The Metalanguage λProlog and Its Implementation

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The Role of Metalanguages

Many computational tasks involve the manipulation of linguistic objects:

- prototyping programming languages
- implementing compilers and program development systems
- manipulating mathematical expressions
- realizing (interactive) proof systems

Emerging applications involve the *integration* of many of these computations.

Can programming language support be provided for such activities?

Metalanguages and Logic Programming

Prolog-like languages contain two features important to symbolic computation:

- First-order terms generalize traditional abstract syntax

$$B \wedge C$$

$$\Downarrow$$

$$\mathrm{and}(\widehat{B},\widehat{C})$$

- Horn clauses naturally translate structural operational semantics rules

$$\frac{\Gamma \vdash B \qquad \Gamma \vdash C}{\Gamma \vdash B \wedge C}$$

$$\Downarrow$$

prove(Gamma,and(B,C)) :-
  prove(Gamma,B), prove(Gamma,C).

An Inadequacy of Traditional Abstract Syntax

Binding notions are not supported in the syntax representation.

The 'first-order' rendition of the formula $\forall x P(x)$:

$$all(x, \widehat{P(x)})$$

Respecting scope issues becomes the programmer's burden with such a representation.

For example consider instantiating the outer quantifier in

$$all(x,\ all(y,\ Q(x,y)))$$

with the term $f(y)$.

In general, 'proper' substitution can be a complex operation to capture correctly.

Higher-Order Treatment of Syntax

Scoping notions arise in many symbolic structures:

- Quantified formulas in non-classical logic
- Side conditions in inference rules
- Proofs for implicational and universal statements
- Binding and bound variable occurrences in programs

A common core of binding related operations apply to all these situations.

A uniform treatment of these aspects can be provided by incorporating binding into syntax representation.

Structure of the Rest of the Talk

- Higher-Order Abstract Syntax in λProlog
- Issues in Realizing the Metalanguage Features
- Structure of the *Teyjus* Implementation
- Concluding Remarks

Higher-Order Abstract Syntax in λProlog

Richer view of object language syntax is supported through the following new features:

- Using lambda terms as data structures
- Incorporating an understanding of lambda conversion into unification
- Allowing for *GENERIC* goals

$$\forall x G$$

"Solve G after replacing x with a new constant"

- Allowing for *AUGMENT* goals

$$D \texttt{ => } G$$

"Add D to program before solving G"

Representing the Lambda Calculus

Term formation through application and abstraction has to be captured.

The HOAS approach:

- Use constructors to distinguish between object language application and abstraction
- Use λProlog abstraction to represent object language binding

Thus

$$\overline{(M\ \ N)} \quad \longrightarrow \quad (\mathit{app}\ \overline{M}\ \overline{N})$$

$$\overline{(\text{lambda (x) M})} \quad \longrightarrow \quad (\mathit{abs}\ \lambda x\ \overline{M})$$

Representing Functional Programs

- Introduce new constructors to represent programming language primitives
- Utilize λProlog abstraction to translate object language binding
- Use syntactic de-sugaring and the basic translation scheme to render programs into terms

An Example

*fact m n = if (m = 0) then n else (fact (m - 1) (m * n))*

$\Updownarrow$

fact =
(fixpt (f)
(lambda (m) (lambda (n)
*(if (m = 0) then n else (f (m - 1) (m * n))))))*

$\downarrow$

fact =
(fix λf
(abs λm(abs λn
(cond (eq m 0) n
(app (app f (minus m 1)) (times m n))))))

Usefulness of HOAS Representation

Primitives in λProlog provide direct support for logical operations on "program terms"

- Lambda conversion rules build in an understanding of binding structure and substitution
- Higher-order unification is a useful tool for examining program structure
- Scoping devices support recursion over binding structure

Pattern Recognition through Unification

Program terms may contain substitutible variables.

However, substitutions for these variables must respect scope restrictions.

For instance, the 'pattern'

$$(abs\ \lambda x(abs\ \lambda y(C\ x)))$$

can match with

$$(abs\ \lambda x(abs\ \lambda y(less\ x\ 0)))$$

but not with

$$(abs\ \lambda x(abs\ \lambda y(less\ x\ y)))$$

Thus, unification provides a sophisticated means for dependency analyses.

Recognizing (Binary) Tail Recursive Functions

Consider the following "template"

(fix λf (abs λx (abs λy
(cond (C x y) (H1 x y) (app (app f (H2 x y)) (H3 x y))))))

Notice that *C*, *H1*, *H2* and *H3* *cannot* be instantiated so as to depend on *f*, *x* or *y*.

Thus, this term recognizes only those recursive two argument 'conditional' programs in which

- there is *no* recursive call in the condition or *then* branch, and
- the value returned in the *else* branch is *completely* determined by recursive call.

Such programs must be tail recursive.

