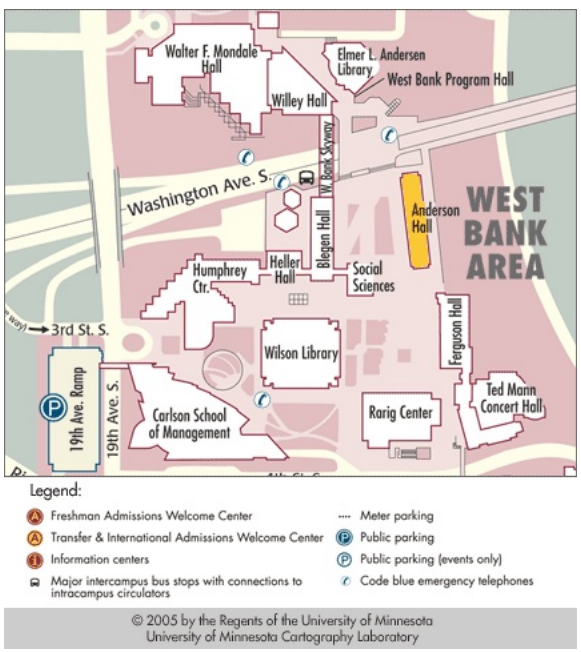
# Math 1371 – Lecture 9

Bryan Mosher

Wednesday, October 3, 2007

## 1 Nuts and bolts

1. The first exam is tomorrow. **All students in this lecture will take the exam in Anderson Hall, room 250, on the West Bank.** Make sure you know where this is before the time of the exam.




2. The exam will cover the sections of the text up to and including **6349 Differentials**, except for **6345 Related Rates**. The emphasis will be on the problems that you encounter on the worksheets, in the homework, and on the review sheets you will see tomorrow and Thursday.

3. Office hours this week: MW 11-12, and **by appointment all day Thursday**. I'll also be stopping in to the workshop sections.

## 2 The main point from Lecture 8 on Monday

The second derivative is the derivative of the derivative of a function. It is the acceleration of a moving particle in applications. The second derivative can be found by implicit differentiation when a function is defined implicitly by an equation.

## 3 What's happening today

1. Finish second derivative examples
2. Review

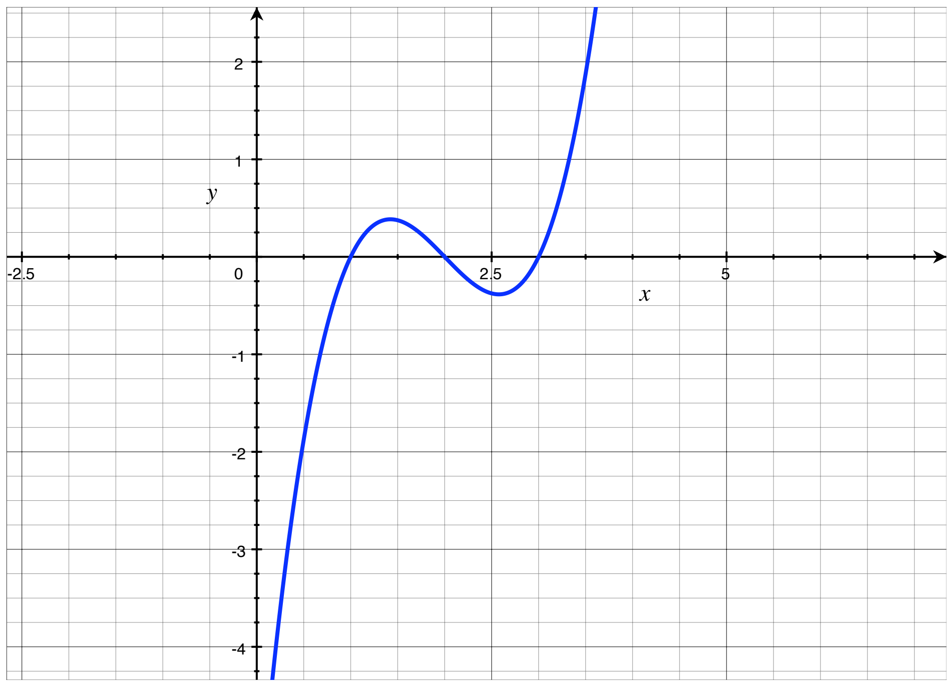
## 4 Second derivatives

**Example 1.** Suppose that a particle is moving along a straight line with position described by the function

$$x(t) = t^3 - 6t^2 + 11t - 6,$$

in feet at time $t$ seconds.

