Table 1. Minimal absolute values of discriminants of number fields of degree n with $2r_2$ complex conjugate fields.

n	r_2=0	r_2=1	r_2=2	r_2=3	r_2=4
1	1				
2	5	3			
3	49	23			
4	725	275	117		
5	14,641	4,511	1,609		
6	300,125	92,779	28,037	9,747	
7	20,134,393	2,306,599	612,233	184,607	
8	282,300,416	?	?	?	1,257,728

Table 2. Minimal root-discriminants of number fields of degree n with $2r_2$ complex conjugate fields.

n	r_2=0	r_2=1	r_2=2	r_2=3	r_2=4
1	1				
2	2.236	1.732			
3	3.659	2.844			
4	5.189	4.072	3.289		
5	6.809	5.381	4.378		
6	8.132	6.728	5.512	4.622	
7	11.051	8.110	6.710	5.653	
8	11.385	?	?	?	5.787

Table 3. Small root-discriminants of totally complex fields and the best known lower bounds. (For $n \leq 8$, the root-discriminants are known to be minimal for each degree.)

n	$D^{1/n}$	GRH bound	unconditional bound
2	1.732	1.722	1.722
4	3.289	3.263	3.254
6	4.622	4.592	4.557
8	5.787	5.734	5.659
10	6.793	6.726	6.600
14	8.426	8.371	8.122
20	10.438	10.270	9.805
32	13.181	12.912	12.002
48	15.472	15.225	13.772

Table 4. Small root-discriminants of totally real fields and the best known lower bounds. (For $n \leq 8$, the root-discriminants are known to be minimal for each degree.)

n	$D^{1/n}$	GRH bound	unconditional bound
1	1	0.997	0.997
2	2.236	2.225	2.223
3	3.659	3.630	3.610

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4	5.189	5.124	5.067
5	6.809	6.640	6.523
6	8.182	8.143	7.941
7	11.051	9.611	9.301
8	11.385	11.036	10.596
9	12.869	12.410	11.823

Table 5. Contributions of prime ideals and zeros to discriminant bounds for some fields. (See Section 6 for detailed explanation.)

D	n	r_2	deficiency	ideals	zeros	norms
20134393	7	0	0.1397	0.0761	0.0636	7,17,23,37,43
1257728	8	4	0.0092	0.0013	0.0079	16
1265625	8	4	0.0100	0.0074	16,16	
282300416	8	0	0.0311	0.0183	0.0128	16,41,47,47,47,47,49,49
309593125	8	0	0.0427	0.029	0.0398	3,41,41
9685993193	9	0	0.0364	0.0143	0.0221	27,31,41,43,67,67,73,73,79,79,79,79

REFERENCES

References are grouped by topic, and are arranged approximately chronologically within each group. For most areas only papers published after 1970 are listed. References to earlier ones can be found in the publications listed here, especially in the book of Narkiewicz [A1].

A - Books and surveys

1. W. NARKIEWICZ, *Elementary and Analytic Theory of Algebraic Numbers*, PWN-Polish Scientific Publishers, Warsaw 1974. MR 50# 268. (Very complete references for work before 1973.)

2. L. C. WASHINGTON, *Introduction to Cyclotomic Fields*, Springer 1982. MR 85g:11001.

3. J. MARTINET, *Méthodes géométriques dans la recherche des petits discriminants*, pp. 147-179, Séminaire Théorie des Nombres, Paris, 1983-84, C. Goldstein éd., Birkhäuser Boston, 1985. MR 88h:11083.

4. A. M. ODLYZKO, *Bounds for discriminants and related estimates for class numbers, regulators and zeros of zeta functions: a survey of recent results,* Sém. de Théorie des Nombres, Bordeaux 2 (1990), 119-141.

B - Analytic lower bounds for discriminants

1. H.M. STARK, *Some effective cases of the Brauer-Siegel theorem,* Invent. math. 23 (1974), 135-152. MR 49 # 7218.

2. H.M. STARK, *The analytic theory of algebraic numbers,* Bull. Am. Math. Soc. 81 (1975), 961-972. MR 56 # 2961.

3. A.M. ODLYZKO, *Some analytic estimates of class numbers and discriminants,* Invent. math. 29 (1975), 275-286. MR 51 # 12788.

