MATH 2243: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

PROFESSOR: GREGG MUSIKER PRACTICE MIDTERM EXAM 3 (APRIL 25TH, 2013)

Student Name Teaching Assistant and Section Number

$$\text{Test Scores: } \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \text{Total} \\ & & & & & \\ \end{bmatrix}$$

INSTRUCTIONS: Books and notes are not allowed. Only scientific (NOT graphing) calculators are allowed. No cell phones are allowed, even as a calculator. Write *complete solutions* to all problems, explaining your method or reasoning, for full credit. Feel free to do first what you know best. You have 50 minutes. Good luck!

REMARK: This Practice Midterm is only given as a sampling of possible problems. You are responsible for all material covered in lectures and on the Homeworks. The upcoming Midterm will cover Sections 5.2, 5.4, 5.5, 5.6, 6.1, 6.2, 6.3 (excluding predator-prey models), 7.1, 7.2, and 7.3.

You are encouraged to work on this Practice Midterm between Monday's lecture and Wednesday's. The professor will go over solutions to this Practice Midterm Wednesday in Lecture.

1 **(25 points)** Use *the method of variation of parameters* to find a particular solution to the second order differential equation

$$y'' - 5y' + 6y = 2e^x.$$

(Solving the problem correctly by other mthods will receive only 15 points credit.)

2 **(20 points)** Let A be the 3-by-3 matrix $\begin{bmatrix} 3 & 0 & 2 \\ -2 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$, with characteristic polynomial which factors as $-(\lambda - 1)^2(\lambda - 3)$. Determine whether or not A is diagonalizable, and if so find a P and D such that $A = P^{-1}DP$.

3 **(10 points)** Consider the differential equation

$$y'' + 7y' + 12y = e^{-3x} + cos(x).$$

When utilizing the method of *undetermined coefficients* to find a particular solution, which of the following terms should be included in the undetermined form? (Circle all correct answers.)

$$(a)x,\ (b)x^2,\ (c)xcos(x),\ (d)xsin(x),\ (e)cos(x),\ (f)sin(x),\ (g)e^{-3x},\ (h)xe^{-3x},\ (i)x^2e^{-3x}.$$

4 **(15 points)** Let A be the matrix $\begin{bmatrix} -1 & -6 \\ 2 & 6 \end{bmatrix}$. Compute A^8.

5 **(20 points)**

a) Consider the system of first-order differential equations

$$\begin{aligned} x_1' &= -21x_1 - 18x_2 \\ x_2' &= 16x_1 + 13x_2 \end{aligned}$$

Describe the general solution for $x_1(t)$ and $x_2(t)$. Solve the initial value problem $x_1(0) = 2$ and $x_2(0) = 3$.

b) a) Consider the system of first-order differential equations

$$\begin{aligned} x_1' &= 12x_1 + 16x_2 \\ x_2' &= -10x_1 - 12x_2 \end{aligned}$$

Describe the general solution for $x_1(t)$ and $x_2(t)$.

6 **(10 points)**

Consider the matrix $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ -3 & -10 & 2 & 0 \\ 4 & 13 & -2 & 0 \end{bmatrix}$. What are its eigenvalues? Is it diagonalizable? Explain your reasoning.