(1)

Math 3592H Fall 2016

9/7/2016 Syllabus items :

- Office hours? 9:05 M,W?? +4pm M,W
- Text: Hubbard & Hubbard – read it before/after lectures! (They have more time/space)
- HW: Collaborate, but understand (and disclose)
- Quizzes: Hopefully straightforward, easy 20% of...

HW	25%
Quizzes	20%
Midterm1	15%
Midterm2	15%
Final	25%
	100%

- Ask questions, come to office hours!

Chapter 0 – Don't worry too much about it; we'll return to it at points. Perhaps skim §§0.0, 0.1, 0.2, 0.3, 0.7 on $\mathbb{C}$

Chapter 1 Vectors matrices & derivatives

§1.1 Points & vectors in $\mathbb{R}^n$

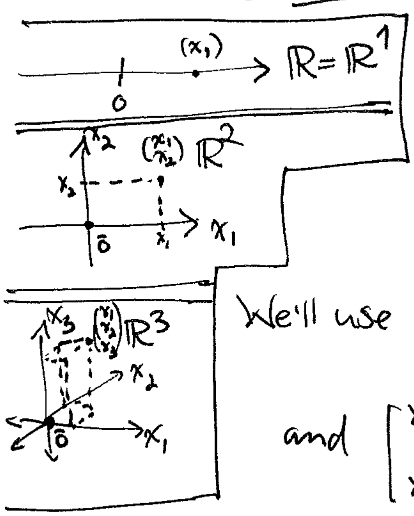
DEFINITION: $\mathbb{R}^n := \left\{ \text{all points } \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} : x_i \in \mathbb{R} \right\}$

"the set of..."

:= means I'm defining it here

: "such that"

$= \left\{ \text{all vectors } \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} : x_i \in \mathbb{R} \right\}$



We'll use $\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ more when think of them as starting points (or tails of vectors)

and $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ more when thinking of them as difference vectors (or directions)

I may mix them up! (I tend to default to $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$)

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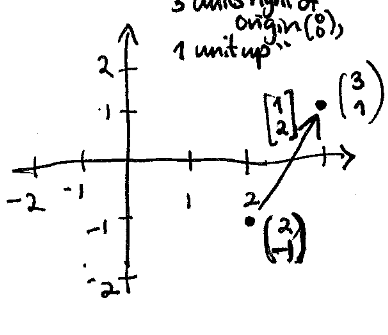
e.g. in $\mathbb{R}^2$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

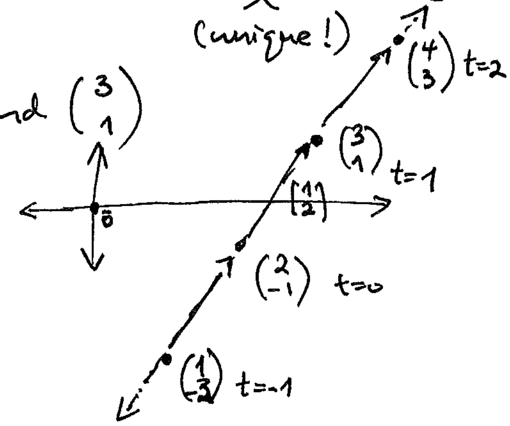
"3 units right of origin $\binom{0}{0}$, 1 unit up"

"over 1, up 2"



e.g. $\left\{ \begin{pmatrix}2\\-1\end{pmatrix} + t\begin{bmatrix}1\\2\end{bmatrix} : t\in\mathbb{R}\right\}$ parametrizes the (unique!) line in $\mathbb{R}^2$

that passes through $\begin{pmatrix}2\\-1\end{pmatrix}$ and $\begin{pmatrix}3\\1\end{pmatrix}$

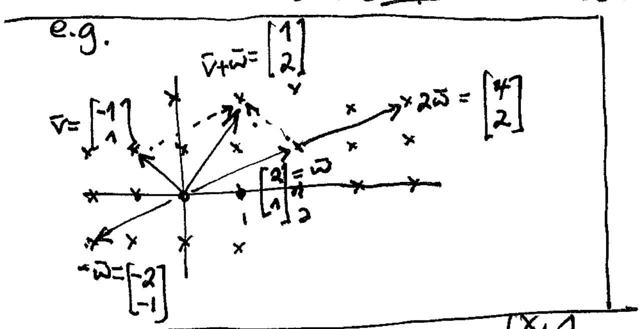


It makes sense to add vectors coordinatewise: $\bar v = \begin{pmatrix}v_1\\ \vdots\\ v_n\end{pmatrix}$, $\bar w = \begin{bmatrix}w_1\\ \vdots\\ w_n\end{bmatrix}$

have $\bar v + \bar w = \begin{bmatrix}v_1+w_1\\ \vdots\\ v_n+w_n\end{bmatrix}$

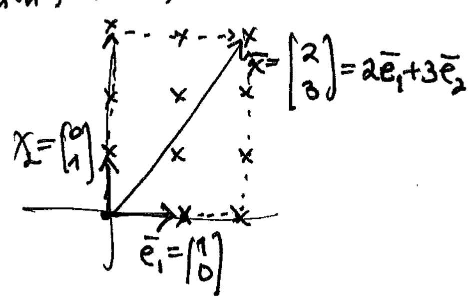
and scale them coordinatewise $c\bar v = \begin{bmatrix}cv_1\\ \vdots\\ cv_n\end{bmatrix}$ for $c\in\mathbb{R}$

c is called a scalar



And every vector $\bar x = \begin{bmatrix}x_1\\ \vdots\\ x_n\end{bmatrix}$ is uniquely expressible as a linear combination of the
$(\bar x = c_1\bar v_1 + \dots + c_m\bar v_n,\ c_i\in\mathbb{R})$

