Study guide
First midterm
Math 5535, Fall 2009

This guide should give you some ideas of important topics that could appear on the exam.

1. Basic ideas of periodic points (section 9.1).
   (a) Calculate orbits
   (b) Verify points are fixed, periodic, eventually fixed, or eventually periodic.
   (c) Determine the number of periodic orbits from information about the fixed points of f and its powers.

2. Graphical method of iteration (section 9.2)
   (a) Find fixed points from a graph
   (b) Determine dynamics and basins of attractions from graphical iteration (cobwebbing or stair stepping)
   (c) Understand and be able to use Theorem 9.2.4 to prove the basin of attraction must contain a certain interval.

3. Determine fixed points or periodic points and their stability (section 9.3).
   (a) Determine fixed points of f or its powers
   (b) Determine stability (attracting, repelling, etc.) based on the derivative of f or its powers.
   (c) Determine which fixed points of f^n must go together to form period-n orbits based on the derivative of f^n at those points. (See problems 9.3.11 or 9.3.12.)
   (d) Additional example problems: 9.3.4 or 9.3.6 or maybe even 9.3.8.
   (e) Addition food for thought: Can two consecutive fixed points of a continuous function be stable?
   (f) If $f'(x) = 1$ at a fixed points, what do you know about that fixed point's stability? What information would allow you resolve any ambiguity?

4. Schwarzian derivative (section 9.4)
   - You only need to know the basic idea. Be able to calculate it (you have a sheet of notes) and understand it means a basin of attraction must contain a critical point or extend to $+\infty$ or extend to $-\infty$.

5. Bifurcations (Section 9.5)
   (a) Understand the conditions for a tangential (saddle-node) bifurcation and for a period-doubling bifurcation, including how to use them to test for the existence of such bifurcations.

(b) Be able to explain the consequences of the bifurcation.

(c) In particular, be able to sketch what the bifurcation diagram must look like around the bifurcations point. The bifurcation diagram should look like 9.5.2 or 9.5.4.

(d) Be able to pick out a tangential (saddle-node) bifurcation from the behavior of the graph of f. Be able to pick out a period-doubling bifurcation from the behavior of the graph of f combined with the behavior of the graph of f^2.

(e) Here's an example problem to think about. Let $f(x) = ax^2 + x + b$ for parameters a and b.

   i. What are the fixed points of $f(x)$? The number of fixed points should depend on the values of a and b. So, you should break down your answer into cases depending on the values of a and b.

   ii. Analyze the stability (attracting, repelling, etc.) of each fixed point. It may be that the stability depends on the value of a and b. So, you can write down conditions for a and b for the fixed point to be attracting or repelling.

   iii. Let a be a fixed positive number (it might help to initially think of setting a to a particular value such as $a = 33$). Can you find a value of b where there is a saddle-node (tangential) bifurcation? Be sure to demonstrate that all conditions for the bifurcation are met.

   iv. Sketch a bifurcation diagram showing the fixed points of f (just for b and x near the bifurcation point).