

Math 1371 – Lecture 8

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1 Nuts and bolts

1. The first exam is on Thursday. **All students in this lecture will take the exam in Anderson Hall, room 250, on the West Bank.** Make sure you know where this is before the time of the exam.



2. The exam will cover the sections of the text up to and including **6349 Differentials**, except for **6345 Related Rates**. The emphasis will be on the problems that you encounter on the worksheets, in the homework, and on the review sheets you will see tomorrow and Thursday.
3. By Tuesday evening, send me email with your suggestions for review problems to discuss during lecture on Wednesday.
4. Office hours this week: MW 11-12, and **by appointment all day Thursday**. I'll also be stopping in to the workshop sections.

2 The main point from Lecture 7 on Wednesday

We can find the derivatives of functions that are defined implicitly by equations through the method of **implicit differentiation**, without solving the equation explicitly for one of the variables.

The main ideas behind implicit differentiation:

- View each side of the equation as a function of x ; their derivatives must also be equal.
- View y as a function of x ; then expressions like y^2 require the chain rule to differentiate.

3 What's happening today

1. Hyperbolic functions
2. Higher derivatives
3. By discussing these topics, we will be reviewing older topics.

4 Hyperbolic functions

The *hyperbolic sine* function is defined to be

$$\sinh(x) = \frac{e^x - e^{-x}}{2}.$$

The *hyperbolic cosine* function is defined to be

$$\cosh(x) = \frac{e^x + e^{-x}}{2}.$$

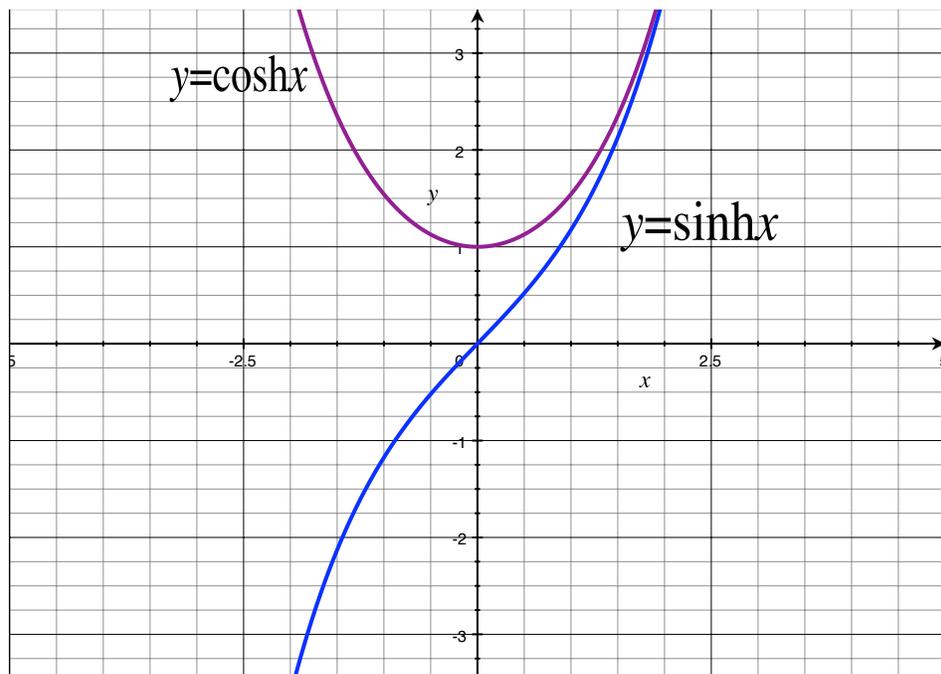
The other four hyperbolic functions are defined as are their trig. counterparts:

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)}$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)}$$



By working with these hyperbolic functions, we can practice

- finding derivatives using the chain rule

- finding derivatives of inverse functions

Example 1. Find the derivatives of $\sinh(x)$ and $\tanh(x)$.

