Math 3593H Honors Math II
Quiz 4, Thursday April 20, 2017

Instructions:
20 minutes, closed book, no electronic devices,
but an 8.5×11 page of notes is OK.
There are three problems, worth a total of 20 points.

1. Let $\bar{F}$ be the vector field on $\mathbb{R}^3$ defined by $\bar{F}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} x^2 \\ y^3 \\ z^4 \end{bmatrix}$

(a) (3 points) Write down the associated work 1-form $W_{\bar{F}}$ in $A^1(\mathbb{R}^3)$.

(b) (3 points) Write down the associated flux 2-form $\Phi_{\bar{F}}$ in $A^2(\mathbb{R}^3)$.

2. (7 points) Parametrize $C \subset \mathbb{R}^3$ via the map from $U = (1,2) \subset \mathbb{R}$

$$\begin{array}{ccc} U & \xrightarrow{\bar{\gamma}} & C \\ t & \longmapsto & \begin{pmatrix} t \\ t^2 \\ t^3 \end{pmatrix}, \end{array}$$

and orient C via $\bar{\gamma}$, that is, $C = [\bar{\gamma}(U)]$. Calculate $\int_C x^2 z^2 \; dy$.

3. (7 points) Prove or disprove:
The parametrization of the strict upper-halfplane

$$M = \{ \left(\begin{smallmatrix} x \\ y \end{smallmatrix} \right) : y > 0 \} \ \subset \mathbb{R}^2$$

via the polar coordinate map from

$$U := \{ \left(\begin{smallmatrix} r \\ \theta \end{smallmatrix} \right) : r > 0 \text{ and } 0 < \theta < \pi \} \ \subset \mathbb{R}^2$$

given by

$$\begin{array}{ccc} U & \xrightarrow{\bar{\gamma}} & M \\ \begin{pmatrix} r \\ \theta \end{pmatrix} & \longmapsto & \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} \end{array}$$

is **order-preserving**, when U, M are both given their standard orientations as open subsets of $\mathbb{R}^2$.