Seven Puzzles You Think You Must Not Have Heard Correctly

with solutions

Peter Winkler

Dedicated to Martin Gardner on the occasion of the Seventh Gathering for Gardner, March 2006.

A typical mathematical puzzle sounds tricky but solvable—if not by you, then perhaps by the genius down the hall. But sometimes the task at hand is so obviously impossible that you are moved to ask whether you understood the problem correctly, and other times, the task seems so trivial that you are sure you must have missed something.

Here, I have compiled seven puzzles[1] which have often been greeted by words similar to "Wait a minute—I must not have heard that correctly." Some seem too hard, some too easy; after you've worked on them for a while, you may find that the hard ones now seem easy and vice versa.

Note: You won't need to know any mathematics to understand the puzzles, but a certain degree of mathematical sophistication is required to solve some of them. For mysterious reasons, I have arranged the puzzles so that those requiring the least math are at the end; if you are math-challenged, you might want to tackle the puzzles in reverse order.

1 Names in Boxes

The names of 100 prisoners are placed in 100 wooden boxes, one name to a box, and the boxes are lined up on a table in a room. One by one, the prisoners are led into the room; each may look in at most 50 boxes, but must leave the room exactly as he found it and is permitted no further communication with the others.

The prisoners have a chance to plot their strategy in advance, and they are going to need it, because unless *every single prisoner finds his own name* all will subsequently be executed.

Find a strategy for them which which has probability of success exceeding 30%.

Comment: If each prisoner examines a random set of 50 boxes, their probability of survival is an unenviable $1/2^{100} \sim 0.00000000000000000000000000008$. They could do worse—if they all look in the *same* 50 boxes, their chances drop to zero. 30% seems ridiculously out of reach—but yes, you heard the problem correctly.

2 Boxes in Boxes

At many train stations, post offices and currier services around the world, the cost of sending a rectangular box is determined by the sum of its dimensions; that is, length plus width plus height. Prove that you can't "cheat" by packing a box into a cheaper box.

Comment: Of course the assumption here is that the cost *increases* as length + width + height increases. But note, by packing diagonally you can certainly have an inside box whose length (greatest dimension) exceeds that of the outside one.

[1] all new to me since the publication of my book, *Mathematical Puzzles: A Connoisseur's Collection*, AK Peters Ltd 2004—but look for these and many more in Volume II.