CORRECTIONS TO FIRST PRINTING OF

Olver, P.J., *Equivalence, Invariants, and Symmetry*,

Last modified: January 4, 2021

*** On back cover, line 17–18, change
prospective geometry
to
projective geometry

*** page xv, add to acknowledgements

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*** page 22, Theorem 1.28, line 3, change
. . . all \( t, s \in \mathbb{R} \) where the equation is defined.
to
. . . all \( t, s \in V \) where \( V \subset \mathbb{R}^2 \) is a connected open subset of the \((t, s)\) plane containing \((0, 0)\) consisting of points where the equation is defined.

*** page 32, line 12-13, change
an (necessarily unique)
to
a (necessarily unique)

*** page 32, line before Definition 2.1, change
structure
to
structure

*** page 36, line before Example 2.9, change

\( \text{GL}(2) \)
to
\( \text{GL}(2, \mathbb{C}) \).

*** page 39, Example 2.13, change the first two occurrences of

\( \text{PSL}(n, \mathbb{R}) \)
to
\( \text{PGL}(n, \mathbb{R}) \).
Also append to the last sentence

\[ \text{PSL}(n, \mathbb{R}) = \text{SL}(n, \mathbb{R})/\{ \pm \mathbb{I} \} \] is equal to the connected component of \( \text{PGL}(n, \mathbb{R}) \) containing the identity.

\[ \text{PSL}(n, \mathbb{R}) = \text{SL}(n, \mathbb{R})/\{ \pm \mathbb{I} \} \]

\[ \text{PSL}(n, \mathbb{R}) = \text{SL}(n, \mathbb{R})/\{ \pm \mathbb{I} \} \]

page 51, equation (2.14), change

\[ C^k_{ij} = -C^k_{ij} \]

to

\[ C^k_{ji} = -C^k_{ij} \]

page 55, lines 4–5, change

\[ G_H = \{ g | gHg^{-1} \subset H \} \] has Lie algebra

to

\[ G_H = \{ g | gHg^{-1} \subset H \} \] is a normal subgroup with Lie algebra

page 61, line 31, change

there is a scalar function \( h_v(t) \) such that

to

there is a function \( h_v : \mathbb{R}^k \to \mathbb{R}^k \) such that

page 65, Example 2.80, line 8, change

\[ v(HF) = 0 \]

to

\[ v(H) = 0. \]

page 73, line 9, change

\[
\begin{pmatrix}
  a^{-1} da & a^{-1}(a db - b da) \\
  0 & 1
\end{pmatrix}
\]

to

\[
\begin{pmatrix}
  a^{-1} da & a^{-1} db \\
  0 & 0
\end{pmatrix}
\]

page 85, equation (3.18), change

\[ 1 + th_v(x) + \frac{1}{2} t^2 v(h_v) + \cdots \]

to

\[ 1 + th_v(x) + \frac{1}{2} t^2 [v(h_v) + h_v^2] + \cdots \]

page 87, equation (3.21), change

\[ \sigma([v,w]) = \hat{w}(\sigma(v)) - \hat{v}(\sigma(w)) \]

to

\[ \sigma([v,w]) = \hat{v}(\sigma(w)) - \hat{w}(\sigma(v)) \]
In order to formulate a general theorem governing the existence of relative invariants for sufficiently regular group actions, we consider the extended group action (3.15) on the bundle \(E = M \times U\) and its dual version \((x, v) \mapsto (g \cdot x, \mu(g, x)^{-T})\) on the dual bundle \(E^* = X \times U^*\). The key remark is that there is a one-to-one correspondence between relative invariants of weight \(\mu\) and linear absolute invariants of the dual action. Specifically, a linear function \(J(x, v) = \sum_{\alpha=1}^{n} R_{\alpha}(x) v^\alpha\) is an invariant of the dual action on \(E^*\) if and only if the vector-valued function \(R(x) = (R_1(x), \ldots, R_q(x))^T\) is a relative invariant of weight \(\mu\).

Therefore, we need only produce a sufficient number of linear invariants of the extended action. Moreover, if \(J(x, v)\) is any invariant of the extended group action, then it is not hard to prove that its linear Taylor polynomial is also an invariant, and hence provides a relative invariant for the multiplier representation. Thus, the only question is how many independent relative invariants can be constructed in this manner.

I do not know a general theorem that counts the number of relative invariants of multiplier representations that do not satisfy the hypotheses of Theorem 3.36.

