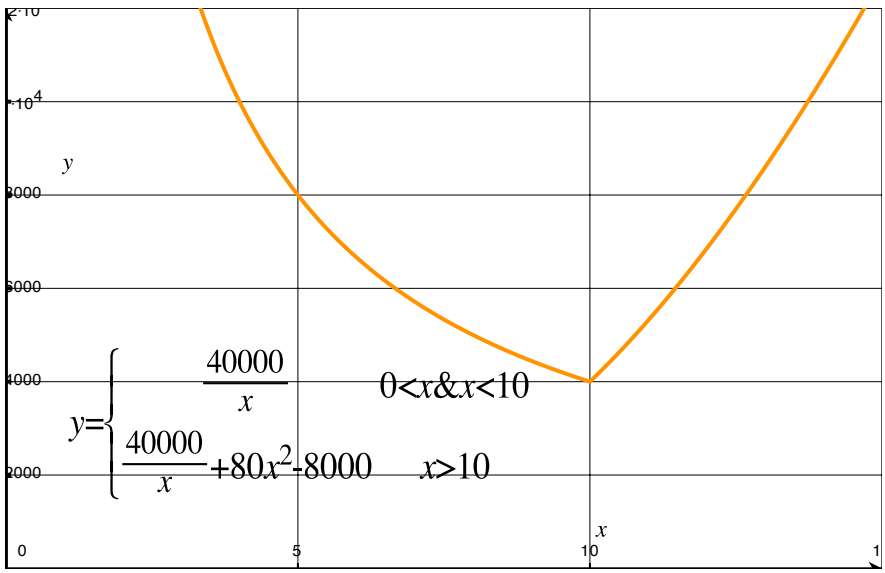
Example 1. Suppose that $f(x)$ is a continuous function with the following information given:

- $f'(x) < 0$ for $-\infty < x < -2$ and $2 < x < \infty$, $f'(x) > 0$ for $-2 < x < 0$ and $0 < x < 2$, and f' is undefined only at $-2, 0, 2$.
- $f''(x) > 0$ for $-\infty < x < -2$, $-2 < x < -1$, $1 < x < 2$, and $2 < x < \infty$, $f''(x) < 0$ for $-1 < x < 0$ and $0 < x < 1$, and f'' is undefined only at $-2, 0, 2$.

Sketch a possible graph for $f(x)$, determine all local extrema, and tell HOW YOU KNOW that each is a local extremum (first or second derivative test?).

4 Optimization

Example 2. We want to manufacture a rectangular storage container with a square top and bottom that encloses 1000 cubic feet. The material for the sides costs \$10 per square foot, and the material for the top and bottom is free for the first 200 square feet of material, and then for larger top and bottom they cost \$40 per extra square foot of material after 200. Find the dimensions of the container that minimize the cost of material to make the container.



5 Related rates

Example 3. Two cars head east from the MN-SD border at the same time on parallel straight roads that are 10 miles apart. Car A is going 70 mph, and car B is going 60 mph. One hour into the trip, what is the rate of change of their distance apart?