MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

PROFESSOR: GREGG MUSIKER PRACTICE MIDTERM EXAM 2 (MARCH 21ST, 2013)

Student Name Teaching Assistant and Section Number

$$\text{Test Scores: } \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \text{Total} \\ & & & & & \\ & & & & & \end{bmatrix}$$

INSTRUCTIONS: Books and notes are not allowed. Only scientific (NOT graphing) calculators are allowed. No cell phones are allowed, even as a calculator. Write *complete solutions* to all problems, explaining your method or reasoning, for full credit. Feel free to do first what you know best. You have 50 minutes. Good luck!

REMARK: This Practice Midterm is only given as a sampling of possible problems. You are responsible for all material covered in lectures and on the Homeworks. The upcoming Midterm will cover Sections 3.5, 3.6, 4.2, 4.3, 4.4, 4.5, 4.7, 5.1, and 5.3.

You are encouraged to work on this Practice Midterm between Monday's lecture and Wednesday's. The professor will go over solutions to this Practice Midterm Wednesday in Lecture.

1 **(20 points)** Using the adjoint matrix, or otherwise, compute the inverse of the matrix $\begin{bmatrix} 4 & 5 & 6 & 0 \\ 2 & 0 & 2 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Make sure to explain your work and or reasoning.

2 (a) **(10 points)** Determine if the vectors $\mathbf{v}_1 = (1, -1, 2, 3), \mathbf{v}_2 = (2, 3, 4, 1), \mathbf{v}_3 = (1, 1, 2, 1)$, and $\mathbf{v}_4 = (4, 1, 8, 7)$ form a basis of $\mathbb{R}^4$ or not. Explain your reasoning either way.

2 (b) **(10 points)** Determine whether or not the subset of polynomials $\{a_0 + a_1x + a_2x^2 + a_3x^3$, with $a_i \in \mathbb{R}$, satisfying the condition $a_0a_1 = a_2 + a_3\}$ is a subspace of P_3. Explain your reasoning either way.

3 (a) **(10 points)** Verify by computation or give an explanation why $e^{-3x}cos(4x)$ and $e^{-3x}sin(4x)$ are two solutions to the the differential equation $y'' + 6y' + 25y = 0$.

3 (b) **(10 points)** Use the Wronskian or otherwise to show that $e^{3x}cos(4x)$ and $e^{3x}sin(4x)$ are linearly independent as differentiable functions. Explain your reasoning.

4 **(20 points)** Compute the general solution $y(x)$ to the differential equation $y'''' - 6y''' + 10y'' - 6y' + 9y = 0$. Explain your work.

Remark: I did not get to all of the relevant examples in Lecture Monday. Thus, the following examples will be discussed in Section on Tuesday.

Hint: Recall the **rational root test** from algebra: if a polynomial $r^d + a_{d-1}r^{d-1} + \cdots + a_2r^2 + a_1r + a_0$ has a root $r = c$, where c is an integer, then c must be a factor of a_0.

Related Examples: i) Solve the general solution to $y''' + y' - 10y = 0$.

Solution: The differential equation $y''' + y' - 10y = 0$ has characteristic equation $r^3 + r - 10 = 0$. The only possible integer roots are ± 1, ± 2, ± 5, and ± 10. Trying a few of them, we see that $r = 2$ is a root ($2^3 + 2 - 10 = 0$). Thus we then proceed with synthetic division, dividing $r^3 + r - 10$ by $(r - 2)$ to obtain $r^3 + r - 10 = (r - 2)(r^2 + 2r + 5)$.

We find the additional two roots via the quadratic formula: $r^3 + r - 10$ has real root $r_1 = 2$ and two complex roots $-1 \pm 2i$.

We conclude that the general solution to $y''' + y' - 10y = 0$ is

$$y(x) = c_1e^{2x} + c_2e^{-x}cos(2x) + c_3e^{-x}sin(2x).$$

ii) Solve the IVP $y''' + 3y'' - 10y' = 0$ with $y(0) = 7$, $y'(0) = 0$, $y''(0) = 70$.

Solution: We have the characteristic equation $r^3 + 3r^2 - 10r = 0$. We factor this as $r(r^2 + 3r - 10) = r(r + 5)(r - 2) = 0$ which has roots $r_1 = 0$, $r_2 = 2$, $r_3 = -5$. We thus have the general solution

$$y(x) = c_1e^{0x} + c_2e^{-5x} + c_3e^{2x} = c_1 + c_2e^{-5x} + c_3e^{2x}.$$

Using the initial conditions, we get the linear system

$$\begin{aligned} y(0) &= c_1 + c_2 + c_3 = 7 \\ y'(0) &= -5c_2 + 2c_3 = 0 \\ y''(0) &= 25c_2 + 4c_3 = 70 \end{aligned}$$

with solutions $c_1 = 0$, $c_2 = 2$, and $c_3 = 5$. We thus get the particular solution $y(x) = 2e^{-5x} + 5e^{2x}$ for this IVP.

iii) Compute the general solution for $9y''''' - 6y'''' + y''' = 0$.

Solution: We have the characteristic equation $9r^5 - 6r^4 + r^3$, which factors as $r^3(9r^2 - 6r + 1) = r^3(3r - 1)^2$. We thus have the roots $0, 0, 0, 1/3, 1/3$ with multiplicities.

Thus the general solution is $y(x) = c_1 + c_2x + c_3x^2 + c_4e^{x/3} + c_5xe^{x/3}$.

5 **(20 points)** Consider the matrix $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 6 & 10 & 4 \end{bmatrix}$. Find a basis for each of the subspaces:

(a) The nullspace (solution space) $\text{Null}(\mathbf{A}) = \{\mathbf{x} \in \mathbb{R}^4 \mid \mathbf{Ax} = \mathbf{0}\}$.

(b) The row space $\text{Row}(A)$.

(c) The column space $\text{Col}(A)$.