Math 3592H Honors Math I
Midterm exam 1, Thursday October 6, 2016

Instructions:
50 minutes, closed book and notes, no electronic devices.
There are four problems, worth a total of 100 points.

1. (48 points total; 8 points each part)
For these vectors in $\mathbb{R}^3$,

$$\overline{u} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \overline{v} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \overline{w} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix},$$

compute the following via dot and cross products. Your answers are allowed to contain unevaluated inverse trigonometric functions.

(i) The length of $\overline{u}$.

(ii) The angle between $\overline{u}, \overline{v}$.

(iii) The length of the projection of $\overline{v}$ orthogonally (perpendicularly) onto the line spanned by $\overline{u}$.

(iv) The area of the parallelogram in $\mathbb{R}^3$ spanned by $\overline{u}$ and $\overline{v}$, that is, having vertices $\{\overline{0}, \overline{u}, \overline{v}, \overline{u} + \overline{v}\}$.

(v) Some vector in $\mathbb{R}^3$ orthogonal (perpendicular) to both $\overline{u}$ and $\overline{v}$.

(vi) The volume of the parallelepiped (slanted box) in $\mathbb{R}^3$ spanned by $\overline{u}, \overline{v}, \overline{w}$, that is, having vertices $\{\overline{0}, \overline{u}, \overline{v}, \overline{w}, \overline{u}+\overline{v}, \overline{u}+\overline{w}, \overline{v}+\overline{w}, \overline{u}+\overline{v}+\overline{w}\}$.

2. (21 points total; 7 points each part)

Assuming that $f(x) = \sin(x)$ is continuous, and $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$, compute with proof and/or explanations the values of the following limits of functions $g : \mathbb{R}^3 \to \mathbb{R}$ as $\overline{\mathbf{x}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ approaches $\overline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

(i)

$$\lim_{\overline{\mathbf{x}}\to\overline{0}} 3x^2 + 5y + z$$

(ii)

$$\lim_{\overline{\mathbf{x}}\to\overline{0}} \sin(3x^2 + 5y + z)$$

(iii)

$$\lim_{\overline{\mathbf{x}}\to\overline{0}} \frac{\sin(3x^2 + 5y + z)}{3x^2 + 5y + z}$$

3. (15 points total) Consider the matrix $A = \begin{bmatrix} 0 & a & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{bmatrix}$.

(i) (5 points) Compute A^2 and A^3.

(ii) (10 points) Compute $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$

4. (16 points total; 8 points each part) Prove or disprove:

(a) An arbitrary union of (possibly infinitely many) closed sets is closed.

(b) If $\lim_{k\to\infty} a_k = L$ in $\mathbb{R}$, and $a_k \le M$ for all k, then $L \le M$.