Applied Linear Algebra

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Corrections to Student Solution Manual

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1.2.4 (*d*) $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -1 & 3 \\ 3 & 0 & -2 \end{pmatrix}, \;\; \mathbf{x} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \;\; \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$

(*f*) $\mathbf{b} = \begin{pmatrix} -3 \\ -5 \\ 2 \\ 1 \end{pmatrix}.$

1.4.15 (a) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$

1.8.4 (*i*) $a \neq b$ and $b \neq 0$; (*ii*) $a = b \neq 0$, or $a = -2, \; b = 0$; (*iii*) $a \neq -2, \; b = 0$.

1.8.23 (*e*) $(0, 0, 0)^T$;

2.5.5 (*b*) $\mathbf{x}^\star = (1, -1, 0)^T, \quad \mathbf{z} = z\left(-\frac{2}{7}, -\frac{1}{7}, 1\right)^T$;

2.5.42 True. If $\ker A = \ker B \subset \mathbb{R}^n$, then both matrices have n columns, and so $n - \operatorname{rank} A = \dim \ker A = \dim \ker B = n - \operatorname{rank} B$.

3.4.22 (*v*) Change “null vectors” to “null directions”.

4.4.27 (a) Change “the interpolating polynomial” to “an interpolating polynomial”.

4.4.52 (*b*) $z = \frac{3}{5}(x - y)$.

(The solution given is for the square $S = \left\{ 0 \leq x \leq 1, \; 0 \leq y \leq 1 \right\}$.)

5.1.14 One way to solve this is by direct computation. A more sophisticated approach is to apply the Cholesky factorization (3.70) to the inner product matrix: $K = M M^T$. Then, $\langle \mathbf{v} \,;\mathbf{w} \rangle = \mathbf{v}^T K \mathbf{w} = \widehat{\mathbf{v}}^T \widehat{\mathbf{w}}$ where $\widehat{\mathbf{v}} = M^T \mathbf{v}$, $\widehat{\mathbf{w}} = M^T \mathbf{w}$. Therefore, $\mathbf{v}_1, \mathbf{v}_2$ form an orthonormal basis relative to $\langle \mathbf{v} \,;\mathbf{w} \rangle = \mathbf{v}^T K \mathbf{w}$ if and only if $\widehat{\mathbf{v}}_1 = M^T \mathbf{v}_1$, $\widehat{\mathbf{v}}_2 = M^T \mathbf{v}_2$, form an orthonormal basis for the dot product, and hence of the form determined in Exercise 5.1.11. Using this we find:
(a) $M = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{2} \end{pmatrix}$, so $\mathbf{v}_1 = \begin{pmatrix} \cos\theta \\ \frac{1}{\sqrt{2}} \sin\theta \end{pmatrix}$, $\mathbf{v}_2 = \pm \begin{pmatrix} -\sin\theta \\ \frac{1}{\sqrt{2}} \cos\theta \end{pmatrix}$, for any $0 \le \theta < 2\pi$.

5.4.15 $p_0(x) = 1, \quad p_1(x) = x, \quad p_2(x) = x^2 - \frac{1}{3}, \quad p_4(x) = x^3 - \frac{9}{10}\,x.$
(The solution given is for the interval $[0,1]$, not $[-1,1]$.)

5.5.6 (*ii*) (*c*) $\begin{pmatrix} \frac{23}{43} \\ \frac{19}{43} \\ -\frac{1}{43} \end{pmatrix} \approx \begin{pmatrix} .5349 \\ .4419 \\ -.0233 \end{pmatrix}.$

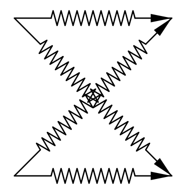
5.6.20 (*c*) The solution corresponds to the revised exercise for the system

$$x_1 + 2x_2 + 3x_3 = b_1, \quad x_2 + 2x_3 = b_2, \quad 3x_1 + 5x_2 + 7x_3 = b_3, \quad -2x_1 + x_2 + 4x_3 = b_4.$$

For the given system, the cokernel basis is $(-3,1,1,0)^T$, and the compatibility condition is $-3b_1 + b_2 + b_3 = 0$.