4. A.M. ODLYZKO, *Lower bounds for discriminants of number fields,* Acta Arith. 29 (1976) 275–297. MR 53 # 5531.

5. A.M. ODLYZKO, *Lower bounds for discriminants of number fields. II.* Tohoku Math. J. 29 (1977), 209-216. MR 56 # 309.

6. A.M. ODLYZKO, *On conductors and discriminants,* pp. 377-407, *Algebraic Number Fields,* (Proc. 1975 Durham Symp.), A. Fröhlich, ed., Academic Press 1977. MR 56 #11961.

7. J.-P. SERRE, *Minorations de discriminants,* note of October 1975, published on pp. 240-243 in vol. 3 of Jean-Pierre SERRE, Collected Papers, Springer 1986.

8. A.M. ODLYZKO, *Discriminant bounds,* tables dated Nov. 29, 1976 (unpublished). Some of these bounds are included in Ref. B12.

9. G. POITOU, *Minorations de discriminants (d'après A.M. Odlyzko),* Séminaire Bourbaki, Vol. 1975/76 28ème année, Exp. No. 479, pp. 136-153, Lecture Notes in Math. #567, Springer 1977. MR 55 #7995.

10. G. POITOU, *Sur les petits discriminants,* Séminaire Delange-Pisot-Poitou, 18e année : (1976/77), Théorie des nombres, Fasc. 1, Exp. No. 6, 18pp., Secrétariat Math., Paris, 1977. MR 8li:12007.

11. F. DIAZ Y DIAZ, *Tables minorant la racine n-ième du discriminant d'un corps de degré n,* Publications Mathématiques d'Orsay 80.06. Université de Paris-Sud, Département de Mathématique, Orsay, 1980. 59 pp. MR 82i:12007. (Some of these bounds are included in Ref. B12.)

12. J. MARTINET, *Petits discriminants des corps de nombres,* pp. 151-193, Journées Arithmétiques 1980, J.V. Armitage, ed., Cambridge Univ. Press 1982. MR 84g:12009.

See also A3,D9,E4,E6,E10,E16.

C. Constructions of fields with small discriminants

1. H.W. LENSTRA, *Euclidean number fields of large degree,* Invent. math. 38 (1977), 237-254. MR 55#2836.

2. J. MARTINET, *Tours de corps de classes et estimations de discriminants,* Invent. math. 44 (1978), 65-73. MR 57 #275.

3. J. MARTINET, *Petits discriminants,* Ann. Inst. Fourier (Grenoble) 29, no 1 (1979), 159-170. MR 81h:12006.

4. R. SCHOOF, *Infinite class field towers of quadratic fields,* J. reine angew. Math. 372 (1986), 209–220. MR 88a:11121.

See also A3,B12.

D. Bounds for class numbers

1. J.M. MASLEY, *Odlyzko bounds and class number problems,* pp. 465-474, Algebraic Number Fields (Proc. Durham Symp., 1975), A. Fröhlich, ed., Academic Press 1977. MR 56 #5493.

2. J.M. MASLEY, *Class numbers of real cyclic number fields with small conductor,* Compositio Math. 37 (1978), 297-319. MR 80e:12005.

3. J.M. MASLEY, *Where are number fields with small class numbers?,* pp. 221-242, Number Theory, Carbondale 1979, Lecture Notes in Math. #751, Springer, 1979. MR 81f:12004.

4. J. HOFFSTEIN, *Some analytic bounds for zeta functions and class numbers,* Invent. math. 55 (1979), 37-47. MR 80k:12019.

5. J.M. MASLEY, *Class groups of abelian number fields,* pp. 475-497, Proc. Queen's Number Theory Conf. 1979, P. Ribenboim, ed., Queen's Papers in Pure and Applied Mathematics no. 54, Queen's Univ., 1980, MR 83f:12007.

