Math 4707 Intro to combinatorics and graph theory
Spring 2008, Vic Reiner
Midterm exam 1- Due Wednesday March 5, in class

Instructions: This is an open book, open library, open notes, open web, take-home exam, but you are *not* allowed to collaborate. The instructor is the only human source you are allowed to consult.

1. (20 points total; 10 points each part) DNA strings can be thought of as ordered sequences of 4 possible nucleotides:

- A (=adenine)
- C (=cytosine)
- G (=guanine)
- T (=thymine).

(a) How many such sequences are there of length 1000?
(b) How many such sequence are there of length 1000 containing an equal number of each of the 4 possible nucleotides A, C, G, T?

2. (20 points) Exercise 3.8.5 on p. 62 of our text by L, P and V. (**Warning:** A brute force solution calculating all the values $k\binom{99}{k}$ for $k = 0, 1, 2, \ldots, 99$ will be given no credit, but is fine to check your answer!)

3. (20 points total) You are supposed to choose 100 appetizers for the office party, from 4 distinct varieties offered by the caterer.

(a) (5 points) How many ways are there for you to choose them?

(b) (10 points) Suppose that exactly one of the 4 varieties is vegetarian, and you have been told that must choose *at least* 20 out of the 100 appetizers from this variety. Now how many ways are there for you to choose the appetizers?

(c) (5 points) Exactly how many bits (binary digits) are required to write the number which is the answer from part (a)?

4. (20 points) Find a simple explicit formula for the sequence a_n defined by the recurrence

$$a_n = a_{n-1} + 2a_{n-2} - 2$$

for $n \geq 2$, with initial conditions

$$a_0 = 2, \;\; a_1 = 3.$$

Your answer should write a_n as a function of n involving no summation or product ($\sum$ or $\prod$) symbols, only operations like $+, -, \times, /$ and exponentiation.
(Hint: before doing anything else, you might just tabulate the first few values, such as $a_0, a_1, \ldots, a_7$, and try to guess the formula for a_n.)

5. (20 points total)

(a) (5 points) How many paths are there in the plane $\mathbb{R}^2$ going from $(0, 0)$ to $(40000, 44000)$ taking unit length steps in either the north or east direction at each step?

(b) (15 points) How many such paths are there which avoid passing through any of these three 3 "bad" points

$$(10, 11), \;\; (200, 220), \;\; (3000, 3300)?$$