Limitations of Template Matching

Unfortunately, templates alone have limited applicability.

For example, what if

- the recursive call is in the *then* branch of conditional?
- there are embedded conditionals?

Thus, our template will not recognize tail recursiveness of the following program:

gcd x y =
  if (x = 1) then 1
  else if (x < y) then (gcd y x)
    else if (x = y) then x else (gcd (x - y) y)

Worse still, there is no *finite* set of templates covering *all* mentioned cases and recognizing *only* tail recursive programs.

Recursion over Conditional Structure

However, a satisfactory *recursive* description of such program terms *can be* provided:

- A program with *no* recursive calls

  $tr\ (fix\ \lambda f(abs\ \lambda x(abs\ \lambda y(H\ x\ y)))).$

- A program comprising *only* a recursive call

  $tr\ (fix\ \lambda f(abs\ \lambda x(abs\ \lambda y\ (app\ (app\ f\ (H\ x\ y))\ (G\ x\ y))))).$

- A conditional program with *no* recursion in the test *and* with 'tail recursive' *then* and *else* branches

$$
\begin{array}{l}
tr\ (fix\ \lambda f(abs\ \lambda x(abs\ \lambda y \\
\quad (cond\ (C\ x\ y)\ (H1\ f\ x\ y)\ (H2\ f\ x\ y))))) \texttt{ :-} \\
\qquad tr\ (fix\ \lambda f(abs\ \lambda x(abs\ \lambda y(H1\ f\ x\ y)))), \\
\qquad tr\ (fix\ \lambda f(abs\ \lambda x(abs\ \lambda y(H2\ f\ x\ y)))).
\end{array}
$$

Recursion Over Binding Structure

Recognizing tail-recursiveness of *arbitrary* arity functions requires an explicit recursion over *binding* structure:

- Given an expression of the form

  $$(fix\ (\lambda f\ F))$$

  analyze F after replacing f with a new constant whose occurrences must be restricted.

- Given an expression of the form

  $$(abs\ (\lambda x\ R))$$

  analyze R after replacing x with a new constant whose usage can be arbitrary.

- Check the eventual "first-order" structure for satisfaction of usage constraints.

Can be realized using *GENERIC*, *AUGMENT* and application.

A Recognizer for Tail Recursive Functions

Assume that *(term T)* succeeds just in case T is a 'program term.'

tr (fix M) **:-** $\forall f$ *((recfn f)* **=>** *(trfn (M f)))*.

trfn (abs R) **:-** $\forall x$ *((term x)* **=>** *(trfn (R x)))*.
trfn R **:-** *trbody R*.

trbody (cond C M N) **:-** *term C, trbody M, trbody N*.
trbody (app M N) **:-** *trbody M, term N*.
trbody M **:-** *recfn M*.

Representation of Lambda Terms

Lambda terms are being used as *data structures*.

Thus, the representation should satisfy the following criteria:

- Structure should be accessible
- Equality under renaming should be easy to determine
- The operation of β-reduction should be efficiently supported

A Complication: In the context of interest, it may be necessary to look *inside* abstractions.

A Consideration in Beta Reduction

Support for *laziness* in reduction substitutions could be useful:

- Provides the basis for combining structure traversals in reductions
- Actual substitution may sometimes be delayed to a point where it becomes unnecessary

Explicit treatment of substitution is an essential ingredient to realizing such benefits.

Actual Lambda Term Representation

The representation used in *Teyjus* has the following characteristics:

- Utilizes the deBruijn scheme for eliminating (bound variable) names
- Based on an explicit substitution notation called the *suspension* notation
- Uses a demand driven approach to reduction and substitution, thereby interleaving these with comparison operations
- Exploits annotations indicating closedness status of terms
- Implements reduction using a graph-based scheme

Dealing with GENERIC

Idea of instantiating with new constant may be used

However, there is interference with usual treatment of free (existential) variables