6. J. MARTINET, *Sur la constante de Lenstra des corps de nombres,* Sém. Théorie des Nombres de Bordeaux 1979-1980, Exp. #17, 21 pp., UNIV. BORDEAUX 1980. MR 83b:12007.

7. F.J. van der LINDEN, *Class number computations of real abelian number fields,* Math. Comp. 39 (1982), 693-707. MR 84e:12005.

8. A. LEUTBECHER and J. MARTINET, *Lenstra's constant and Euclidean number fields,* Astérisque 94 (1982), 87-131. MR 85b:11090.

9. A. LEUTBECHER, *Euclidean fields having a large Lenstra constant,* Ann. Inst. Fourier (Grenoble) 35, no.2 (1985), 83-106. MR 86j:11107.

10. J. HOFFSTEIN and N. JOCHNOWITZ, *On Artin's conjecture and the class number of certain CM fields,* Duke Math. J., 59 (1989), 553-563.

11. J. HOFFSTEIN and N. JOCHNOWITZ, *On Artin's conjecture and the class number of certain CM fields-II,* Duke Math. J., 59 (1989), 565-584.

12. K. YAMAMURA, *Determination of imaginary abelian number fields with class number one,* Math. Comp. 62 (1994), 899-921.

13. K. YAMAMURA, *The maximal unramified extensions of the imaginary quadratic number fields with class number two,* J. Number Theory 60 (1996), 42-50.

14. K. YAMAMURA, *Maximal unramified extensions of imaginary quadratic number fields of small conductor,* to appear.

See also B1,B2,B5.

E. Bounds for regulators and norms of ideals in ideal classes

1. M. POHST, *Regulatorabschätzungen für total reelle algebraische Zahlkörper,* J. Number Theory 9 (1977) 459-492. MR 57 #268.

2. G. GRAS and M.-N. GRAS, *Calcul du nombre de classes et des unités des extensions abéliennes réelles de Q,* Bull. Sci. Math. 101 (2) (1977), 97-129. MR 58 #586.

3. M. POHST, *Eine Regulatorabschätzung,* Abh. Math. Sem. Univ. Hamburg 47 (1978), 95-106. MR 58 #16596.

4. R. ZIMMERT, *Ideale kleiner Norm in Idealklassen und eine Regulatorabschätzung,* Invent. math. 62 (1981), 367-380. MR 83g:12008.

5. G. POITOU, *Le théorème des classes jumelles de R. Zimmert,* Sém. de Théorie des Nombres de Bordeaux 1983-1984, Exp. #86b:11003.) (Listed in MR 86b:11003.)

6. J. OESTERLÉ, *Le théorème des classes jumelles de Zimmert et les formules explicites de Weil*, pp. 181-197, Sém. Théorie des Nombres, Paris 1983-84, C. Goldstein, ed., Birkhäuser Boston, 1985.

7. J. SILVERMAN, *An inequality connecting the regulator and the discriminant of a number field*, J. Number Theory 19 (1984), 437-442. MR 86c:11094.

8. T.W. CUSICK, *Lower bounds for regulators*, pp. 63-73 in Number Theory, Noordwijkerhout 1983, H. Jager, ed., Lecture Notes in Math. # 1068, Springer 1984. MR 85k:11052.

9. A.-M. BERGE and J. MARTINET, *Sur les minorations géométriques des régulateurs*, pp. 23-50, Séminaire Théorie des Nombres, Paris 1987-88, C. Goldstein, ed., Birkhäuser Boston, 1990.

10. E. FRIEDMAN, *Analytic formulas for regulators of number fields*, Invent. math., 98 (1989), 599-622.

11. M. POHST and H. ZASSENHAUS, *Algorithmic Algebraic Number Theory*, Cambridge Univ. Press., 1989.

12. R. SCHOOF and L.C. WASHINGTON, *Quintic polynomials and real cyclotomic fields with large class numbers*, Math. Comp. 50 (1988), 543-556.