5.7.2 (a) (*i*) and 5.7.2 (*c*) (*i*) To avoid any confusion, delete the superfluous last sample value in the first equation, which become (*a*) (*i*) $f_0 = 2$, (*c*) (*i*) $f_0 = 6$,

6.2.1 (*b*) The solution given in the manual corresponds to the revised exercise with incidence matrix $\begin{pmatrix} 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$. For the given matrix, the solution is



6.3.5 (*b*)

$$\begin{aligned} \tfrac{3}{2} u_1 - \tfrac{1}{2} v_1 - u_2 &= f_1, \\ -\tfrac{1}{2} u_1 + \tfrac{3}{2} v_1 &= g_1, \\ -u_1 + \tfrac{3}{2} u_2 + \tfrac{1}{2} v_2 &= f_2, \\ \tfrac{1}{2} u_2 + \tfrac{3}{2} v_2 &= g_2. \end{aligned}$$

8.3.21 (a) $\begin{pmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{8}{3} & \frac{1}{3} \end{pmatrix}$, 8.5.26 (b) should be 8.5.26 (c).

11.2.8 (d) $$f'(x) = 4\,\delta(x+2) + 4\,\delta(x-2) + \begin{cases} 1, & |\,x\,| > 2, \\ -1, & |\,x\,| < 2, \end{cases}$$
$$= 4\,\delta(x+2) + 4\,\delta(x-2) + 1 - 2\,\sigma(x+2) + 2\,\sigma(x-2),$$
$$f''(x) = 4\,\delta'(x+2) + 4\,\delta'(x-2) - 2\,\delta(x+2) + 2\,\delta(x-2).$$

11.2.31 (a)
$$u_n(x) = \begin{cases} x\,(1-y), & 0 \le x \le y - \frac{1}{n}, \\ -\frac14\,n\,x^2 + \left(\frac12\,n - 1\right)x\,y - \frac14\,n\,y^2 + \frac12\,y + \frac12\,x - \frac{1}{4\,n}, & |\,x - y\,| \le \frac{1}{n}, \\ y\,(1-x), & y + \frac{1}{n} \le x \le 1. \end{cases}$$

11.3.3 (c) (i) $u_\star(x) = \frac12\,x^2 - \frac52 + x^{-1}$,

(ii) $\mathcal{P}[\,u\,] = \displaystyle\int_1^2 \left[\,\tfrac12\,x^2\,(u')^2 + 3\,x^2\,u\,\right] dx, \qquad u'(1) = u(2) = 0,$

(iii) $\mathcal{P}[\,u_\star\,] = -\frac{37}{20} = -1.85,$

(iv) $\mathcal{P}[\,x^2 - 2\,x\,] = -\frac{11}{6} = -1.83333, \quad \mathcal{P}\big[\,-\sin\frac12\,\pi x\,\big] = -1.84534.$

11.5.7 (b) For $\lambda = -\omega^2 < 0$, $$G(x,y) = \begin{cases} \dfrac{\sinh\omega\,(y-1)\,\sinh\omega\,x}{\omega\sinh\omega}, & x < y, \\ \dfrac{\sinh\omega\,(x-1)\,\sinh\omega\,y}{\omega\sinh\omega}, & x > y; \end{cases}$$

for $\lambda = 0$, $$G(x,y) = \begin{cases} x(y-1), & x < y, \\ y(x-1), & x > y; \end{cases}$$

for $\lambda = \omega^2 \ne n^2\pi^2 > 0$, $$G(x,y) = \begin{cases} \dfrac{\sin\omega\,(y-1)\,\sin\omega\,x}{\omega\sin\omega}, & x < y, \\ \dfrac{\sin\omega\,(x-1)\,\sin\omega\,y}{\omega\sin\omega}, & x > y. \end{cases}$$

11.5.9 (c) (ii) Replace $\displaystyle\int_a^b$ by $\displaystyle\int_1^2$.

