Math 3592H Honors Math I

Instructions: **Quiz 2, Thursday Oct. 20, 2016**

15 minutes, closed book and notes, no electronic devices.
There are three problems, worth a total of 20 points.

1. (8 points total; 4 points each part)

Consider the function $\mathbf{f} : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $f \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ z^2 \end{pmatrix}$.

(a) Write down the matrix representing $D\mathbf{f}(\mathbf{a})$ at $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(b) Compute the directional derivative of $\mathbf{f}$ at $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ in the direction of the unit vector $\mathbf{v} = \begin{pmatrix} 3/5 \\ 4/5 \\ 0 \end{pmatrix}$.

2. (8 points total; 4 points each part)
Assume $\mathbf{f} : \mathcal{U} \to \mathbb{R}^{100}$ is defined on an open subset $\mathcal{U}$ of $\mathbb{R}^3$, and differentiable at some point $\mathbf{a} \in \mathcal{U}$.

(a) What are the dimensions of the Jacobian matrix $J\mathbf{f}(\mathbf{a})$?

(b) On what subset of points $\mathbf{x} \in \mathbb{R}^3$ is the derivative $D\mathbf{f}(\mathbf{a})(\mathbf{x})$ defined?

3. (4 points total) Prove or disprove:
The function $\mathbf{f} : \mathrm{Mat}(n,n) \to \mathrm{Mat}(n,n)$ sending $X \in \mathrm{Mat}(n,n)$ to

$$f(X) = I + 6X - 5X^2$$

is differentiable on all of $\mathrm{Mat}(n,n)$, and at $X = A$, its derivative $D\mathbf{f}(A) : \mathrm{Mat}(n,n) \to \mathrm{Mat}(n,n)$ is

$$D\mathbf{f}(A)(H) = 6H - 5(AH + HA).$$