Math 5588
February 10, 2005

Homework Assignment # 2

1. (*a*) Find two linearly independent power series solutions to the differential equation

$$u'' + x\,u + 2\,u = 0 \qquad \text{at the point} \qquad x_0 = 0.$$

(*b*) Write down the power series solution to the initial value problem $u(0) = 1, u'(0) = -1$. (*c*) What is the radius of convergence of your power series solution in part (*b*)? Can you justify this? (*d*) Let $p_n(x)$ be the polynomial of degree n obtained by summing the first n terms in the power series solution from part (*b*). Graph $p_2(x), p_4(x)$ and $p_6(x)$. Compare their graphs with the actual solution. How large does n need to be so that $p_n(x)$ is a reasonable approximation to $u(x)$ on the interval $-2 \le x \le 2$?

2. Find the general real solution to the differential equations
(a) $x^2u'' + 3\,x\,u' + 5\,u = 0$, (*b*) $(x-1)^2u'' + 8\,(x-1)\,u' + 12\,u = 0$.

3. Consider the differential equation

$$2\,x\,u'' + u' + x\,u = 0.$$

(*a*) Prove that $x = 0$ is a regular singular point; (*b*) Find two independent series solutions in powers of x.

4. The Chebyshev differential equation is

$$(1-x^2)u'' - x\,u' + m^2u = 0.$$

(*a*) Classify all $x_0 \in \mathbb{R}$ as to (*i*) regular point; (*ii*) regular singular point; (*iii*) irregular singular point. (*b*) Show that, if m is an integer, the equation has a polynomial solution of degree m, known as a Chebyshev polynomial. Write down the Chebyshev polynomials of degrees $1, 2$ and 3. (*c*) For $m = 1$, find two linearly independent solutions around the point $x_0 = 1$.

Due: Tuesday, February 22

Text: Walter A. Strauss, *Partial Differential Equations: an Introduction*, John Wiley & Sons, New York, 1992.