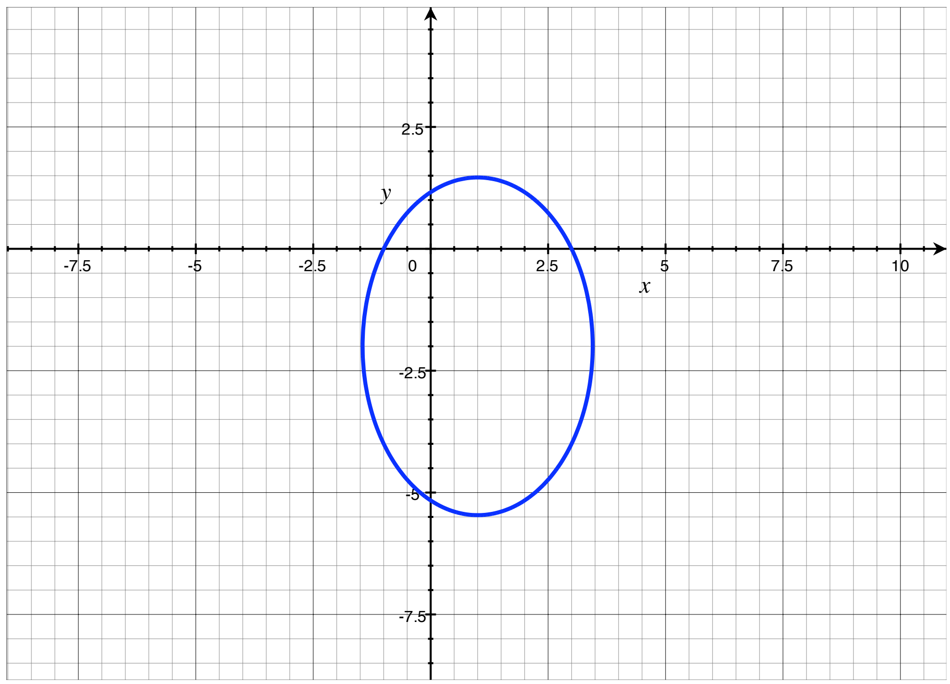
Find the times at which the velocity is zero, and find the (sign of the) acceleration at those times.




**Example 2.**
We can use implicit differentation to find second derivatives.
Given the equation

$$y^2 + 4y + 2x^2 - 4x = 6,$$

which defines $y$ implicitly as functions of $x$, find where the graph has horizontal and vertical tangent lines, and find $\frac{d^2y}{dx^2}$ at those points.




## 5 Review

Do not spend time memorizing derivatives of basic functions. If you need it, it will be given to you.

Let's focus on problems of the following types; this is not an exhaustive list:

- "sparse data" chain rule problems
- finding secant lines and tangent lines
- using the definition of derivative directly

**Example 3.** Let $f(x)$, $g(x)$, and $h(x)$ be functions that satisfy the data in the table. Find $\frac{dy}{dx}$ when

$$y = \frac{e^{f(\sqrt{g(2x)})}}{h(\sin x)}$$

and then evaluate at $x = 0$.

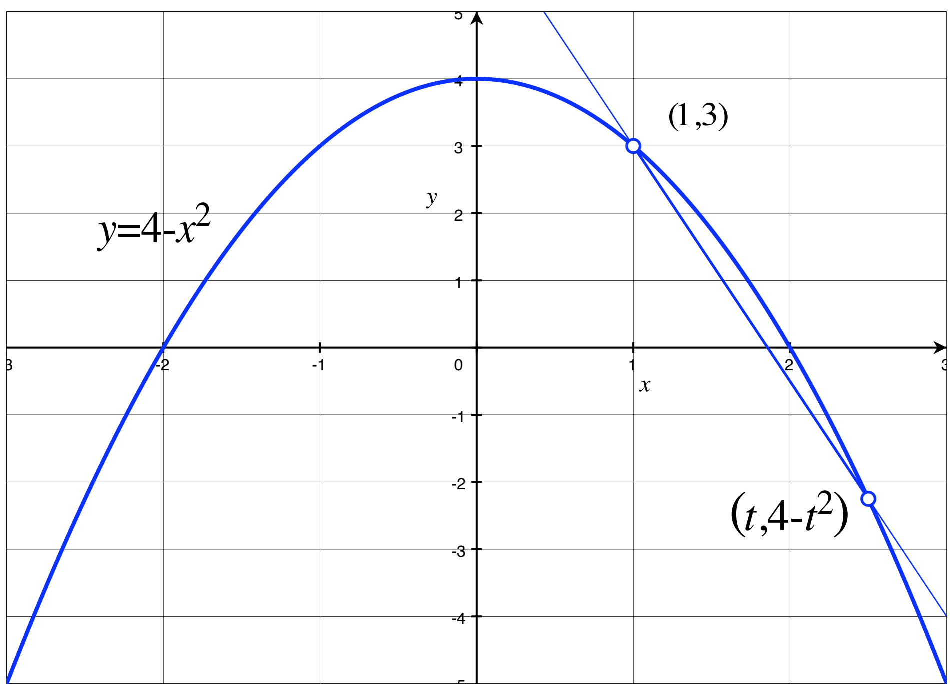
| x | f(x) | f’(x) | g(x) | g’(x) | h(x) | h’(x) |
|---|---|---|---|---|---|---|
| 0 | 3 | -2 | 4 | -1 | 1 | 2 |
| 2 | 2 | e | 2 | -2 | 0 | 8 |

Notice that only the following data is relevant.

| x | f(x) | f’(x) | g(x) | g’(x) | h(x) | h’(x) |
|---|---|---|---|---|---|---|
| 0 | | | **4** | **-1** | **1** | **2** |
| 2 | **2** | **e** | | | | |

**Example 4.** Find the equation of the secant line to the graph of the equation $y = 4 - x^2$ that passes through the points $(1, 3)$ and $(t, 4 - t^2)$.

Find the equation of the tangent line to the graph at $x = 1$.




**Example 5.** Find $\frac{dy}{dx} = f'(x)$ directly, from the definition of the derivative, when

$$y = f(x) = \frac{x+1}{x^2}.$$

**Example 6.** Evaluate

$$\lim_{t \to -3} \frac{15 - t - 2t^2}{6 + 2t}.$$