Math 5587
September 14, 2004

Homework Assignment # 1

Exercises: Strauss p. 9, #1, 3, 7 (assuming a, b, c are constant.)

1. (a) Find the characteristic curves for the differential equation $u_t + (x^2 - 1)u_x = 0$.
(b) Write down the solution to the initial value problem $u(0, x) = e^{-x^2}$.
(c) Graph the solution at $t = 1, t = 10$ and $t = 100$. Explain what you observe.

2. Explain how the sign of the wave velocity influences the direction of wave motion.

3. (a) Prove that the characteristic curves of the transport equation $u_t + c(x)\, u_x = 0$ equation cannot cross each other. (b) Prove that if $x = g(t)$ is a characteristic curve, then so are the translated curves $x = g(t + \delta)$ for any δ. (c) Prove that each non-vertical characteristic curve is the graph of a monotone function. (d) Explain why a wave cannot reverse its direction.

4. Let us solve the transport equation $u_t + c(t, x)\, u_x = 0$ with time-varying wave speed. The corresponding characteristic equation is $\dfrac{dx}{dt} = c(t, x)$ and its solutions $x(t)$ define the characteristic curves of the partial differential equation. (a) Prove that any solution $u(t, x)$ is constant on the characteristic curves. (b) Let $\xi(t, x) = k$, where k is an arbitrary constant, be the general solution to the characteristic equation. Prove that $u(t, x) = p(\xi(t, x))$, where $p(s)$ is any differentiable function, defines a solution to the transport equation. (c) Solve the initial value problem $u_t + (2t - 1)\, u_x = 0$, $u(0, x) = \dfrac{1}{1 + x^2}$. Explain what happens to the solution as t increases.

Due: Tuesday, September 21

Text: Walter A. Strauss, *Partial Differential Equations: an Introduction*, John Wiley & Sons, New York, 1992.