13. A.-M. BERGÉ and J. MARTINET, *Notions relatives de régulateurs et de hauteurs*, Acta Arith. 54 (1989), 155-170. MR 90m:11167.

14. A.-M. BERGÉ and J. MARTINET, *Minorations de hauteurs et petits régulateurs relatifs*, Sém. Théorie des Nombres Bordeaux 1987-88, Exp.#11, Univ. Bordeaux 1988.

15. A. COSTA and E. FRIEDMAN, Ratios of regulators in totally real extensions of number fields, to be published.

16. E. FRIEDMAN and N.-P. SKORUPPA, *Explicit formulas for regulators and ratios of regulators of number fields*, manuscript in preparation.

F. Determination of minimal discriminants

1. P. CARTIER and Y. ROY, *On the enumeration of quintic fields with small discriminants*, J. reine angew. Math 268/269 (1974), 213-215. MR 50 # 2119.

2. M. POHST, *Berechnung kleiner Diskriminanten total reeller algebraischer Zahlkörper*. J. reine angew. Math. 278/279 (1975), 278-300. MR 52 # 8085.

3. M. POHST, *The minimum discriminant of seventh degree totally real algebraic number fields*, pp. 235-240, Number theory and algebra, H. Zassenhaus, ed., Academic Press 1977. MR 57 # 5952.

4. J. LIANG and H. ZASSENHAUS, *The minimum discriminant of sixth degree totally complex algebraic number field*, J. Number Theory 9 (1977), 16-35. MR 55 # 305.

5. M. POHST, *On the computation of number fields of small discriminants including the minimum discriminants of sixth degree fields*, J. Number Theory 14 (1982), 99-117. MR 83g:12009.

6. M. POHST, P. WEILER, and H. ZASSENHAUS, *On effective computation of fundamental units*, Math. Comp. 38 (1982), 293-329. MR 83e:12005b.

7. D.G. RISH, *On algebraic number fields of degree five*, Vestnik Moskov. Univ. Ser. I Mat. Mekh. (1982), no 2, 76-80. English translation in Moscow Univ. Math. Bull. 37 (1982), no. 99-103. MR 83g:12006.

8. F. DIAZ Y DIAZ, *Valeurs minima du discriminant des corps de degré 7 ayant une seule place réelle,* C.R. Acad. Sc. Paris 296 (1983), 137-139. MR 84i:12004.

9. F. DIAZ Y DIAZ, *Valeurs minima du discriminant pour certains types de corps de degré 7,* Ann. Inst. Fourier (Grenoble) 34, no 3 (1984), 29-38. MR 86d:11091.

10. K. TAKEUCHI, *Totally real algebraic number fields of degree 5 and 6 with small discriminant,* Saitama Math. J. 2(1984), 21-32. MR 86i:11060.

11. H.J. GODWIN, *On quartic fields of signature one with small discriminant. II,* Math. Comp. 42 (1984), 707-711. *Corrigendum,* Math. Comp. 43 (1984), 621. MR 85i:11092a, 11092b.

12. F. DIAZ Y DIAZ, *Petits discriminants des corps de nombres totalement imaginaires de degré 8,* J. Number Theory 25 (1987), 34-52.

13. S.-H. KWON and J. MARTINET, *Sur les corps résolubles de degré premier,* J. reine angew. Math. 375/376 (1987),12-23. MR 88g:11080.

14. F. DIAZ Y DIAZ, *Discriminants minima et petits discriminants des corps de nombres de degré 7 avec cinq places réelles,* J. London Math. Soc. (2) 38 (1988), 33-46.

15. J. BUCHMANN and D. FORD, *On the computation of totally real quartic fields of small discriminant,* Math. Comp. 52 (1989), 161-174.

16. S.-H. KWON, *Sur les discriminants minimaux des corps quaternioniens,* preprint 1987.

17. P. LLORENTE and J. QUER, *On totally real cubic fields with discriminant $D < 10^7$,* Math. Comp. 50 (1988), 581-594.

