Corrections to the Corrected Printing and Paperback Edition of

Olver, P.J., *Applications of Lie Groups to Differential Equations*,

Last updated: February 21, 2023

*** On page 10, line 6, change
\[ \phi \circ \tilde{\phi}^{-1} : \mathbb{R} \to \mathbb{R} \]
to
\[ \phi^{-1} \circ \tilde{\phi} : \mathbb{R} \to \mathbb{R} \]

*** On page 19, line 17, change
\[ x \in V_0 = \{ x : |x| < \frac{1}{2} \} \]
to
\[ x \in V_0 = \{ -1 < x < \frac{1}{3} \} \]

*** On page 36, line 9, change
\[ \varepsilon, \theta \in V \]
to
\[ (\varepsilon, \theta) \in V \]

*** On page 43, line 23, change
smooth vector fields
to
smooth vector fields

*** On page 52, Example 1.58, change
Lie proved that

to
Lie proved, [4], that

*** On page 67, line 15, change
sort

to
short

*** On page 70, Exercise 1.8, change
in polar coordinates,

to
in polar coordinates with \( r > 0 \),
On page 72, Exercise 1.24(b), change

\[ h \subset g \] has the property

to

\[ h \subset g \] is a normal subalgebra (or ideal), meaning that it has the property

On pages 79–81, the ends of the proofs of Proposition 2.6 and Theorem 2.8 that refer back to (1.40) are subtly flawed, in that, under our definition of a local group action, even when \( g = g_1 \cdot g_2 \) with \( g, g_1, g_2 \in G_x \), there is no guarantee that \( g_1 \in G_{g_2 \cdot x} \), i.e., that \( g_1 \cdot (g_2 \cdot x) \) is defined even though \( g \cdot x = (g_1 \cdot g_2) \cdot x \) is defined.

However, a more direct argument, that avoids this difficulty, is to replace the end of the proof of Proposition 2.6 by the following:

\[ \vdots \text{Conversely, if (2.1) holds everywhere, then} \]

\[ \frac{d}{d\varepsilon} \zeta(\exp(\varepsilon v) x) = 0 \]

where defined, and hence \( \zeta(\exp(\varepsilon v) x) = \zeta(x) = c \) is constant for \( \varepsilon \) in the connected interval containing 0 in \( \{ \varepsilon \in \mathbb{R} \mid \exp(\varepsilon v) \in G_x \} \). Using the fact that the exponential map is a local diffeomorphism from a neighborhood of 0 in \( g \) to a neighborhood of \( e \in G_x \), we conclude that \( \zeta(g \cdot x) = c \) for all \( g \) in an open neighborhood of the identity in \( G_x \). Now, set \( \tilde{G}_x = \{ g \in G_x \mid \zeta(g \cdot x) = c \} \). Applying the preceding argument at the point \( g \cdot x \) for any \( g \in \tilde{G}_x \) proves that \( \tilde{G}_x \) is open, while continuity proves that it is closed in \( G_x \). Thus, by connectivity, \( \tilde{G}_x = G_x \), and the result follows. \( \square \)

Similarly, replace the end of the proof of Theorem 2.8 by:

\[ \vdots \text{We have thus shown that if } x_0 \text{ is a solution to } F(x) = 0, \text{ and } v \text{ is an infinitesimal generator of } G, \text{ and } \varepsilon \text{ is sufficiently small, then } \exp(\varepsilon v) x_0 \text{ is also a solution. As in the proof of Proposition 2.6, one then shows that } \tilde{G}_x = \{ g \in G_x \mid F(g \cdot x_0) = 0 \} \text{ is both open and closed in } G_x, \text{ and hence, by connectivity, } \tilde{G}_x = G_x. \] \( \square \)

- Thanks to Colin James Stockdale Klaus for correspondence on this point.

On page 83, line 14, change

follow
to

following

On page 93, line 8, change

functon
to

function
*** On page 106, line -5, the term inside the parentheses should be $\Xi_\varepsilon(x)$. Thus, the equation should read
\[
\sum_l \frac{\partial^2 \Xi_{\varepsilon}^k}{\partial \tilde{x}_j \partial \tilde{x}_l} (\Xi_\varepsilon(x)) \frac{d \Xi_l}{d \varepsilon}(x) = 0,
\]

*** On page 111, line 3, change $J \tilde{f}_\varepsilon(x)$ to $J \tilde{f}_\varepsilon(\tilde{x})$

*** On page 123, line 5, change $\tau \frac{\partial}{\partial \tau}$ to $\tau \frac{\partial}{\partial t}$

*** On page 151, line after (2.110), change normal subalgebra of $\mathfrak{g}$ to normal subalgebra (ideal) of $\mathfrak{g}$

*** On page 163, line -3, change differential equations to differential equations

*** On page 167, line 2, change $\left(\begin{array}{c} p+k-1 \\ k \end{array}\right) \cdot l$ to $\left(\begin{array}{c} p+k \\ k \end{array}\right) \cdot l$

*** On page 167, Definition 2.83, change differential equations to differential equations

*** On page 169, line -9, change Exercise 2.32 to Exercise 2.33

• Thanks to Mariano Hermida de La Rica for the preceding four corrections.
On page 187, line 8, change

\[ \frac{\partial \delta}{\partial \hat{x}} + \frac{\partial \delta}{\partial y} + \frac{\partial \delta}{\partial v} \frac{\partial \eta}{\partial x} \]

to

\[ \frac{\partial \delta}{\partial \hat{x}} \frac{\partial}{\partial \hat{x}} + \frac{\partial \delta}{\partial y} \frac{\partial \eta}{\partial x} + \frac{\partial \delta}{\partial v} \frac{\partial \eta}{\partial x} \]

Thanks to Wen-xiu Ma and his class for catching this and several other errors.