A general theorem that counts the number of relative invariants of multiplier representations in all cases can be found in the recent paper by M. Fels and the author, “On relative invariants”, Math. Ann. 308 (1997), 701–732.

\[
\begin{align*}
v_0 &= -na_0 \frac{\partial}{\partial a_0} - (n-2)a_1 \frac{\partial}{\partial a_1} + \cdots + (n-2)a_{n-1} \frac{\partial}{\partial a_{n-1}} + na_n \frac{\partial}{\partial a_n}, \\
v_+ &= na_0 \frac{\partial}{\partial a_1} + (n-1)a_1 \frac{\partial}{\partial a_2} + \cdots + 2a_{n-2} \frac{\partial}{\partial a_{n-1}} + a_{n-1} \frac{\partial}{\partial a_n}. \\
v_- &= a_1 \frac{\partial}{\partial a_0} + 2a_2 \frac{\partial}{\partial a_1} + \cdots + (n-1)a_{n-1} \frac{\partial}{\partial a_{n-2}} + na_n \frac{\partial}{\partial a_{n-1}}.
\end{align*}
\]
*** page 108, line 24, change
\[ \cot \theta \neq a \]
to
\[ \cot t \neq a \]

*** page 110, Theorem 4.6, line 2, change
\( r \)-dimensional orbits
to
\( s \)-dimensional orbits

*** page 113, line 7, change
\[ \bar{z}_0 = (\bar{x}_0, \bar{u}_0^{(n)}) = (x_0, \bar{f}(x_0)) \]
to
\[ \bar{z}_0 = (\bar{x}_0, \bar{u}_0^{(n)}) = (x_0, \bar{f}(n)(x_0)) \]

*** page 119, equation (4.31), change
\[ \sum_{\#J \geq 0} \]
to
\[ \sum_{\#J = 0}^{n} \]

*** page 119, equation (4.32), change
\[ D_i. \]
to
\[ D_i^{(n)}, \]
and add the following sentence:
where \( D_i^{(n)} \) denotes the order \( n \) truncation of the \( i \)th total derivative, i.e., the summation in (4.18) is just over \( 0 \leq \#J \leq n \).

*** page 120, second line after equation (4.35), change
The Lie algebra (4.14)
to
The Lie algebra (4.35)

*** page 124, first displayed equation, add subscript \( i \) to \( Q \) in first summation
\[ \omega = \sum_{i=1}^{p} Q_i(x, u^{(n)}) \, dx^i + \sum_{\alpha=1}^{q} \sum_{\#J \leq n} P_{\alpha}^{i}(x, u^{(n)}) \, du_\alpha^J \]
page 126, line 12, change

\((\Psi^{(n)})^* \theta\)

to

\(\Psi^* \theta\)

page 142, line 28, change

\(s_0 = 1, s_1 = 2, \ldots, s_{r-3} = s_{r-2} = r - 1\)

to

\(s_0 = 2, s_1 = 3, \ldots, s_{r-3} = s_{r-2} = r - 1\)

page 144, line 10, change

\(a^\nu_{\mu} \xi^i_i\)

to

\(A^\nu_{\mu} \xi^i_i\)

page 148, equation (5.15), change

\(v_0 = x \frac{\partial}{\partial x} - \frac{n}{2} u \frac{\partial}{\partial u}, \quad v_+ = x^2 \frac{\partial}{\partial x} - n x u \frac{\partial}{\partial u}\)

to

\(v_0 = x \frac{\partial}{\partial x} + \frac{n}{2} u \frac{\partial}{\partial u}, \quad v_+ = x^2 \frac{\partial}{\partial x} + n x u \frac{\partial}{\partial u}\)

page 159, lines 5, 15 & 18, change

\(d_{n+1} K_1 \wedge \cdots \wedge d_{n+1} K_r\)

to

\(d_{n+1}[D K_1] \wedge \cdots \wedge d_{n+1}[D K_r]\)

page 171, lines 20 & -8, change

\(n + 2\)

to

\(n + 1\)

page 171, line -7 to -3, delete sentence

Moreover, if the stable ... have order at most \(n + 1\).

page 173, Example 5.52, line 2, after “... via the standard representation”, add

\((x, y, u) \mapsto (\alpha x + \beta y, \gamma x + \delta y, u), \text{ where } \alpha \delta - \beta \gamma = 1\)

page 174, add remark that the referenced formula for the curvature that appears in [106, p. 26] is not quite correct. The denominator should be raised to the power \(3/2\).
\*\*\* page 188, line -2, change
\[ \log x = h(u/x) \]
to
\[ \log x = h(u/x^m) \]
\*\*\* page 190, lines 8–9, change

\[ \text{H-reduced equationsymmetry reduced equation } \Delta/H = 0 \text{ admits the corresponding normalizer subgroup } G_H = \{ g | g \cdot H \cdot g^{-1} \subset H \} \text{ as a symmetry group.} \]
to
\[ \text{H-reduced equation } \Delta/H = 0 \text{ admits the quotient group } G_H/H, \text{ where } G_H = \{ g | g \cdot H \cdot g^{-1} \subset H \} \text{ is the normalizer subgroup, as a symmetry group.} \]

