Math 2263.002 Name:
Summer 2009
Exam 1

There are six numbered problems, each worth 25 points, on three sheets of paper.

1. Write parametric equations for the line of intersection of the planes

$$x - y + 2z = 3$$

and

$$z - x + 2 = 0.$$

2. Find the limit, if it exists, or show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{5x^2y}{x^3+y^3}.$$

3. Let $f(x, y) = 3x^2 - (xy)^2 + 2x$.

   (a) (15) Find an equation of the tangent plane to the graph of f when $(x, y) = (1, -2)$.

   (b) (10) Find an equation of the plane passing through the origin that is parallel to the plane in (a).

4. Let $f(x,y) = x^2 \sin y + ye^{xy}$ and

$$x(s,t) = s + 2t,$$

$$y(s,t) = st.$$

(a) (15) Use the chain rule to write $\frac{\partial f}{\partial t}$. You can write your answer in terms of x, y, s and t.

(b) (10) Evaluate $\frac{\partial f}{\partial t}$ at $(s,t) = (2,1)$. Write an *exact* number, not a decimal approximation.

5. Let

$$f(x, y, z) = \frac{x - y}{z^2}.$$

(a) (6) Find the gradient of f.

(b) (10) Find the directional derivative of f at the point $(-2, 3, -1)$ in the direction of the vector $\langle 2, 1, 3 \rangle$.

(c) (9) Find the maximum rate of change of f at the point $(-2, 3, -1)$, and write a *unit* vector indicating the direction in which that maximum rate of change occurs.

6. Let

$$f(x, y) = 3xy - x^2y - xy^2.$$

(a) (15) Find the local maximum and minimum values and saddle points of f.

(b) (10) Find the absolute maximum and minimum values of f on the triangular region defined by the inequalities $x \geq 0$, $y \geq 0$, and $x+y \leq 3$.