18. J. BUCHMANN, M. POHST and J. v. SCHMETTOW, *On the computation of unit groups and class groups of totally real quartic fields,* Math. Comp. 53 (1989), 387-397.

19. A.-M. BERGÉ, J. MARTINET and M. OLIVIER, *The computation of sextic fields with a quadratic subfield,* Math. Comp. 54 (1990), 869-884.

20. F. DIAZ Y DIAZ, *A table of totally real quintic number fields,* Math. Comp. 56 (1991), 801-808.

21. M. OLIVIER, *Corps sextiques contenant un corps quadratique (I),* Sém. de Théorie des Nombres, Bordeaux 1 (1990), 205-250.

22. M. OLIVIER, *Corps sextiques primitifs,* Ann. Inst. Fourier (Grenoble) 40 (1990), 757-767.

23. J. MARTINET, *Discriminants and permutation groups,* pp. 359-385, Number Theory, R. A. Mollin, ed., De Gruyter, 1990.

24. M. POHST, J. MARTINET and F. DIAZ Y DIAZ, *The minimum discriminant of totally real octic fields,* J. Number Theory 36 (1990), 145-159.

25. D. FORD, *Enumeration of totally complex quartic fields of small discriminant,* pp. 129-138, Computational Number Theory, A. Petho, M. Pohst, H. C. Williams, and H. G. Zimmer, eds., De Gruyter 1991.

26. M. OLIVIER, *The computation of sextic fields with a cubic subfield and no quadratic subfield,* Math. Comp. 58 (1992), 419-432.

27. J. BUCHMANN, D. FORD, and M. POHST, *Enumeration of quartic fields of small discriminant,* Math. Comp. 61 (1993), 873-879.

28. H. FUJITA, *The minimum discriminant of totally real algebraic number field of degree 9 with cubic subfields,* Math. Comp. 60 (1993), 801-810.

29. H. FUJITA, *The minimum discriminant of totally real algebraic number field of degree 9 with cubic subfields. II,* Saitama Math. J. 9 (1991), 9-18.

30. D. FORD and M. POHST, *The totally real A5 extension of degree 6 with minimum discriminant,* Experimental Math. 1 (1992), 231-235.

31. D. FORD and M. POHST, *The totally real A6 extension of degree 6 with minimum discriminant,* Experimental Math. 2 (1993), 231-232.

32. A. SCHWARZ, M. POHST, and F. DIAZ Y DIAZ, *A table of quintic number fields,* Math. Comp. 63 (1994), 361-376.

G. Small zeros of Dedekind zeta functions

1. J. HOFFSTEIN, *Some results related to minimal discriminants,* pp. 185-194, Number Theory, Carbondale 1979, Lecture Notes in Math. # 751, Springer 1979, MR 81d:12005.

2. A. NEUGEBAUER, *On zeros of zeta functions in low rectangles in the critical strip* (in Polish), Ph.D. Thesis, A. Mickiewicz University, Poznan, Poland, 1985.

3. A. NEUGEBAUER, *On the zeros of the Dedekind zeta-function near the real axis,* Funct. Approx. Comment. Math. 16 (1988), 165-167. MR 90b:11122.

4. A. NEUGEBAUER, *Every Dedekind zeta-function has a zero in the rectangle* $1/2 \leq \sigma \leq 1, 0 < t < 60$, Discuss. Math. 7 (1985), 141-144. MR 87i:11167.

5. A.M. ODLYZKO, *Low zeros of Dedekind zeta function,* manuscript in preparation.

H. Other related papers

1. J.-F. MESTRE, *Formules explicites et minorations de conducteurs de variétés algébriques,* Compositio Math. 58 (1986), 209-232. MR 87j:11059.

2. E. FRIEDMAN, *The zero near 1 of an ideal class zeta function,* J. London Math. Soc. (2) 35 (1987, 1-17. MR 88g:11087.

3. E. FRIEDMAN, *Hecke's integral formula,* Sém. Théorie des Nombres de Bordeaux 1987-88, Exp. #5, 23 pp., Univ. Bordeaux 1988.