\*\*\* page 190, line 18, change
\[ \eta \partial_y + \zeta \partial_u + \zeta^y \partial_v \]
to
\[ \eta \partial_y + \zeta \partial_v + \zeta^y \partial_v \]

\*\*\* page 190, line 22, change
\[ v = \partial_y \]
to
\[ v = \partial_v \]

\*\*\* page 192, formula (6.32), change
\[ (1 + u_x)^{3/2} \]
to
\[ (1 + u_x^2)^{3/2} \]

\*\*\* page 192, displayed formula after (6.32), change
\[ (1 + \theta^2 \tau^2) \]
to
\[ (1 + \tau^2 \theta^2)^{3/2} \]

\*\*\* page 195, line -4, change

Alternatively, \( x = w_{uu}/w_u \), where \( w \) is an arbitrary solution . . .
to

Alternatively, \( w = x_{uu}/x_u \) is an arbitrary solution . . .

\*\*\* page 198, line 9, change
\[ y = f(x) \]
to
\[ w = f(x) \]
*** page 198, equation (6.56), change
\[ y \]
to
\[ w \]

*** page 201, equation (6.61), change
\[
\begin{vmatrix}
\xi_1 & \varphi_1 & 1 & \cdots & \varphi_1^{r-1} \\
\xi_2 & \varphi_2 & 1 & \cdots & \varphi_2^{r-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\xi_r & \varphi_r & 1 & \cdots & \varphi_r^{r-1}
\end{vmatrix}
\]

\[ = 0. \]
to
\[
\begin{vmatrix}
\xi_1 & \varphi_1 & 1 & \cdots & \varphi_1^{r-2} \\
\xi_2 & \varphi_2 & 1 & \cdots & \varphi_2^{r-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\xi_r & \varphi_r & 1 & \cdots & \varphi_r^{r-2}
\end{vmatrix}
\]

\[ = 0. \]

*** page 213, equation (6.84), change all p’s to f’s:
\[
\eta(x) = |f^n(x)|^{(1-n)/(2n)} \exp \left\{ \int_x^x f^{n-1}(y) dy \right\}. \quad (6.84)
\]

*** page 218, line -2, change
\[ f^k(x) = W^k(x) \]
to
\[ f^k(x) = (-1)^k W^k(x) \]

*** page 226, line 6, change
\[ P(t, x, u^{(2n)}) \]
to
\[ R(t, x, u^{(2n)}) \]

*** page 231, lines -4 & -1, change
\[ E(T) \]
to
\[ E(T) \]
Determine the conservation laws associated with the point symmetries found in Exercise 6.16.

since the precise connection between symmetries and conservation laws has not been discussed in this book. (See, however, [186].)

However, I do not know . . . I. Anderson, [7].


\[(x, v_y, v_{yy}, \ldots)\]
\[(y, v_y, v_{yy}, \ldots)\]

\[a_4 = 0\]
\[a_4 = a_5 = 0\]

\[\bar{a}_6 \omega^3 = a_6 \omega^3\]
\[\bar{a}_6 \bar{\omega}^3 = a_6 \omega^3\]

\[\tilde{\alpha}^\kappa = \sum_{j=1}^{r} z_{j}^\kappa(x) \theta^j\]
\[\tilde{\alpha}^\kappa = \sum_{j=1}^{m} z_{j}^\kappa \theta^j\]

\[\sum_{k=1}^{r} z_{j}^\kappa \theta^j\]
\[\sum_{j=1}^{m} z_{j}^\kappa \theta^j\]
*** page 309, equation (10.12), change
\[ \sum_{i=1}^{p} z_i^\kappa \theta^i \]
to
\[ \sum_{i=1}^{m} z_i^\kappa \theta^i \]

*** page 339, line 6, delete first arc length

*** page 341, line -3, change
\[ I_4 \]
to
\[ I_5 \]

*** page 349, line -12, change
\[ \alpha^1 - T_{12}^1 \theta^1 \wedge \theta^2 - T_{13}^1 \theta^1 \wedge \theta^3 \]
to
\[ \alpha^1 - T_{12}^1 \theta^2 - T_{13}^1 \theta^3 \]

*** page 367, line 10, change
manifolds \( M \)
to
manifolds \( M \) and \( \overline{M} \)

*** page 368, equation (11.30), change
\[ = T \omega^1 \wedge \omega^2 \wedge \omega^3 = T\Omega. \]
to
\[ = T \omega^1 \wedge \omega^2 \wedge \omega^3. \]

