Math 5285 Honors abstract algebra
Fall 2007, Vic Reiner
Midterm exam 2- Due Wednesday November 14, in class

Instructions: This is an open book, open library, open notes, open web, take-home exam, but you are *not* allowed to collaborate. The instructor is the only human source you are allowed to consult.

1. (15 points total) Artin's Chapter 2 Miscellaneous Problems # 3 on p. 77.

2. (15 points total) Artin's Chapter 2 Miscellaneous Problems # 4 on p. 77.

3. (20 points total; 10 points each part)
(a) Prove that a group of order 45 must be abelian.
(b) Exhibit an explicit example of a group of order 45 which is *not* cyclic, with proof that it is not cyclic.

4. (20 points total; 10 points each part) Artin's Exercise 3.2.15 on p. 105.

5. (15 points total) Believing that 547 is prime, use Euclid's algorithm to find the multiplicative inverse $\overline{10}^{-1}$ of $\overline{10}$ in the finite field $\mathbb{F}_{547} (:= \mathbb{Z}/547\mathbb{Z})$.
(Using a brute force exhaustive search will earn no credit on this problem, but is fine as a check.)

6. (15 points total) Find a basis over the field $\mathbb{F}_7$ for the subspace

$$\ker A := \{X \in \mathbb{F}_7^3 : AX = \mathbf{0}\}$$

of $\mathbb{F}_7^3$, where $A \in \mathbb{F}_7^{2\times 3}$ is the matrix

$$A = \begin{bmatrix} \overline{0} & \overline{1} & \overline{2} \\ \overline{3} & \overline{4} & \overline{5} \end{bmatrix}.$$

Show your work, that is, don't just write down an answer.