Program: $\forall x\, p(x, x)$

Goal: ?- $\forall x\, p(Y, x)$

$$\Big\downarrow \; c/x$$

?- $p(Y, c)$

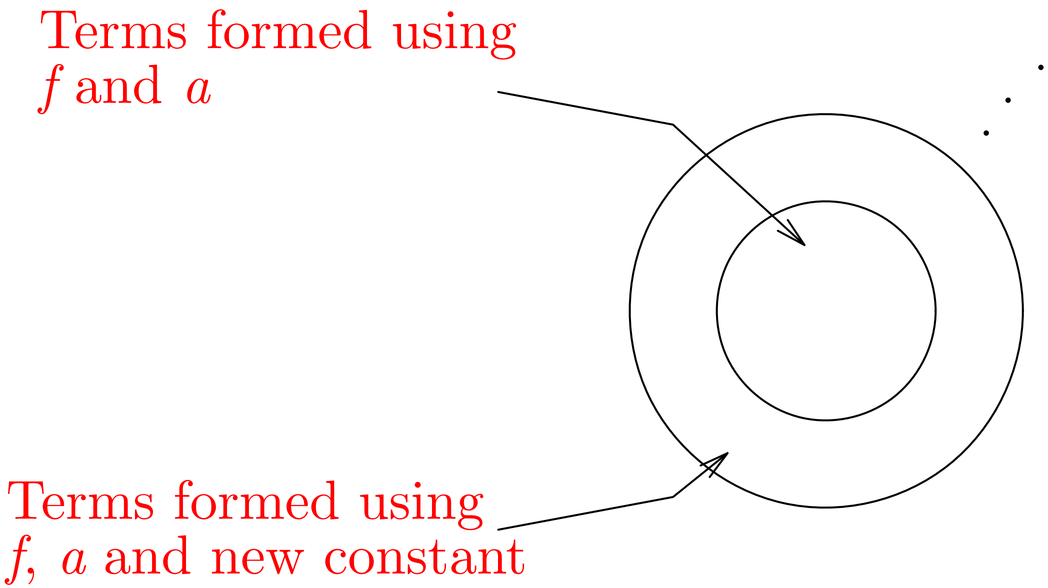
Unification has to be somehow constrained to cause failure in this situation

A Possible Solution

Maintain term universes as hierarchy

Introduce new levels in the hierarchy when processing GENERIC

$$\forall y\; p(a, f(X), y)$$



Label constants to determine 'place' in hierarchy

Label variables to constrain possible instantiations

Details of the Solution

The scheme can be realized as follows:

- Maintain the current highest universe level in a special register
- Translate GENERIC into register increment on entry and decrement on success
- Label constants and variables with register value at creation
- When binding a variable, check also the consistency of labelling

Most actions can be realized though low-level instructions.

Realizing Higher-Order Unification

Multiple most general unifiers may exist.

For example, consider the problem

$$(F\ 1) = (g\ 1\ 1)$$

This problem has four distinct unifiers:

$$F \mapsto \lambda x(g\ x\ x)$$
$$F \mapsto \lambda x(g\ x\ 1)$$
$$F \mapsto \lambda x(g\ 1\ x)$$
$$F \mapsto \lambda x(g\ 1\ 1)$$

An implementation must correspondingly manifest a branching character.

Moreover, it should be able to sometimes suspend unification problems to avoid redundant search.

Treatment of Higher-Order Unification

Our implementation of this operation has the following characteristics:

- Supports compilation of first-order like processing
- Attempts to exploit determinism in unification
- Provides an explicit representation for unification problems that supports sharing
- Has efficient mechanisms for realizing branching in unification

Higher-Order Pattern Unification

Decidability and unicity properties hold when existential variables are applied to distinct variables universally quantified within their scope.

For example,

$$\forall v \exists X \forall u \exists Z \forall w \, ((X \ u) = (v \ (Z \ w)))$$

has the solution

$$X \mapsto \lambda x (v \ (Y \ x))$$

$$Z \mapsto \lambda x (Y \ u)$$

where Y is a new variable existentially quantified at the same level as X.

Generating this substitution involves *pruning* and *raising* steps.

A new algorithm that does these steps on the fly has been developed.

Dealing with AUGMENT

Two new issues arise in a sequential implementation with a central program:

- Incremental programs changes must be modelled

Thus, solving the goal

$$D \texttt{ => } G$$

This involves adding and removing code

- Backtracking behavior requires old programs to be remembered

For example, consider the goal

$$(D_1 \texttt{ => } G_1(X)) \wedge (D_2 \texttt{ => } G_2(X))$$

Both code access and context switching must be efficient.

An Implementation Scheme

An efficient implementation can be realized using the following ideas:

- Represent program via an *implication point record* (IPR) containing
  - access function to new code layer
  - pointer to previous IPR
- Compile *AUGMENT* goals into creation and 'removal' of IPRs
- Maintain a special *program* register pointing to most recent IPR
- At choice point creation, store also the contents of program register

The scheme permits compilation of the antecedents of *AUGMENT* goals.

Bringing it All Together

The *Teyjus* system embodies a solution to all the problems and comprises three parts:

- An abstract machine that supports, low-level, λProlog relevant operations
- A compiler for translating to abstract machine programs
- A loader for realizing modularity notions with separate compilation

The abstract machine has been realized through a software emulator.

The entire system has been implemented in C.

Directions of Ongoing Research

- Improved support for modularity
- Compiled treatment of higher-order pattern unification based language
- Evaluation of choices in the representation of lambda terms
- Modularization of implementation technology

Resources

- The λProlog web page

  `http://www.cse.psu.edu/~dale/lProlog/`

- The *Teyjus* web page

  `http://teyjus.cs.umn.edu/`

- Papers providing the basis for this talk

  `http://www.cs.umn.edu/~gopalan/papers.html`