*** page 372, lines 13–16, change

However, I do not know any naturally occurring examples exhibiting this phenomenon, and, moreover, the prolongation procedure to be discussed below will handle this (remote) possibility as well.)
to

However, the prolongation procedure to be discussed below will handle this possibility as well; an example is the equivalence problem for a parabolic evolution equation analyzed in [69].)
page 375, line 5, change (12.3) to (12.1)

page 394, lines 16 & 21, change (11.6) to (11.7)

page 394, line 22, change vector S to matrix S

page 395, equation (12.52), change
\[ w = \alpha + S \theta, \quad \text{or explicitly,} \quad w^i = \alpha^i + \sum_{j=1}^{m} S^i_j \theta^j \]
to
\[ w = \alpha - S \theta, \quad \text{or explicitly,} \quad w^i = \alpha^i - \sum_{j=1}^{m} S^i_j \theta^j \]

page 406, equation (12.73), change
\[ Q_p \hat{D}_x Q_{pp} 6Q_{uu} \]
to
\[ Q_p \hat{D}_x Q_{pp} + 6Q_{uu} \]

page 411, lines 12–13, change
\[ c(x, y, \varphi(x, y)) \frac{\partial \varphi}{\partial x} = a(x, y, \varphi(x, y)), \]
\[ c(x, y, \varphi(x, y)) \frac{\partial \varphi}{\partial y} = b(x, y, \varphi(x, y)). \]
to
\[ c(x, y, \varphi(x, y)) \frac{\partial \varphi}{\partial x} = -a(x, y, \varphi(x, y)), \]
\[ c(x, y, \varphi(x, y)) \frac{\partial \varphi}{\partial y} = -b(x, y, \varphi(x, y)). \]
page 423, equation (14.4), change

\[ \Phi(t, w) \]
to

\[ \Phi(t, s) \]

There is, however, a four-parameter group action obtained by including the additional generator \( z \partial_y \), whose associated one-parameter group \( (x, y, z) \mapsto (x, y + \mu z, z) \) can be recovered from the previous group transformations by taking commutators.

Moreover, one cannot include these vector fields in a finite-dimensional Lie algebra, since \([v_2, v_3] = v_4 = z \partial_y\), \([v_4, v_3] = v_5 = z^2 \partial_y\), and so on, hence the successive commutators span an infinite-dimensional Lie algebra of vector fields.

Relative invariants correspond to linear invariants \( J(x, u) = R(x) \cdot u = \sum_{\alpha=1}^{q} R_\alpha(x) u^\alpha \) of the extended action, . . .

Relative invariants of the dual action on \( E^* = X \times U^* \) correspond to linear invariants \( J(x, u) = \sum_{\alpha=1}^{q} R_\alpha(x) u^\alpha \) of the extended action, . . .

page 437, line -9, change

... the rank zero case in Theorem 4.24.

to

... the rank zero case in Theorem 4.18.

page 442, Figure 5, change

L

to

M

page 446, line 5, change

... restrictions of \( \theta \) to \( U \) and \( V \), so that

to

... restrictions of \( \theta \) to \( U \) and \( \tilde{U} \), so that

page 472, Table 1, Case 1.8, column 3, change

\( a(1) \times \mathbb{C}^k \)

to

\( \mathbb{C} \times (\mathbb{C} \times \mathbb{C}^k) \)
*** page 475, Table 6, Case 6.1:

It would be better to replace $\alpha$ by $\beta$ in this entry. A good exercise is to determine the relation between $\alpha$ and $\beta$ in the complex equivalence between Case 6.1 and Case 1.7 (for $k = 1$).

*** page 475, Table 6, Cases 6.2 and 6.3, column 5, change both from

1.1
to
1.2

*** pages 477, 478, 480, 484, 486 & 487, update the following references:


*** page 479, refs [37–38], change

Complètes
to
Complètes

*** page 479, refs [37–42], change

Gauthiers
to
Gauthier

*** page 483, reference [128], change

$dx/dy$
to
$dy/dx$
*** page 500, change
Galois, E., 3
to
Galois, E., 4

*** page 501, change
Morikawa, H., 217, [170–172]
to
Morikawa, H., 217, [170]
Morrey, C.B., Jr., 346, [171]
Mostow, G.D., 41, 61, [172]

*** page 503, add the following to the end of the Author Index.
Zhitomirskii, M.Y., 31, [232], [233]

*** page 504, change two entries
affine-invariant arc length, 339
to
affine-invariant arc length, 241, 339