I. Other papers cited in the text

1. E. LANDAU, *Zur Theorie der Heckeschen Zetafunktionen, welche komplexen Charakteren entsprechen,* Math. Zeit. 4 (1919), 152-162. Reprinted on pp. 176-186 of vol. 7, Edmund Landau : Collected Works, P.T. Bateman, et al., eds., Thales Verlag.

2. E. LANDAU, *Einführung in die elementare und analytische Theorie der algebraischen Zahlen und der Ideale,* 2nd ed., Göttingen, 1927. Reprinted by Chelsea, 1949.

3. R. REMAK, *Über die Abschätzung des absoluten Betrages des Regulators eines algebraischen Zahlkörpers nach unten,* J. reine angew. Math. 167 (1931), 360-378.

4. R.P. BOAS and M. KAC, *Inequalities for Fourier transforms of positive functions,* Duke Math. J. 12 (1945), 189-206, MR 6-265.

5. A.P. GUINAND, *A summation formula in the theory of prime numbers,* Proc. London Math. Soc. (2) 50 (1948), 107-119. MR 10, 104g.

6. A.P. GUINAND, *Fourier reciprocities and the Riemann zeta- function,* Proc. London Math. Soc. (2) 51 (1949), 401-414. MR 11, 162d.

7. A. WEIL, *Sur les "formules explicites" de la théorie des nombres premiers,* Comm. Sem. Math. Univ. Lund, tome supplémentaire (1952), 252-265, MR 14, 727e.

8. R. REMAK, *Über Grössenbeziehungen zwischen Diskriminante und Regulator eines algebraischen Zahlkörpers,* Compos. Math. 10 (1952), 245-285. MR 14, 952d.

9. R. REMAK, *Über algebraische Zahlkörper mit schwachem Einheitsdefekt,* Compos. Math. 12 (1954), 35-80. MR 16, 116a.

10. A. WEIL,*Sur les formules explicites de la théorie des nombres,* Izv. Akad. Nauk SSSR Ser. Mat. 36 (1972), 3-18. MR 52 # 345. Reprinted in A. Weil, Oeuvres Scientifiques, vol. 3, pp. 249-264, Springer 1979.

10. H.-J. BESENFELDER, *Die Weilsche "Explizite Formel" und temperierte Distributionen,* J. reine angew. Math. 293/294 (1977), 228-257. MR 57 # 254.

11. H.-J. BESENFELDER, *Die Weilsche "Explizite Formel" und temperierte Distributionen,* J. reine angew. Math. 293/294 (1977), 228-257. MR 57 #254.

12. J.-P. SERRE, note on p. 710 in vol. 3 of Jean-Pierre SERRE, Collected Papers, Springer 1986.

13. H. COHEN and H.W. LENSTRA, Jr., *Heuristics on class groups of number fields,* pp. 33-62 in Number Theory, Noordwijkerhout 1983, H. Jager, ed., Lecture Notes in Math. # 1068, Springer 1984. MR 85j:11144.

14. J.-P. SERRE, *Sur le nombre des points rationnels d'une courbe algébrique sur un corps fini,* C.R. Acad. Sci. Paris 296 (1983), ser. I, 397-402. MR 85b: 14027. Reprinted on pp. 658-663 in vol. 3 of Jean-Pierre Serre, Collected Papers, Springer 1986.

15. A.M. ODLYZKO and H.J.J. te RIELE, *Disproof of the Mertens conjecture,* J. reine angew. Math. 357 (1985), 138-160. MR 86m:11070.

16. J.-M. FONTAINE, *Il n'y a pas de variété abélienne sur* Z, Invent. math. 81 (1985), 515-538. MR 87g:11073.

17. H. COHEN and J. MARTINET, *Etude heuristique des groupes de classes des corps de nombres,* J. reine angew. Math. 404 (1990), 39-76. MR 91k:11097.

18. A. BOREL and G. PRASAD, *Finiteness theorems for discrete subgroups of bounded covolume in semi-simple groups,* Publ. Math. I.H.E.S. 69 (1989), 119-171.

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