For practice, find the derivatives of the other hyperbolic functions. (Notice that the derivative of $\cosh(x)$ is not $-\sinh(x)$, as you might expect by analogy with \cos ; it is $\sinh(x)$.)

To summarize:

$$\frac{d}{dx} \sinh(x) = \cosh(x)$$

and

$$\frac{d}{dx} \tanh(x) = \operatorname{sech}^2(x).$$

Example 2. Find the inverse function of $\sinh(x)$.

To summarize:

We have the following explicit expression for the inverse of the hyperbolic sine:

$$\operatorname{arcsinh}(x) = \ln(x + \sqrt{x^2 + 1}),$$

and we have

$$\frac{d}{dx} \operatorname{arcsinh}(x) = \frac{1}{\sqrt{1 + x^2}}.$$

5 Higher derivatives

The second derivative arises in applications most commonly as the acceleration of a moving particle.

The acceleration is the second derivative of the position function. If the position function is $x(t)$, then the acceleration function is written $x''(t)$ or

$$\frac{d^2x}{dt^2}.$$

Generally, for a function $y = f(x)$, the second derivative is written $f''(x)$ or

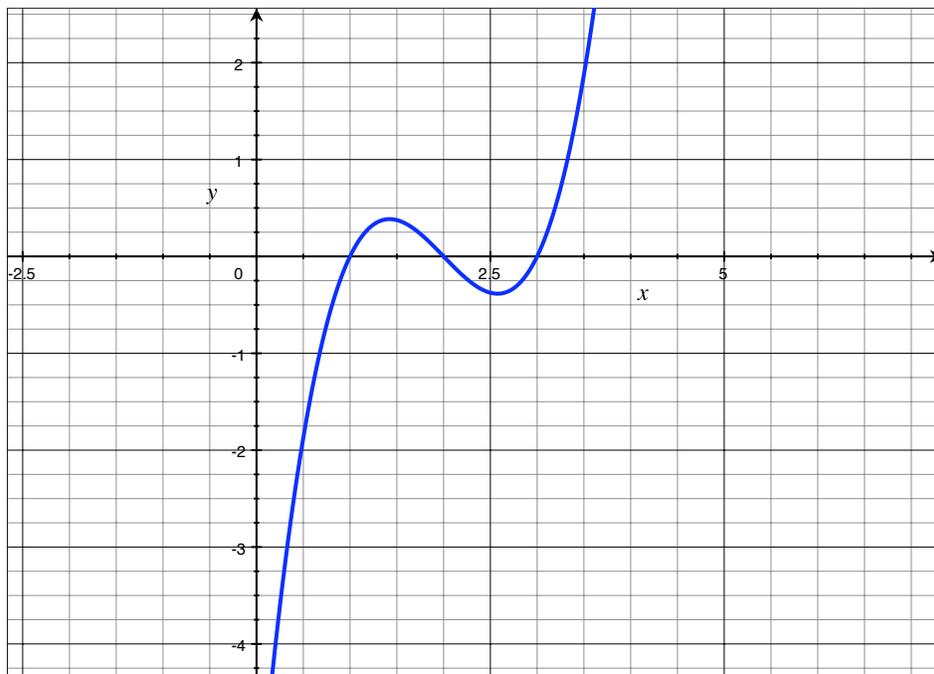
$$\frac{d^2y}{dx^2}.$$

Example 3. Suppose that a particle is moving along a straight line with position described by the function

$$x(t) = t^3 - 6t^2 + 11t - 6,$$

in feet at time t seconds.

Find the times at which the velocity is zero, and find the (sign of the) acceleration at those times.



6 Example 4.

We can use implicit differentiation to find second derivatives.

Given the equation

$$y^2 + 4y + 2x^2 - 4x = 6,$$

which defines y implicitly as functions of x , find where the graph has horizontal and vertical tangent lines, and find $\frac{d^2y}{dx^2}$ at those points.

