Study guide for the second midterm

Math 5485, Fall 2008

1. Basic ideas (Chapter 1)
   (a) Order of convergence
   (b) Floating point numbers systems, arithmetic and roundoff error
   (c) Well-conditioned versus ill-conditioned
2. Rootfinding of scalar equations (Chapter 2)
   (a) Basic ideas
      - Multiplicity of roots
   (b) Bisection method, False position, Newton's method, Secant Method
      i. Requirements to guarantee convergence
      ii. Order of convergence (including requirements to achieve this)
      iii. Compute a few iterations and check convergence
      iv. Given word problem, formulate root problem and find root to tolerance
   (c) Fixed point iteration in general
      i. Requirements for existence of fixed point
      ii. Requirements to guarantee convergence fixed point iteration scheme
      iii. Conditions that determine order of convergence
      iv. Compute a few iterations and check convergence
      v. Appropriate stopping conditions
   (d) Accelerating convergence
      i. Aitken's Δ^2-Method and Steffensen's Method
         A. When they apply
         B. How well they accelerate
      ii. Restoring quadratic convergence to Newton's method
   (e) Roots of polynomial
      i. Polynomial deflation
      ii. Laguerre's method
3. Systems of equations (Chapter 3)
   (a) Basic linear algebra (such as in section 3.0)
   (b) Gaussian elimination
      i. Row operations
      ii. Operation count (and why better than Gauss-Jordan or multiplying by inverse)

iii. Partial pivoting and scaled partial pivoting

(c) LU decomposition

i. Via Gaussian elimination

ii. Via direct factorization

- Note that we did not cover how to do pivoting here, but in general, it is necessary.

iii. Know what special matrices don't require pivoting strategies.

iv. Cholesky decomposition

- Special case of direct factorization for symmetric positive definite matrices

v. Factorization of tridiagonal matrices

(d) Norms, error estimates, and condition numbers

i. Understand and be able to calculate l_2 and l_∞ vector and matrix norms.

ii. Predict error estimates from condition number.

(e) Iterative methods

i. Condition on iteration matrix for convergence.

ii. Understand when iterative methods may outperform direct methods.

iii. Basic ideas of Jacobi, Gauss-Seidel, and SOR method

- Don't worry about their convergence properties.

(f) Newton's method for nonlinear systems of equations

i. How to use it

ii. Why it's slow

4. Eigenvalues and eigenvectors

(a) Gerschgorin Circle Theorem

(b) Power method

i. Why it works in general

ii. How to calculate it (nonsymmetric and symmetric)

iii. Don't worry about detailed conditions for which it works

(c) Inverse power method

i. How it follows from power method

ii. Use it with Gerschgorin Circle Theorem or to find smallest eigenvalue.

(d) Deflation

i. How to transform matrix to remove eigenvalue

- Effect of this transformation on eigenvectors and other eigenvalues

ii. Wielandt Deflation and Hotelling Deflation

iii. Problems with using deflation to compute all eigenvalues.