Pre-class quiz solutions

Math 5447, Mathematical Neuroscience, Fall 2008

1. Solve the equation $dx/dt = 1 - x$ with initial condition $x(0) = 3$.

$$t = \int \frac{dx}{1-x} = -\log 1 - x + C_1$$
$$1 - x = Ce^{-t}$$
$$x(t) = 1 - Ce^{-t}$$
$$3 = x(0) = 1 - C \Rightarrow C = -2$$
$$x(t) = 1 + 2e^{-t}$$

2. Given the system of equations $dx/dt = -x - y/2 + c$ and $dy/dt = x - y$, solve for the steady state as a function of c.

   Find steady states:

$$x + y/2 = c \text{ and } x = y \Rightarrow 3x/2 = c \Rightarrow x = y = 2c/3$$

   Steady state is $(x, y) = (2c/3, 2c/3)$.

   For stability, find eigenvalues of

$$A = \begin{bmatrix} -1 & -1/2 \\ 1 & -1 \end{bmatrix}$$

   They are $\lambda_{\pm} = -1 \pm i\frac{\sqrt{2}}{2}$.

   Since the real part of both eigenvalues is negative, the steady state is asymptotically stable.

3. Given the system of equations $dx/dt = -x + 10/(1 + 4y)$ and $dy/dt = x - y$, find all equilibria and determine their stability.

   Find steady states (equilibria):

$$x = 10/(1 + 4y) \text{ and } x = y \Rightarrow x(1 + 4x) = 10 \Rightarrow 4x^2 + x - 10 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{161}}{8}$$

   Steady states are $(x_1, y_1) = (-1 + \sqrt{161}, -1 + \sqrt{161})/8$ and $(x_2, y_2) = (-1 - \sqrt{161}, -1 - \sqrt{161})/8$.

   Jacobian matrix is

$$J = \begin{bmatrix} -1 & \frac{-40}{(1+4y)^2} \\ 1 & -1 \end{bmatrix}$$

   Plug in first steady state and calculate eigenvalues get $\lambda_{\pm} \approx -1 \pm 0.9241i$, so steady state is asymptotically stable.

   Plug in second steady state and calculate eigenvalues get $\lambda_{\pm} \approx -1 \pm 1.0822i$, so steady state is asymptotically stable.