On page 202, lines 13–14, change

linear system of ordinary differential equations,

to

linear, constant coefficient system of ordinary differential equations,

On page 207, line -2, change

Example 2.64

to

Example 2.44

On page 212, on both line 12 and line 13, change

\[ M/G \]

to

\[ M/G^2 \]

On page 214, line 13, change

\[ \tilde{F}(x) = F[\pi(x)] \]

to

\[ F(x) = \tilde{F}[\pi(x)] \]

On page 215, line -1, change

\[ \tilde{F} = \tilde{F}(\pi_1, \ldots, \pi_{m-s}) \]

to

\[ \tilde{F} = \tilde{F}(\pi_1, \ldots, \pi_{m-s}) \]
*** On page 217, line 6, change
\[ F(R, K) = 0 \]
to
\[ \hat{F}(R, K) = 0 \]

*** On page 238, Exercise 3.7, change the formula to
\[ R = \left( \frac{t^2 E}{p_0} \right)^{1/5} h \left[ p_0 \left( \frac{t^6}{p_0^4 E^2} \right)^{1/5} \right] \]

- Thanks to Kameron Decker Harris for spotting this.

*** On page 273, line -7, change
technique
to

technique

*** On page 280, in the table, change
\[ I_x = x D - y A + \frac{1}{2} x u u_t + t M_x \]
to
\[ I_x = x D + y A + \frac{1}{2} x u u_t + t M_x \]

- Thanks to Gehrt Hartjen for checking through this table and the table on page 340 in his Mathematics Diplomarbeit in Aachen, 2001.

*** On page 290, line -4, change
Exercise 2.33
to
Exercise 2.35

*** On page 331, insert minus sign after equals sign in second displayed equation:
\[ D^*_\Delta Q = -q_t - q_{xx} + u q_x \]

*** On page 340, in line 4 of the table, change
\[ -y u_{xxx} + x u_{xyy} + u_{xy} \]
to
\[ -y u_{xxx} + x u_{xyy} + u_{xy} \]
On page 340, in line 5 of the table, change
\[ u_{xx}(yu_{yt} + \frac{1}{2} u_t) - u_{yy}(xu_{xt} + \frac{1}{2} u_t) \]
to
\[-u_{xx}(yu_{yt} + \frac{1}{2} u_t) + u_{yy}(xu_{xt} + \frac{1}{2} u_t)\]

On page 350, lines -8 to -7, change
the their Fréchet
to
their Fréchet

On page 364, Theorem 5.92, first line, change
\[ P[u] \in A^p \]
to
\[ P[u] \in A^q \]
• Thanks to Thomas von Schroeter for spotting this.

On page 366, line -7, change
\[ J \setminus I \]
to
\[ I \setminus J \]
• Thanks to Rob Thompson for catching this and several other errors.

On page 381, Exercise 3.16a:
The system does not, in fact have a recursion operator, although there is a recursive
formula for generating the higher order symmetries. On the other hand, the related system
\[ u_t = u_{xx} + v^2, \quad v_t = v_{xx}, \]
does admit a recursion operator. Details can be found in: Beukers, F., Sanders, J.A., and
396-408.

On page 381, Exercise 3.16b:
A proof that the Bakirov system has only one generalized symmetry can now be found
in: Beukers, F., Sanders, J.A., and Wang, J.P., One symmetry does not imply integrability,
On page 387, Exercise 5.51, change

homogeneous system of differential equations
to
homogeneous, totally nondegenerate system of differential equations

Also, delete the first sentence in the Hint, which is not valid when the conservation law is not polynomial.

- Thanks to Kostya Druzhkov for correspondence on this exercise.

On page 397, equation (6.17), change closing } to ) :

\[ \{ F, H \}(x) = \langle x; [ \nabla F(x), \nabla H(x) ] \rangle, \quad x \in \mathfrak{g}^* , \tag{6.17} \]

On page 420, line -8, change

procedure.
to
procedure.

On page 427, lines 23–24, change

Recently, Weinstein, [3], proposed the less historically motivated term “Casimir function” for these objects, . . .
to


- Thanks to Phil Morrison for alerting me to these earlier references.

On page 430, Exercise 6.16:

One needs to assume that \( \frac{\partial^2 L}{\partial u_n^2} \neq 0 \) in order to (locally) solve the final equation for \( u_n \) in terms of \( p, q \).

- Thanks to Thomas von Schroeter for alerting me to this omission.

On page 469, change title of second reference by H. Bateman to

On dissipative systems and related variational principles

On page 501, change

adjoint 314, 465
to

adjoint 314, 329, 343, 465