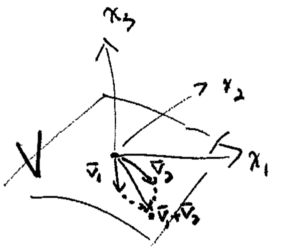
standard basis vectors $\bar e_i = \begin{bmatrix}0\\ \vdots\\ 0\\ 1\\ 0\\ \vdots\\ 0\end{bmatrix} \leftarrow i^{th}$ coordinate



namely $\bar x = \begin{bmatrix}x_1\\ \vdots\\ x_n\end{bmatrix} = x_1\bar e_1 + x_2\bar e_2 + \dots + x_n\bar e_n$

$= x_1\begin{bmatrix}1\\0\\ \vdots\\ 0\end{bmatrix} + x_2\begin{bmatrix}0\\1\\0\\ \vdots\\ 0\end{bmatrix} + \dots + x_n\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix}$

(3) Certain subsets of $\mathbb{R}^n$ will be easiest to deal with, and arise frequently...



DEF'N: A (vector or linear) subspace $V \subset \mathbb{R}^n$ is a subset of vectors $\bar{v}$ which is

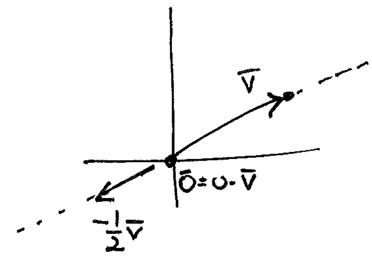
(a) nonempty $V \neq \emptyset$

(b) closed under $+$, that is, $\bar{v}_1, \bar{v}_2 \in V \underset{\text{implies}}{\Longrightarrow} \bar{v}_1+\bar{v}_2 \in V$

(c) closed under scalar multiplication, that is,

$$\bar{v} \in V,\ c \in \mathbb{R} \Rightarrow c\bar{v} \in V$$

(In particular, (a),(c) $\Longrightarrow$ $\bar{0} = 0 \cdot \bar{v}$ $\forall \bar{v} \in V$
$\in V$
so V contains the origin)



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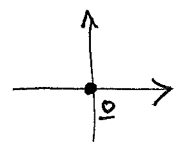
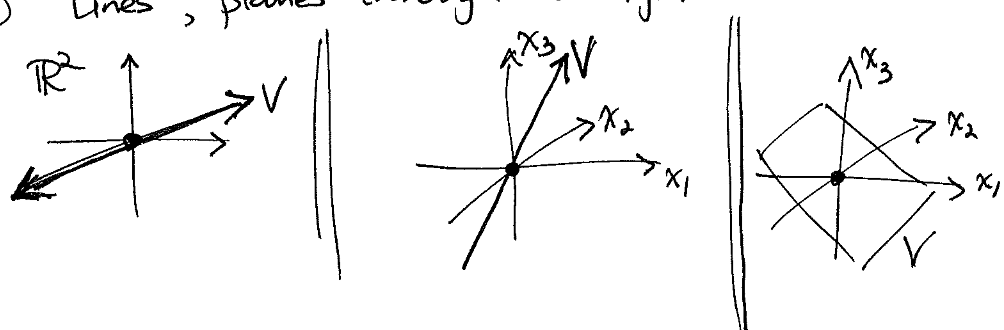
EXAMPLES:

① $\{\bar{0}\}$, $\mathbb{R}^n$ are trivial subspaces inside $\mathbb{R}^n$



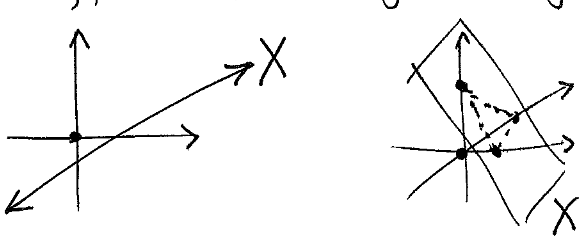
② Lines, planes through the origin



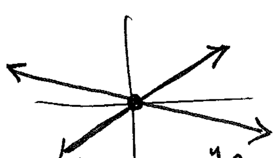
③ See EXERCISE 1.1.9

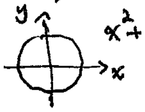
NON-EXAMPLES:

④ Lines, planes not through the origin $X \subset \mathbb{R}^n$ (Why not?)



⑤ Union of two lines through origin (Why not?)



⑥ Unit circle  $x^2+y^2=1$ (Why not?) (Lots of non-examples!)